"Light Sail" Acceleration Reexamined

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The dynamics of the acceleration of ultrathin foil targets by the radiation pressure of superintense, circularly polarized laser pulses is investigated by analytical modeling and particle-in-cell simulations. By addressing self-induced transparency and charge separation effects, it is shown that for "optimal" values of the foil thickness only a thin layer at the rear side is accelerated by radiation pressure. The simple "light sail" model gives a good estimate of the energy per nucleon, but overestimates the conversion efficiency of laser energy into monoenergetic ions.

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Radiation pressure acceleration (RPA) of ultrathin solid targets by superintense laser pulses has been proposed as a promising way to accelerate large numbers of ions up to "relativistic" energies, i.e., in the GeV/nucleon range [1–9]. The simplest model of this acceleration regime is that of a "perfect" (i.e., totally reflecting) plane mirror boosted by a light wave at perpendicular incidence [10], which is also known as the "light sail" (LS) model. The LS model predicts the efficiency η, defined as the ratio between the mechanical energy of the mirror over the electromagnetic energy of the light wave pulse, to be given by

$$\eta = 2\beta/(1+\beta), \qquad \beta = V/c, \tag{1}$$

where V is the mirror velocity; hence, RPA becomes more and more efficient ($\eta \to 1$) as $\beta \to 1$. Heuristically, Eq. (1) can be explained by the conservation of the number of "photons" N of the light wave reflected by the moving mirror in a small time interval: each photon has energy $\hbar\omega$, thus the total energy of the incident and reflected pulses are given by $N\hbar\omega$ and $N\hbar\omega_r$, where $\omega_r = \omega(1-\beta)/(1+\beta)$ due to the Doppler effect, and the energy transferred to the mirror is given by their difference $[2\beta/(1+\beta)]N\hbar\omega$.

The predictions of the LS model are very appealing for applications, but one may wonder to what extent this picture is appropriate to describe the acceleration of a solid target by a superintense laser pulse. In this Letter, we reexamine the LS model with the help of simple modeling and particle-in-cell (PIC) simulations. We address issues outside the model itself, such as the effects of nonlinear reflectivity and charge depletion, and on this basis we explain a few features observed in simulations. Our main result is that the LS model is accurate in predicting the ion energy but overestimates the corresponding conversion efficiency, i.e., the fraction of the laser pulse energy transferred *into quasimonoenergetic ions*, due to the fact that only a layer of the foil *at its rear side* is accelerated by RPA.

Our analysis is confined to a one-dimensional (1D) approach for the sake of simplicity and because multidimensional simulations showed that a "quasi-1D" geometry has to be preserved in the acceleration stage (by using flattop intensity profiles) to avoid early pulse transmission due to the expansion of the foil in the radial direction [11]. Circularly polarized pulses are used to reduce electron heating [12], an approach followed by several groups for efficient acceleration of thin foils [2–6,11]. We do not consider intensities high enough that ions become relativistic within the first laser cycle; this condition may affect the early stage of charge depletion (e.g., by narrowing the temporal scale separation between ions and electrons) and lead to different estimates [1,6].

The LS model is based on the following equation of motion for the foil:

$$\frac{d}{dt}(\beta\gamma) = \frac{2I(t-X/c)}{\rho\ell c^2}R(\omega')\frac{1-\beta}{1+\beta}, \tag{2}$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $dX/dt = V$, I is the light wave intensity, ρ and ℓ are the mass density and thickness of the foil, $R(\omega')$ is the reflectivity in the rest frame of the foil, and $\omega' = \omega\sqrt{(1-\beta)/(1+\beta)}$. For suitable expressions of $R(\omega')$, the final velocity β_f can be obtained from Eq. (2) as a function of the pulse fluence $\mathcal{F} = \int I dt$. For $R = 1$, one obtains

$$\beta_f = \frac{(1+\mathcal{E})^2 - 1}{(1+\mathcal{E})^2 + 1}, \qquad \mathcal{E} = \frac{2\mathcal{F}}{\rho\ell c^2} = 2\pi\frac{Z}{A}\frac{m_e}{m_p}\frac{a_0^2\tau}{\zeta}. \tag{3}$$

In the last equality we wrote the fluence in dimensionless units as $a_0^2\tau$, where $a_0 = \sqrt{I/m_e c^3 n_c}$ is the dimensionless pulse amplitude and τ is the pulse duration in units of the laser period, and introduced the parameter $\zeta = \pi(n_0/n_c)(\ell/\lambda)$ which characterizes the optical properties of a subwavelength plasma foil [13]. In these equations, n_0 is the initial electron density, $n_c = \pi m_e c^2/e^2\lambda^2$ is the cutoff density, and λ is the laser wavelength. In practical units, $n_c = 1.1 \times 10^{21}\ \mathrm{cm}^{-3}\ [\lambda/\mu\mathrm{m}]^{-2}$ and $a_0 = (0.85/\sqrt{2})(I\lambda^2/10^{18}\ \mathrm{W\,cm}^{-2}\ \mu\mathrm{m}^2)^{1/2}$ for a circularly polarized laser pulse. Using Eq. (3) it is found that with a

$1\ \mathrm{ps} = 10^{-12}$ s, $1\ \mathrm{PW} = 10^{12}$ W laser pulse and a 10 nm target of $1\ \mathrm{g\,cm^{-3}}$ density, $\sim$1 GeV per nucleon may be obtained. As the LS model assumes the target to be a *perfect* mirror (i.e., rigid and totally reflecting), it implies that *all* the ions are accelerated to the same velocity and the spectrum is perfectly monoenergetic.

Figure 1 shows a parametric study of the ion spectrum versus ℓ and a_0 from PIC simulations. For all runs, $n_0 = 250n_c$, $Z/A = 1/2$, and the pulse has a flattop envelope with 1 cycle rise and fall times and 8 cycles plateau. For each value of a_0 and for ℓ less than a threshold value ℓ_{opt} we observe a narrow spectral peak, whose energy increases with decreasing ℓ and is in very good agreement with the predictions of the LS model, assuming $R = 1$. A typical lineout of the spectrum is shown in Fig. 2(a). For $\ell > \ell_{\mathrm{opt}}$, the peak disappears and a thermal-like spectrum is observed. This is correlated with an almost complete expulsion of the electrons from the foil in the forward direction at the beginning of the interaction, leading to a Coulomb explosion of the ions.

The results of Fig. 1 show that the LS model is useful for quantitative predictions of the ion energy, but also suggest several questions of interest both for the basic physics of RPA and its applications. How is ℓ_{opt} determined? Does the reflectivity of the foil and relativistic effects on the latter play a role? As the radiation pressure tends to separate electrons from ions, does the foil remain neutral before and/or after the acceleration stage? Moreover, as shown in Fig. 2(b), the "monoenergetic" peak contains just a fraction of the total number of ions, and such fraction depends on ℓ and a_0. This is different from the assumption of the LS model, which assumes all the ions in the foil to move coherently with the foil, and may sound surprising, since the peak energy is in agreement with the LS formula where the *whole* mass of the foil, including low-energy ions out of spectral peak, is used. In the following we provide answers to the questions above by discussing effects not included in the simplest LS model, i.e., beyond the description of the foil as a perfect, rigid mirror.

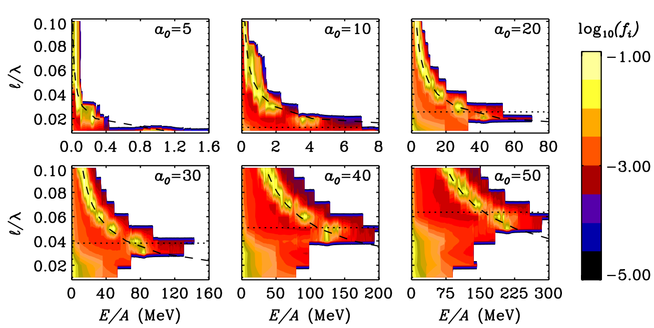

FIG. 1 (color online). Parametric study of the ion energy spectra versus laser amplitude a_0 and foil thickness ℓ. The contours of $\log_{10} f_i(E)$ are shown, with $f_i(E)$ the energy per nucleon distribution normalized to unity. For all runs, $n_0 = 250n_c$, $Z/A = 1/2$, $\tau = 9$. The dashed line shows the prediction of the LS model for the ion energy. The dotted horizontal line marks ℓ_{opt} given by the $\zeta = a_0$ condition.

First we discuss effects related to the reflectivity R of the plasma foil. For very high intensities, electrons oscillate with relativistic momenta in the laser field, leading to a nonlinear dependence of R upon a_0. An explicit expression can be found analytically by using the model of a deltalike "thin foil" [13], i.e., a plasma slab located at $x = 0$ with electron density $n_e(x) = n_0 \ell \delta(x)$. The expression obtained for R in the rest frame of the foil is very well approximated by

$$R \simeq \begin{cases} \zeta^2/(1+\zeta^2) & (a_0 < \sqrt{1+\zeta^2}) \\ \zeta^2/a_0^2 & (a_0 > \sqrt{1+\zeta^2}). \end{cases} \quad (4)$$

A threshold for self-induced transparency of the foil may thus be defined as $a_0 = \sqrt{1+\zeta^2} \simeq \zeta$ when $\zeta \gg 1$, i.e., in most cases of interest. According to Eq. (4), the total radiation pressure P_{rad} on the target

$$P_{\mathrm{rad}} = 2RI/c = 2m_e c^3 n_c a_0^2 R \quad (5)$$

becomes *independent* upon a_0 for $a_0 > \zeta$. Thus, the maximum radiation pressure is obtained for $a_0 \lesssim \zeta$, and in this condition typically $R \simeq 1$ for solid densities. This suggests that the optimal thickness ℓ_{opt} is determined by the condition $a_0 \simeq \zeta$, in good agreement with the simulation results in Fig. 1 and as also found by other studies [8,14].

The nonlinear reflectivity of the thin foil is determined by the transverse motion of electrons (in the foil plane).

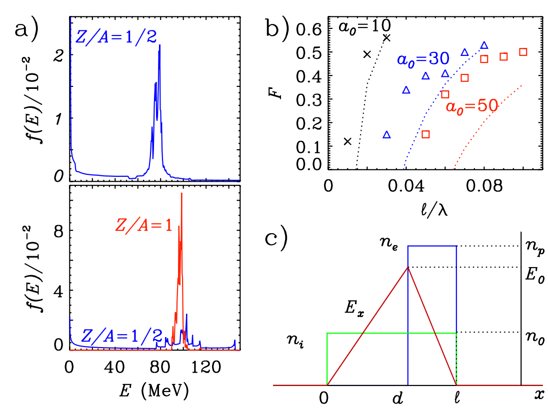

FIG. 2 (color online). (a) Ion energy spectra (in energy per nucleon) from a simulation with $a_0 = 30$ and a $\ell = 0.04\lambda$ thick foil of a single ion species with $Z/A = 1/2$ (top) and one with the same parameters but where ions in a thin surface layer (0.01λ) at the rear side are replaced by protons (bottom). (b) Fraction of ions contained in the spectral peak versus the target thickness ℓ for three values of $a_0 = 10$ (black crosses), 30 (blue triangles), and 50 (red squares). The dashed lines correspond to Eq. (8) for F. All other parameters for both (a) and (b) are the same as in Fig. 1. (c) Approximate profiles of ion (n_i, green line) and electron (n_e, blue line) densities and of the electrostatic field (E_x, red line) in the early stage of the interaction, before ions move.

However, for a thin but "real" target the radiation pressure tends to push electrons also in longitudinal direction, and may remove them from the foil. Let us compare $P_{\rm rad}$ with the electrostatic pressure $P_{\rm es}$ that would be generated if all electrons would be removed from the foil. The condition

$$P_{\rm rad} \geq P_{\rm es} = 2\pi(en_0\ell)^2 \quad (6)$$

corresponds to the threshold for the removal of all electrons from the foil. However, when Eq. (4) is used for $a_0 \leq \sqrt{1+\zeta^2}$, Eq. (6) reduces to $a_0 \geq \sqrt{1+\zeta^2}$, while for $a_0 \geq \sqrt{1+\zeta^2}$ we find that $P_{\rm rad} = P_{\rm es}$ holds. It is thus possible to produce a density distribution where *all electrons pile at the rear surface of the foil*. In fact, if $a_0 \lesssim \zeta$ and $R \simeq 1$, the laser pulse compresses the electron layer while keeping R constant since the product $n_e\ell$ does not change during the compression; at the same time almost no electrons are ejected from the rear side because the ponderomotive force vanishes there if $R \simeq 1$, and so does the electrostatic field: the qualitative profiles of the electron density and of the electric field are shown in Fig. 2(c). Since for a_0 close to ζ the equilibrium between the electrostatic and radiation pressures occurs only when the depth d of the region of charge depletion is close to ℓ, electrons are compressed in a very thin layer. The depletion depth d may be estimated from the equilibrium condition

$$P_{\rm es} = 2\pi(en_0 d)^2 \simeq 2I/c \quad (7)$$

when $R \simeq 1$, which yields $d \simeq \ell(a_0/\zeta) \lesssim \ell$. It is worth pointing out that these considerations are appropriate for a circularly polarized laser pulse; for linear polarization, all electrons may be expelled for a transient stage under the action of the $\mathbf{J}\times\mathbf{B}$ force whose peak value per unit surface exceeds $2RI/c$ due to its oscillating component. Complete expulsion of electrons for $a_0 > \zeta$ has been discussed in Ref. [15].

The snapshots from a PIC simulation shown in Fig. 3 for a case with $a_0 = 30$ and $\zeta = 31.4$ confirm the scenario outlined above. The electron density n_e reaches values (out of scale in Fig. 3) up to tens of the initial density. A very high resolution $\Delta x = \lambda/2000$ is used to resolve the density spike properly. For a laser pulse with flattop envelope the density spiking at the rear side of the foil is particularly evident, but we verified that it occurs also for a "$\sin^2$" envelope. Similar features were observed also in Refs. [5,8], but not discussed in detail.

The electron compression in a thin layer during the initial "hole boring" stage has important consequences for the later acceleration stage. Let us refer to the approximate field profiles in the initial stage, sketched in Fig. 2(c), which were the basis of the model presented in Ref. [12]. This model suggests that only the ions located initially in the electron compression layer ($d < x < \ell$) will be bunched and undergo RPA (via a "cyclic" acceleration as discussed in Refs. [2–4]) because for these ions only the electrostatic pressure balances the radiation pressure, while

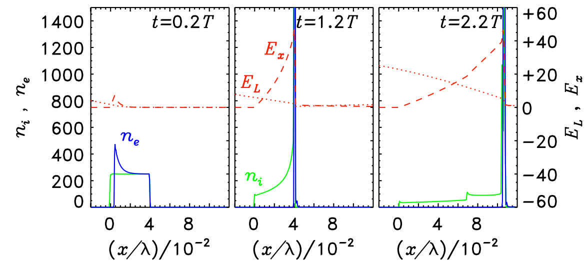

FIG. 3 (color online). Snapshots from a 1D PIC simulation of the interaction of a laser pulse with a thin plasma slab. The ion density n_i (green line), the electron density n_e (blue line), the longitudinal electric field E_x (red dashed line), and the pulse field amplitude $E_L = \sqrt{E_y^2 + E_z^2}$ (red dotted line) are shown. The target left boundary is at $x = 0$ where the pulse impinges at $t = 0$. Times are normalized to the laser period T, fields to $m_e\omega c/e$, and densities of n_c. The laser pulse has amplitude $a_0 = 30$ and the foil thickness is $\ell = 0.04\lambda$. All other parameters are the same as Fig. 1.

the ions in the electron depletion layer ($0 < x < d$) will be accelerated via Coulomb explosion, i.e., by their own space-charge field. This is exactly what is observed in the PIC simulations, both in the density profiles (see Fig. 3 at $t = 2.2T$) and in the ion spectra. This effect also explains how RPA with circularly polarized pulses may work also in double layer targets [14], if the thickness of a thin layer on the rear side matches $\ell_{\rm eff} = \ell - d$. Figure 2 shows ion spectra for the same simulation of Fig. 3 and for a simulation with the same parameters, but where ions in a surface layer of 0.01λ thickness have been replaced by protons. A fraction of heavier ions is also accelerated to the same energy per nucleon as the protons, a typical feature of RPA of a thin plasma foil.

As an additional consequence of the piling up of electrons at the rear surface, the portion of the foil which is boosted by the laser pulse is negatively charged due to the excess of electrons. However, the simulations show that when the laser pulse is over the excess electrons detach from the foil and move in the backward direction, so that the accelerated layer is eventually neutral. This is important to avoid a later Coulomb explosion of the layer and to preserve a monoenergetic spectrum. During the acceleration, the longitudinal field at the surface of the accelerated layer is almost constant, implying that the charge there contained is also constant. It is thus possible to estimate the fraction F of accelerated ions from the initial equilibrium condition, Eq. (7), as

$$F \simeq \ell_{\rm eff}/\ell \simeq (1 - a_0/\zeta). \quad (8)$$

The agreement with data in Fig. 2(b) is qualitative, with large deviations as F becomes significantly smaller than 1. As explained below, a lower bound on F is determined by energy conservation.

A simple argument of force balance also explains why the energy of the spectral peak in Fig. 2 is in very good agreement with the predictions of the LS model where the *initial* value ℓ of the foil thickness is used, while only a layer of thickness $\ell_{\text{eff}} < \ell$ is accelerated via RPA. Let us refer again to the profiles of Fig. 2. The equilibrium condition for electrons implies

$$\frac{2I}{c} \doteq \int_d^\ell eE_0\left(\frac{\ell - x}{\ell - d}\right)n_p dx = \frac{1}{2}en_0E_0\ell. \quad (9)$$

The electric field pushes ions in the compression layer $d < x < \ell$, exerting a total pressure

$$P_c = \int_d^\ell eE_0\left(\frac{\ell - x}{\ell - d}\right)n_i dx = \frac{2I}{c}\left(\frac{\ell - d}{\ell}\right), \quad (10)$$

where we used Eq. (9) and assumed $R = 1$. The equation of motion for the ion layer, in the early stage, can be thus written as

$$\frac{d}{dt}[\rho(\ell - d)\gamma\beta] = \frac{P_c}{c} = \frac{2I}{c}\left(\frac{\ell - d}{\ell}\right), \quad (11)$$

which is trivially equivalent to

$$\frac{d}{dt}(\rho\ell\gamma\beta) = \frac{2I}{c}, \quad (12)$$

i.e., to the equation of motion one would write for the *whole* foil. The argument may be applied also when the layer is in motion leading to the same conclusion. Having the same $\beta(t)$ as the whole foil implies that the energy per nucleon and the efficiency (1) will also be the same, but the total kinetic energy will be lower for the thin layer. The rest of the absorbed energy is stored in the electrostatic field and as kinetic energy of the ions in the $x < X(t)$ region. Let us consider, for example, the energy stored in the electrostatic field. At the time t, the field E_x between the initial ($x = 0$) and the actual [$x = X(t)$] positions of the front surface of the foil is given approximately by $E_x = E_0x/X(t)$, where $E_0 = 4\pi en_0d$, corresponding to an electrostatic energy per unit surface

$$U_{\text{es}} = U_{\text{es}}(t) = \int_0^{X(t)} \frac{E_x^2(x, t)}{8\pi}dx, \quad (13)$$

which varies in time as

$$\frac{dU_{\text{es}}}{dt} = \frac{1}{8\pi}E_x^2(X(t))\frac{dX}{dt} = \frac{1}{8\pi}E_0^2\beta c. \quad (14)$$

Dividing (14) by the laser intensity we obtain the "conversion efficiency" into electrostatic energy η_{es}

$$\eta_{\text{es}} = \frac{1}{I}\frac{dU_{\text{es}}}{dt} = 2\beta\left(\frac{d}{\ell}\right)^2\left(\frac{\zeta}{a_0}\right)^2. \quad (15)$$

If $\zeta \simeq a_0$ and thus $d \simeq \ell$, we would obtain $\eta_{\text{es}} \simeq 2\beta > \eta$ that is unphysical. Thus, the energy stored in the electrostatic field also prevents the accelerated layer thickness to shrink to zero.

In conclusion, we have reexamined the "light sail" model of radiation pressure acceleration of a thin plasma foil. The nonlinear reflectivity of the foil determines the "optimal" condition $\zeta \simeq a_0$, for which the energy in the RPA spectral peak is highest and in good agreement with the LS model formula where the *total* thickness (or the total mass) of the foil enters as a parameter. However, not all the foil is accelerated, but only a thin layer at the rear side of thickness $\ell_{\text{eff}} < \ell$; the apparent paradox is solved by observing that, to keep electrons in a mechanical quasiequilibrium, the electrostatic pressure pushing ions in the accelerated layer is ℓ_{eff}/ℓ times the radiation pressure on electrons, so that the equation of motion for the thin layer is the same as if the whole foil were accelerated. Finally, we showed that the energy stored in the electrostatic field is comparable to the kinetic energy and must be taken into account. For applications, the most relevant consequences and differences with respect to the simplest LS picture are that the number of "monoenergetic" ions is reduced, so that the actual efficiency may be much lower than given by Eq. (1), and that also light ions in a thin layer at the rear surface (e.g., hydrogen impurities) may be accelerated by RPA.

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