

Peccei-Quinn Mechanism in Gravity and the Nature of the Barbero-Immirzi Parameter

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A general argument provides the motivation to consider the Barbero-Immirzi parameter as a field. The specific form of the geometrical effective action allows one to relate the value of the Barbero-Immirzi parameter to other quantum ambiguities through the analog of the Peccei-Quinn mechanism.

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One of the most successful attempts to construct a non-perturbative quantum theory of gravity is *loop quantum gravity* [1–3]. Its classical starting point is the Ashtekar-Barbero canonical formulation of general relativity (GR) [4,5], which, classically, corresponds to a modification of the Hilbert-Palatini (HP) action as demonstrated by Holst [6]. This modification consists in adding to the usual HP action a new term which vanishes on the (half-)shell, i.e., [7]

$$S[e, \omega] = -\frac{1}{16\pi G} \int e_a \wedge e_b \wedge (\star R^{ab} + \beta R^{ab}), \quad (1)$$

where β is a constant known as Barbero-Immirzi (BI) parameter [8]. The parameter β is not fixed by the theory and its origin is still debated [9–14]. The quantum theory is, basically, the result of the Dirac quantization procedure applied to the constraints of classical GR in the Ashtekar-Barbero formulation. Remarkably, by using holonomies and fluxes as fundamental variables in analogy with lattice quantum field theories, the quantum representation algebra can be rigorously formalized [1–3] and a class of geometrical partial observables [2], as the area and volume of a portion of spacetime, can be defined. Their associated operators, once suitably regularized, result to have a discrete spectrum depending on the BI parameter β .

It is important to mention that the action (1) is classically equivalent to the HP one, as the additional term vanishes as soon as the homogeneous second Cartan structure equation is solved. It goes without saying that this is no longer true if a source for torsion is present. The case of fermion fields coupled to gravity is particularly instructive and has been studied in many recent papers [15–19]. Interestingly enough, it is easy to demonstrate that, in the presence of fermionic matter, the contribution coming from the Holst term can be compensated by a further modification in the fermionic sector [18]. Thus, the resulting effective theory is the usual Einstein-Cartan (EC) one plus a modification which can be associated with the so-called Nieh-Yan (NY) topological term [20].

This fact suggests a natural generalization of the Holst action (1) to spacetimes with torsion. Namely, we can assume that the action describing the dynamics of the gravitational field is the following one [9]:

$$\begin{aligned} S_{\text{Grav}}[e, \omega] &= S_{\text{HP}}[e, \omega] + S_{\text{NY}}[e, \omega] \\ &= -\frac{1}{16\pi G} \int e_a \wedge e_b \wedge \star R^{ab} \\ &\quad + \frac{\beta}{16\pi G} \int (T^a \wedge T_a - e_a \wedge e_b \wedge R^{ab}), \quad (2) \end{aligned}$$

where the second term is exactly the NY invariant. By remembering the definition of the torsion two-form $T^a = de^a + \omega^a_b \wedge e^b$, the NY term can be rewritten as a total divergence. Hence, in this new framework, the modification is a true topological term related to Pontryagin classes through the MacDowell-Mansouri connection [21]. The inclusion of matter is straightforward. As noted before, the case of spinor fields is particularly interesting: By generating torsion in spacetime, they can, in fact, reveal the claimed properties of the NY term.

According to the minimal prescription, the dynamics of the matter and gravity coupled system is described by the following action:

$$\begin{aligned} S_{\text{Tot}} &= S_{\text{HP}}[e, \omega] + S_{\text{NY}}[e, \omega] + \frac{i}{2} \int \star e_a \wedge \left(\bar{\psi} \gamma^a D \psi \right. \\ &\quad \left. - \overline{D \psi} \gamma^a \psi + \frac{i}{2} m e^a \bar{\psi} \psi \right). \quad (3) \end{aligned}$$

The covariant derivative D contains the Lorentz valued connection ω^{ab} , according to the following definition: $D\psi = d\psi - \frac{i}{4} \omega^{ab} \Sigma_{ab} \psi$, $\Sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$ being the generators of the Lorentz group. It is easy to demonstrate that pulling back the original action on the solution of the second Cartan structure equation one obtains the EC effective action, with its typical axial-axial four-fermion interaction. In other words, the new modification does not affect the classical equations of motion, even though a source for torsion is present [10,18,22]; but, in general, it can affect the quantum theory introducing P and CP violating effects. It is worth noting that in this framework the BI parameter β seems to play a role analogous to that played by the θ angle of QCD, as already argued in Ref. [11] (see also [9,10]).

Recently, in a certain number of papers [23–26], it has been proposed to promote the BI parameter to be a field rather than a constant. In particular, in Ref. [25], it was suggested that the interaction of the BI field with gravity is

mediated by the NY term. As a result, in the effective theory, the BI field decouples from gravity and behaves exactly as a (pseudo)scalar field, whereas, in the previous models based on the Holst action [23,24], it was $\phi = \sinh\beta$ to play the role of a scalar field rather than β itself, complicating also the effective interaction with fermions [24]. In this perspective, the NY modification appears more natural and, to a great extent, more appealing than the Holst one, though a strong motivation to promote the BI parameter to a field is still missing.

In this Letter, we suggest a possible motivation for considering the hypothesis that β is actually a field. Specifically, it will be demonstrated that a well-known divergent contribution to the chiral anomaly can be reabsorbed if β is a new field of the theory. Moreover, by extracting the vacuum expectation value of the BI field [27], we argue that the analog of the Peccei-Quinn mechanism allows one to relate the constant value of the BI parameter to other possible topological ambiguities, producing an interesting interaction between the physical BI field with gravity. Finally, we discuss some possible physical consequences of this model.

Let us consider the action (3) where the fermionic sector describes ordinary spin-1/2 matter fields interacting with gravity. Specifically, let us assume that the fermionic action describes leptons as well as quarks. It is well known that the quark mass matrices M resulting from the spontaneous breaking of the $SU(2) \times U(1)$ symmetry are neither diagonal nor Hermitian [28,29]. In order to diagonalize M , a chiral rotation is necessary:

$$q_R \rightarrow q'_R = e^{(i/2n_f)\text{Arg det}M} q_R, \quad (4a)$$

$$q_L \rightarrow q'_L = e^{-(i/2n_f)\text{Arg det}M} q_L, \quad (4b)$$

where q_R and q_L describes, respectively, a right- and left-handed quark of any flavor and n_f denotes the number of flavors. It turns out that such a chiral transformation introduces a divergent term in the effective theory.

In this respect, we recall that, in spacetime with torsion, the chiral rotation of the fermionic measure in the Euclidean path-integral generates, besides the usual Pontryagin class, a NY term, which diverges as the square of the regulator [21], i.e.,

$$\delta\psi\delta\bar{\psi} \rightarrow \delta\psi\delta\bar{\psi} \exp\left\{\frac{i}{8\pi^2} \int \alpha [R_{ab} \wedge R^{ab} + 2\Lambda^2(T_a \wedge T^a - e_a \wedge e_b \wedge R^{ab})]\right\}. \quad (5)$$

Above, Λ denotes the regulator, while α is the parameter of the transformation; the imaginary unit i disappears in Minkowski space. The robustness of this result has been confirmed by many independent calculations [30–32], performed by using different regularization procedures; however, it is worth mentioning that in Ref. [33] it has been claimed that, in fact, the divergent term is irrelevant for the chiral anomaly. In this respect, we stress that the main claim in Ref. [33] was based on an untenable observation,

mainly motivated by a classical “on-shell” condition, which, as noted in Ref. [34], cannot be extended to quantum regimes and to general torsional spacetimes. So, according to Eq. (5), the resulting effective action

$$S_{\text{Eff}} = S_{\text{HP}}[e, \omega] + S_D[e, \omega, \psi, \bar{\psi}] + \frac{1}{8\pi^2} \alpha \int R_{ab} \wedge R^{ab} + \frac{1}{16\pi G} \left(\beta + \frac{4G}{\pi} \alpha \Lambda^2 \right) \times \int (T^a \wedge T_a - e_a \wedge e_b \wedge R^{ab})$$

($\alpha = \frac{1}{n_f} \text{Arg det}M$) diverges as soon as we try to remove the regulator.

It is important to note that the divergence can be reabsorbed by promoting the β parameter to be a field and then renormalizing it. According to this observation, let us introduce in the fundamental action the field $\beta(x)$, which interacts with gravity through the NY term, i.e.,

$$S_{\text{Tot}}[e, \omega, \psi, \bar{\psi}, \beta] = S_{\text{HP}}[e, \omega] + S_D[e, \omega, \psi, \bar{\psi}] + \frac{1}{16\pi G} \int \beta(x)(T^a \wedge T_a - e_a \wedge e_b \wedge R^{ab}). \quad (6)$$

By assuming that (6) is the fundamental action for gravity, we can immediately eliminate the possible divergence resulting from the diagonalization of the quark mass matrix by renormalizing the field $\beta(x)$. In fact, by using the invariance of the new action under a shift of $\beta(x)$, without affecting the dynamical content of the theory, we can incorporate the divergence in the definition of a new field $\beta'(x) = \beta(x) + \frac{4G}{\pi} \alpha \Lambda^2$.

In this new framework the BI parameter can be naturally associated to the expectation value of the field $\beta(x)$, namely, $\beta_0 = \langle \beta(x) \rangle$.

Dynamically, the new action (6) is no longer equivalent to the EC one; in fact, the presence of the new field $\beta(x)$ modifies the expression of the torsion two-form, which reflects on the torsionless effective theory. In this respect, by varying action (6) with respect to the Lorentz valued one-form ω^{ab} , we obtain the following structure equation:

$$de^a + \omega^a_b \wedge e^b = T^a = \epsilon^a_{bcd} (\frac{1}{2} \eta^{bf} \partial_f \beta - 2\pi G J_{(A)}^b) e^c \wedge e^d,$$

where $J_{(A)} = e_a J_{(A)}^a = \sum_{f=1}^n e_a \bar{\psi}_f \gamma^a \gamma^5 \psi_f$ is the fermionic axial current (the sum is extended to all of the spin-1/2 matter fields contained in the action and described by the collective symbols ψ and $\bar{\psi}$). The connection ω^{ab} can be easily calculated. We have $\omega^{ab} = {}^\circ\omega^{ab}(e) + K^{ab}(e, \beta, \psi, \bar{\psi})$, where ${}^\circ\omega^{ab}(e)$ is the usual Ricci spin connection, while

$$K^{ab}(e, \beta, \psi, \bar{\psi}) = \epsilon^{abcd} e^c (2\pi G J_{(A)}^d - \frac{1}{2} \eta^{df} \partial_f \beta) \quad (7)$$

is the contortion one-form. It is worth noting that for $\beta = \text{const}$ the solution reduces to the ordinary solution of the EC theory. This fact suggests that, if $\beta = \text{const}$, the NY modification does not affect the classical effective action,

even though a source for torsion, e.g., spinor fields, is considered, exactly as claimed above and here demonstrated as a particular case in a more general framework. More generally, when the parameter β is promoted to be a field, the NY term in the action ceases to be topological generating a modification with respect to the EC theory. Interestingly enough, in order to preserve the usual transformation properties of the torsion tensor under the action of the Lorentz group and, consequently, its geometrical interpretation, the $\beta(x)$ field has to be a pseudoscalar, as can be easily understood by considering expression (7). In other words, the pseudoscalar nature of $\beta(x)$ is not assumed *a priori* but is a geometrical consequence of the theory.

Let us now explicitly extract the effective action. By calculating the torsion contributions contained in the different pieces of the total action and considering a non-vanishing vacuum angle θ , we have

$$S_{\text{Eff}} = S_{\text{HP}}[e, \omega] + S_D[e, \omega, \psi, \bar{\psi}] \\ + \frac{3}{2}\pi G \int dV \eta_{ab} J_{(A)}^a J_{(A)}^b + \frac{1}{2} \int \star d\tilde{\beta}(x) \wedge d\tilde{\beta}(x) \\ + \frac{\tilde{\theta}}{8\pi^2} \int R^{ab} \wedge R_{ab} - \sqrt{3\pi G} \int \star J_{(A)} \wedge d\tilde{\beta}(x), \quad (8)$$

where torsionless geometrical objects are distinguished by a circle [35]. As discussed above, in general, the effective action contains P and CP violating terms, included in the effective action via the parameter $\tilde{\theta} = \theta + \frac{1}{8\pi^2}\alpha$. Remarkably, the divergent contribution discussed before no longer affects the effective action, because it is reabsorbed through the shift symmetry in the field β . For convenience, we have defined the new field $\tilde{\beta}(x) = (3/16\pi G)^{1/2}\beta(x)$, so that it has the dimension of an energy and the kinetic term in the action has the usual numerical factor. The net effect of torsion in the effective action (8) is the presence of the four-fermion interaction as well as the appearance of the kinetic and interacting terms for the new dynamical field $\tilde{\beta}(x)$. In other words, the dynamics of the field $\tilde{\beta}(x)$ is completely determined by the geometrical content of the theory; in particular, torsion generates an interesting physical interaction between the pseudoscalar field $\tilde{\beta}(x)$ and the fermionic axial current $J_{(A)}$, described by the last term of (8). Let us now focus the attention exactly on this interaction term. As stressed before, the axial current fails to be conserved because of the noninvariance of the path-integral measure under chiral rotations [36]. Specifically, considering only gravity, we now have that $d\star J_{(A)} = -\frac{1}{8\pi^2}R^{ab} \wedge R_{ab}$. This allows us to contain the two terms in the last line of the effective action (8) in a single expression, namely,

$$S_{CP}[e, \beta] = \frac{1}{8\pi^2} \int [\tilde{\theta} + \sqrt{3\pi G}\tilde{\beta}(x)] R^{ab} \wedge R_{ab}. \quad (9)$$

The term above is the only CP violating contribution to the effective action. Furthermore, it represents a nontrivial

potential for the field $\tilde{\beta}$, with a stationary point in $\tilde{\theta} + \sqrt{3\pi G}\tilde{\beta}(x) = 0$ preserving P and CP symmetries. By using the same argument valid for the axion (see, e.g., [28]), we can eliminate the term (9) from the effective action, by a chiral transformation affecting the fermionic mass term. Assuming for simplicity that the fermionic contribution to the effective action contains only the up and down quarks, we can easily derive an effective action for the pion and the BI field. For the details we refer the reader to the standard literature, e.g., [28], and to Ref. [26], describing here only the main results. From the quadratic part in $\tilde{\beta}$ of the effective Lagrangian, we can extract the mass of the BI field, namely, $m_\beta \approx 10^{-12}$ eV. The higher orders in the effective Lagrangian represent a potential for the massive field $\tilde{\beta}$. The presence of a nontrivial potential for the BI field fixes its expectation value, i.e., $\beta_0 = -4/3\tilde{\theta}$, and then the value of the BI parameter can be directly related to the $\tilde{\theta}$ ambiguity of the theory, which in general depends on the nontrivial global structure of the local gauge group. Moreover, we note that the physical field $\tilde{\beta}_{\text{phys}} = \tilde{\beta} - \langle \tilde{\beta} \rangle$ interacts with gravity through the Pontryagin density, so that the effective theory resembles a Chern-Simons modified gravity [37], largely studied in the literature (see [38,39], and references therein).

The tiny mass of the BI field $\tilde{\beta}$ and the possible existence of the axion induce to consider a more realistic physical situation. Below, the other interactions are taken into account as well as the necessity to solve the so-called *strong CP problem*.

Discussion.—In order to get some more realistic and interesting physical predictions from this model, we cannot neglect the presence of another pseudoscalar particle $a(x)$, which, through the Peccei-Quinn mechanism [40], naturally solves the strong CP problem (for gravitational analogues, see [41,42]). In other words, to delineate a more complete picture, we have to consider in the effective action also the presence of an additional pseudoscalar degree of freedom, $a(x)$, and take into account that, through the chiral anomaly, both $\tilde{\beta}$ and $a(x)$ interact with the gauge fields.

A general argument suggests that the fields $a(x)$ and $\tilde{\beta}(x)$ naturally interact via the chiral anomaly with the gauge bosons in a linear combination, schematically [43]:

$$\mathcal{L}_{\text{int}} = \left(\frac{\beta}{2g_\beta} + \frac{a}{2g_a}\right) \text{tr}G \wedge G + \left(\frac{\beta}{2f_\beta} + \frac{a}{2f_a}\right) F \wedge F \\ + \left(\frac{\beta}{2r_\beta} + \frac{a}{2r_a}\right) R^{ab} \wedge R_{ab}, \quad (10)$$

where G is the curvature two-form of the $SU(3)$ valued connection one-form associated to the strong interaction, while F is the electromagnetic field strength. The mechanism that solves the strong CP problem and simultaneously generates an anomaly-induced mass is peculiar, and only one linear combination of the two pseudoscalar fields acquires a mass [44]. At an effective level, this fact implies

that, besides the usual QCD term for the massive physical axion (the mass depending on the energy scale of the interaction), one has a massless additional pseudoscalar field Φ , which interacts with the electromagnetic as well as the gravitational field as follows:

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{axion}} + \frac{\Phi}{2f_{\Phi}} F \wedge F + \frac{\Phi}{2r_{\Phi}} R^{ab} \wedge R_{ab}, \quad (11)$$

where f_{Φ}^{-1} and r_{Φ}^{-1} denote the scale of the respective interactions.

The above interaction (11) has interesting physical consequences. The presence of the coupling with photons, in fact, induces a rotation of the polarization angle ε of an electromagnetic wave [45]; particularly interesting in this direction could be the study of the polarization anisotropies of cosmic microwave background as recently claimed in Ref. [43]. Moreover, the same mechanism induces a rotation of the polarization angle of the gravitational waves through the interaction with the gravitational term, as demonstrated in the context of Chern-Simons modified gravity [46,47] (see also [48–50] for applications to cosmology). Interestingly enough, recently a black hole solution for Chern-Simons modified gravity has been found [51], representing a natural scenario where this model can be tested.

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