Simulating Dense QCD Matter with Ultracold Atomic Boson-Fermion Mixtures

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We delineate, as an analog of two-flavor dense quark matter, the phase structure of a many-body mixture of atomic bosons and fermions in two internal states with a tunable boson-fermion attraction. The bosons b correspond to diquarks, and the fermions f to unpaired quarks. For weak b-f attraction, the system is a mixture of a Bose-Einstein condensate and degenerate fermions, while for strong attraction composite b-f fermions N, analogs of the nucleon, are formed, which are superfluid due to the N–N attraction in the spin-singlet channel. We determine the symmetry breaking patterns at finite temperature as a function of the b-f coupling strength, and relate the phase diagram to that of dense QCD.

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Ultracold atomic systems and high density QCD matter, although differing by some 20 orders of magnitude in energy scales, share certain analogous physical aspects, e.g., crossovers from Bose-Einstein condensation (BEC) to BCS [1,2]. Motivated by phenomenological studies of QCD that indicate a strong spin-singlet diquark correlation inside the nucleon [3], we focus here on modeling the transition from color superconducting two-flavor quark matter (2SC) at high density to superfluid hadronic matter at low density in terms of a boson-fermion system, in which small size diquarks are the bosons, unpaired quarks are the fermions, and the extended nucleons are regarded as composite boson-fermion particles. Recent advances in atomic physics have made it possible indeed to realize such systems in the laboratory. In particular, tuning the atomic interaction via a Feshbach resonance allows formation of heteronuclear molecules, as recently observed in a mixture of ^{87}Rb and ^{40}K atomic vapors in a 3D optical lattice [4], and in an optical dipole trap [5].

The analogy is incomplete however. The gluonic attraction in QCD is a function of the baryon density, and thus tuning the coupling strength at fixed density is not possible in dense matter; furthermore, chiral symmetry breaking plays an important role in the QCD transition [6]. With these reservations in mind, we suggest that fuller understanding, both theoretical and experimental, of the boson-fermion mixture, as well as a mixture of three species of atomic fermions, as discussed in [7], can reveal properties of high density QCD not readily observable in laboratory experiments.

We first delineate possible phase structures that can be realized in a mixture of single-component bosons (b) and two-component fermions (f) at finite temperature as a function of the boson-fermion (b-f) interaction. In weakly coupled b-f mixtures, an induced interaction between bosons arising from density fluctuations of fermions modifies the critical temperature of the Bose condensate. In addition, an induced interaction between fermions due to density fluctuations of bosons may lead to superfluidity of the fermions [8]. On the other hand, in strongly coupled mixtures, boson-fermion molecules are formed [9], which may become superfluid [10]. By analyzing the realization of internal symmetries in each phase, we classify the types of phase boundaries. We then discuss the detailed connection with the hadronization phase transition in QCD matter at low temperatures.

We consider a (nonrelativistic) boson-fermion mixture with Hamiltonian density,

$$\mathcal{H} = \frac{1}{2m_b}|\nabla\phi(x)|^2 - \mu_b|\phi(x)|^2 + \frac{1}{2}\bar{g}_{bb}|\phi(x)|^4 + \sum_\sigma\left(\frac{1}{2m_f}|\nabla\psi_\sigma(x)|^2 - \mu_f|\psi_\sigma(x)|^2\right) + \bar{g}_{ff}|\psi_\uparrow(x)|^2|\psi_\downarrow(x)|^2 + \sum_\sigma \bar{g}_{bf}|\phi(x)|^2|\psi_\sigma(x)|^2, \quad (1)$$

where ϕ is the boson and ψ the fermion field. We label the two internal states of the fermions by spin indices $\sigma = \{\uparrow, \downarrow\}$. We focus for simplicity on an equally populated mixture of n bosons and n fermions with $n_\uparrow = n_\downarrow = n/2$.

The bare boson-fermion coupling $\bar{g}_{bf}$ is related to the renormalized coupling g_{bf} and to the s wave scattering length a_{bf} by

$$\frac{m_R}{2\pi a_{bf}} = \frac{1}{g_{bf}} = \frac{1}{\bar{g}_{bf}} + \int_{|\mathbf{k}|\le\Lambda}\frac{d^3k}{(2\pi)^3}\frac{1}{\varepsilon_b(k)+\varepsilon_f(k)}, \quad (2)$$

where $\varepsilon_i(k) = k^2/2m_i$ $(i = b, f)$ is the single-particle kinetic energy, m_R is the boson-fermion reduced mass, and Λ is a high momentum cutoff. We define $r_0 \equiv (2\Lambda/\pi)^{-1}$ as a typical atomic scale. We assume an attractive bare b-f interaction ($\bar{g}_{bf} < 0$), tunable in magnitude, with Λ fixed so that the scattering length a_{bf} can change sign: namely, $a_{bf} \to \bar{g}_{bf}m_R/2\pi$ for small negative $\bar{g}_{bf}$, while $a_{bf} \to r_0$ for large negative $\bar{g}_{bf}$. We keep the bare boson-boson and fermion-fermion interactions fixed and repulsive ($\bar{g}_{bb} > 0$, $\bar{g}_{ff} > 0$); thus, the renormalized

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couplings $g_{bb} = 4\pi a_{bb}/m_b$ and $g_{ff} = 4\pi a_{ff}/m_f$ are always positive. These repulsions are required for the stability of the bosons at weak bare b-f coupling and also for the dominance of two-body molecules (bf) over three-body molecules (bbf) and (bff) at strong bare b-f attraction [11]. Boson-fermion mixtures with an attractive b-f interaction may be realized in three-component cold atomic experiments using, e.g., the hyperfine state $|f = 1, m_f = 1\rangle$ of ^{87}Rb, mixed with the hyperfine states $|9/2, -5/2\rangle$ and $|9/2, -9/2\rangle$ of ^{40}K [12].

We first lay out the phase structure at weak bare b-f coupling, where the dimensionless parameter $\eta \equiv -1/n^{1/3}a_{bf}$ is $\gg 1$. In the absence of b-f attraction ($\eta = +\infty$) with weak b-b repulsion ($n^{1/3}a_{bb} \ll 1$), boson condensation (b-BEC) occurs below the critical temperature, $T_c^0(b\text{-BEC}) \simeq (1 + 1.32n^{1/3}a_{bb} + \cdots)T_0$, with the ideal BEC transition temperature $T_0 \equiv 2\pi[n/\zeta(3/2)]^{2/3}/m_b$ [13]. A weak b-f interaction induces an attraction between bosons via fermion density fluctuations (Fig. 1) given by $U_{bfb}(p, \omega; T) = -\bar{g}_{bf}^2\Pi(p, \omega; T)$, where $\Pi(p, \omega; T)$ is the one-loop fermionic polarization at temperature T. In the static limit, relevant for elastic scattering of bosons,

$$\Pi(p, 0, T) = -2\int \frac{d^3q}{(2\pi)^3}\frac{n_f(|\mathbf{p}+\mathbf{q}|) - n_f(q)}{\varepsilon_f(|\mathbf{p}+\mathbf{q}|) - \varepsilon_f(q)}, \quad (3)$$

with $n_f(p) = 1/[e^{[\varepsilon_f(p)-\mu_f]/T} + 1]$ the Fermi occupation. Then, the characteristic length of the fermion-induced b-b interaction near $T_c(b\text{-BEC})$ is $a_{bfb} \equiv (m_b/4\pi)U_{bfb}(\sqrt{2m_bT_0}, 0; T_0) < 0$, where we set $T = T_0$ and take a typical momentum transfer p equal to the thermal momentum $\sqrt{2m_bT_0}$. For $m_b/m_f = 2$, corresponding to a ^{87}Rb-^{40}K mixture, $k_Fa_{bfb} \simeq -11\eta^{-2}$ with $k_F = (3\pi^2n)^{1/3}$. Since the induced attraction tames the bare b-b repulsion, the transition temperature for b-BEC decreases to

$$T_c(b\text{-BEC}) = [1 + 1.32n^{1/3}(a_{bb} + a_{bfb}) + \cdots]T_0. \quad (4)$$

The replacement $a_{bb} \to a_{bb} + a_{bfb}$ is exact to second order in the b-f interaction; more generally, such a replacement is exact in a large $\mathcal{N}$ extension ($a_{bf}, a_{bb} \sim 1/\mathcal{N}$) where only bubble summations are important [14,15]. Calculation of higher order corrections to $T_c(b\text{-BEC})$ is beyond our present scope. The Bose gas becomes mechanically unstable when the net b-b attraction becomes large enough to overcome the thermal pressure [16]: $3nT/2 + (g_{bb} + U_{bfb})n^2/2 < 0$. For $m_b/m_f = 2$ and $n^{1/3}a_{bb} = 10^{-2}$, the critical values for the instability are $\eta_c^{b\text{-BEC}} \simeq 21$ at $T = 0$ and $\eta_c^{b\text{-BEC}} \simeq 2.2$ near $T_c(b\text{-BEC})$. At high T, thermal pressure stabilizes the system.

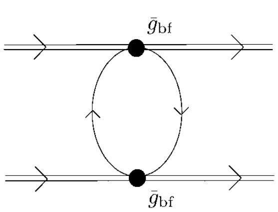

FIG. 1. Fermion-induced interaction between bosons to second order in the boson-fermion interaction. The single lines denote free fermions and the double lines free bosons.

A weak b-f interaction also leads to a boson-induced attraction between fermions similar to the phonon-induced attraction between electrons in metals [8]. The characteristic length of the induced f-f interaction in fully condensed bosons, a_{fbf}, averaged over the Fermi surface, is $a_{fbf} \equiv (\pi/3)^{1/3}(m_bm_f/m_R^2)\ln(1 + x^2)(k_F\eta^2)^{-1}$, with $x = \sqrt{3\pi/4k_Fa_{bb}}$. For $m_b/m_f = 2$, $k_Fa_{fbf} \simeq -4.6\ln(1 + x^2)\eta^{-2}$. A net attraction, $a_{ff} + a_{fbf} < 0$, causes fermionic BCS superfluidity (f-BCS) below a critical temperature [17]

$$T_c(f\text{-BCS}) = \frac{e^\gamma}{\pi}\left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F(a_{ff} + a_{fbf})}\right), \quad (5)$$

where $\varepsilon_f = k_F^2/2m_f$ and γ is Euler's constant. For $m_b/m_f = 2$, f-BCS emerges when $\eta < \eta_c^{f-\text{BCS}} = 2.1[\ln(1 + x^2)/k_Fa_{ff}]^{1/2}$. As the b-f attraction increases, $T_c(f\text{-BCS})$ increases due to the increase of the net f-f attraction.

Let us turn to the regime of strong bare b-f coupling where η is large and negative. Here, bound molecules or *composite fermions*, $N = (bf)$, are formed with a kinetic mass $m_N = m_b + m_f$. We estimate the s wave scattering length of two N's of opposite spins from the diagram sketched in Fig. 2. Integration over the three intermediate states for this process yields the corresponding T-matrix element with zero initial and final momenta,

$$T_{NN}(\mathbf{0}, \mathbf{0}) = 4\int_{|\mathbf{q}|\le\Lambda}\frac{d^3q}{(2\pi)^3}\frac{|\mathcal{M}|^4}{2[\epsilon_N - \varepsilon_b(q) - \varepsilon_f(q)]^3}, \quad (6)$$

where ϵ_N is the binding energy of the composite fermion. The prefactor 4 arises from four possible time orderings of the dissociation and recombination, (12;34) as in Fig. 2, (21;34), (12;43), and (21;43). Using the Schrödinger equation, $[-\nabla^2/2m_R + V(\mathbf{r})]\psi_N(\mathbf{r}) = \epsilon_N\psi_N(\mathbf{r})$, where $V(\mathbf{r})$ is

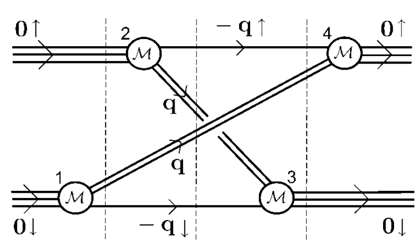

FIG. 2. T-matrix diagram for scattering of composite fermions of opposite spins, arising from exchange of the constituent bosons. The single, double, and triple lines denote free fermions, bosons, and composite fermions, respectively, and the three dashed lines indicate the intermediate states.

the bare b-f potential, $\psi_N(\mathbf{r})$ the internal wave function of the composite fermion, and $\psi_N(\mathbf{q})$ its Fourier transform, we write the matrix element $\mathcal{M}$ for dissociation of N into b and f as $\mathcal{M} = (\epsilon_N - q^2/2m_R)\psi_N(\mathbf{q})$; thus,

$$T_{NN}(\mathbf{0},\mathbf{0}) = 2\int_{|\mathbf{q}|\leq\Lambda} \frac{d^3q}{(2\pi)^3}|\psi_N(q)|^4(\epsilon_N - q^2/2m_R). \quad (7)$$

For a bound state in a zero-range potential, $\psi_N(\mathbf{r}) = e^{-r/a_{bf}}/r\sqrt{2\pi a_{bf}}$ and $\epsilon_N = -1/2m_R a_{bf}^2$, and thus $\mathcal{M} = -\sqrt{2\pi/m_R^2 a_{bf}}$ [18]. Equation (7) yields the s wave scattering length for composite fermions of opposite spins for negative and large η

$$a_{NN} = \frac{m_N}{4\pi}T_{NN}(\mathbf{0},\mathbf{0}) = -\frac{m_N}{2m_R}a_{bf}\Gamma. \quad (8)$$

Here, $0.76 < \Gamma(a_{bf}/r_0) < 1$ in the interval $1 < a_{bf}/r_0 < \infty$. This result is the same in magnitude but opposite in sign from the scattering length between difermion molecules in the same approximation [10]. For $m_b/m_f = 2$, we obtain $k_F a_{NN} \simeq -7.0\Gamma/|\eta|$. The above estimate of the N-N scattering length is the leading order term in the large $\mathcal{N}$ extension of the present boson-fermion model [15]. For finite $\mathcal{N}$, it is corrected by (i) other recombination diagrams [19], (ii) short range b-b and f-f repulsions, and (iii) possible contributions of trimers such as (bff) and (bbf) [11]. We leave the calculation of such corrections for the future and treat Eq. (8) as a first estimate. Equation (8) implies that the low energy effective interaction between composite fermions in the spin-singlet channel is weakly attractive; the stronger the bare b-f coupling, the weaker the N-N interaction. Such an effective attraction causes composite fermions to become BCS-paired (N-BCS) below a transition temperature,

$$T_c(N\text{-BCS}) = \frac{e^\gamma}{\pi}\left(\frac{2}{e}\right)^{7/3}\varepsilon_N e^{\pi/2k_F a_{NN}}. \quad (9)$$

where $\varepsilon_N = k_F^2/2m_N$ is the Fermi energy of the N.

The above analyses suggest possible phase structures of boson-fermion mixtures in the T-η plane, shown in Fig. 3. The system at low temperature for $\eta = \infty$ is a mixture of the b-BEC and degenerate unpaired fermions. As η decreases from the right, the size of the b-BEC region shrinks, Eq. (4). The f-BCS state emerges at $T = T_c(f\text{-BCS})$, Eq. (5). Both effects are caused by the induced attractions. On the other hand, for strong bare b-f coupling (η large and negative), the system at low temperature is in the N-BCS state. As η increases from the left, the size of N-BCS region increases according to Eq. (9).

At intermediate bare b-f coupling ($\eta \sim 0$) where a transition from the b-BEC (and at low T with coexisting f-BCS) phase to N-BCS takes place, the phase diagram has complex structures depending on the relative magnitudes of $\bar{g}_{bb}$, $\bar{g}_{ff}$, and $\bar{g}_{bf}$. When the intrinsic b-b repulsion is weak, uniform b-BEC at low temperature becomes unstable at $\eta = \eta_c^{b\text{-BEC}} > 0$ due to the net b-b attraction, while at high temperature, the system becomes a stable normal gas of bosons and fermions because of thermal pressure [16]. This situation is indicated by the "collapsed" region at $\eta \sim 0$ in Fig. 3(a). For strong bare b-b repulsion, the weak coupling formula (4) is no longer valid, and the b-BEC phase may possibly survive into the region $\eta < 0$ until the bosons become bound in composite fermions as shown in Fig. 3(b). To determine the physical structure of the "collapsed" region requires evaluating the free energy to higher order.

To identify the precise phase boundaries in the region $\eta \sim 0$, we need to solve the system beyond the weak or strong coupling regimes studied here. Rather, we classify the phases of the boson-fermion mixture by their realizations of the internal symmetry of the system. We focus only on the continuous symmetries here. The Hamiltonian density, Eq. (1), has $U(1)_b \times U(1)_{f_\uparrow} \times U(1)_{f_\downarrow}$ symmetry corresponding to independent phase rotations of ϕ, $\psi_\uparrow$, and $\psi_\downarrow$. On the other hand, b-BEC, f-BCS, and N-BCS break $U(1)_b$, $U(1)_{f_\uparrow+f_\downarrow}$, and $U(1)_{b+(f_\uparrow+f_\downarrow)}$ symmetries, respectively. Here, $U(1)_{A\pm B}$ denotes an in-phase rotation of A and B for $(+)$, and an opposite-phase rotation for $(-)$. Therefore, in the coexisting b-BEC and f-BCS phase, the symmetry breaking pattern is $U(1)_b \times U(1)_{f_\uparrow} \times U(1)_{f_\downarrow} \rightarrow U(1)_{f_\uparrow - f_\downarrow}$. On the other hand, the N-BCS phase has the symmetry breaking pattern, $U(1)_b \times U(1)_{f_\uparrow} \times U(1)_{f_\downarrow} \rightarrow U(1)_{b-(f_\uparrow+f_\downarrow)} \times U(1)_{f_\uparrow - f_\downarrow}$. The difference of unbroken symmetries between these two phases implies the existence of a well-defined phase boundary, as indicated in

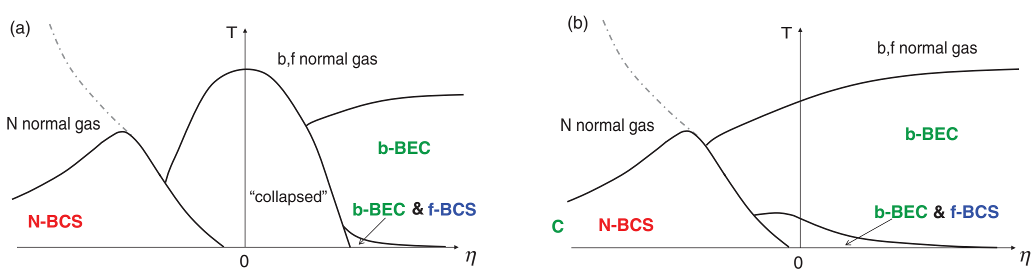

FIG. 3 (color online). Schematic phase structures of boson-fermion mixtures with a tunable boson-fermion interaction. (a) Weak $\bar{g}_{bb}$; in the regime labeled "collapsed" the effective boson-boson interaction is negative. (b) Strong $\bar{g}_{bb}$; a possibly "collapsed" region is not shown.

Fig. 3(b), in contrast to the continuous crossover from BEC to BCS in a two-component Fermi system [20].

Interesting problems remaining for further research on the phase structure of the mixtures include understanding the N-N scattering length beyond leading order in large $\mathcal{N}$, the quantitative description of the phase structure in the intermediate coupling regime, and extensions to spin-dependent b-f interactions and unequal population of bosons and fermions.

The phase structures we find for boson-fermion mixtures of cold atoms display features of those in two-flavor QCD with equal numbers of up (u) and down (d) quarks with three colors (R, G, B). The ground state of this system at high density is a two-flavor color superconductivity (2SC) with s wave spin-singlet pairing, e.g., between uR and dG, in the color antisymmetric and flavor antisymmetric channel. The order parameter for color-symmetry breaking, $SU(3)_c \to SU(2)_c$, is the diquark condensate $\langle b_3 \rangle$ with the diquark operator $b_\gamma = \epsilon_{ij}\epsilon_{\alpha\beta\gamma} q^i_\alpha C\gamma_5 q^j_\beta$; here i, j are flavor and α, β color indices, and C denotes charge conjugation. The gap is of order a few tens of MeV; the remaining quarks, uB and dB, are unpaired and form degenerate Fermi seas [21]. On the other hand, the ground state of two-flavor QCD with equal numbers of u and d quarks at low density is nuclear matter with equal numbers of neutrons and protons, a superfluid state with a pairing gap of a few MeV (see, e.g., [22]); the order parameter for the spontaneous breaking of baryon-number symmetry $U(1)_B$ is the six-quark condensate $\langle N^i_\uparrow N^j_\downarrow \rangle = \langle (b_\alpha q^i_{\alpha,\uparrow}) \times (b_\beta q^j_{\beta,\downarrow}) \rangle$.

If we model the nucleon, of radius $r_N \sim 0.86$ fm, as a bound molecule of a diquark (of radius $r_D \sim 0.5$ fm) and an unpaired quark, we can make the following correspondence between cold atoms and QCD: $b \leftrightarrow$ 2SC-diquarks and $f \leftrightarrow$ unpaired-quark, $N \leftrightarrow$ nucleon, b-f attraction $\leftrightarrow$ gluonic attraction, b-BEC $\leftrightarrow$ 2SC, and N-BCS $\leftrightarrow$ nucleon superfluidity [23]. In particular, the boundary between the b-BEC and N-BCS phases shown in Fig. 3(b) is strongly suggestive of that deduced between the color superconducting quark phase and the superfluid hadronic phase in Refs. [2,6].

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