Testing Astrophysical Models for the PAMELA Positron Excess with Cosmic Ray Nuclei

Philipp Mertsch and Subir Sarkar

Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, United Kingdom

(Received 21 May 2009; revised manuscript received 9 July 2009; published 21 August 2009)

The excess in the positron fraction measured by PAMELA has been interpreted as due to annihilation or decay of dark matter in the Galaxy. More prosaically it has been ascribed to direct production of positrons by nearby pulsars or due to pion production during diffusive shock acceleration of hadronic cosmic rays in nearby sources. We point out that measurements of secondary cosmic ray nuclei can discriminate between these possibilities. New data on the titanium-to-iron ratio support the hadronic source model above and enable a prediction for the boron-to-carbon ratio at energies above 100 GeV.

DOI: 10.1103/PhysRevLett.103.081104 PACS numbers: 98.70.Sa, 96.50.sb, 98.58.Mj

The PAMELA collaboration [1] has reported an excess in the cosmic ray positron fraction, i.e., the ratio of the flux of positrons to the combined flux of positrons and electrons, $\phi_{e^+}/(\phi_{e^+} + \phi_{e^-})$, which is significantly above the background expected from production of positrons and electrons during propagation of cosmic ray protons and nuclei in the Galaxy [2]. It has been noted that the observed rise in the positron fraction between ~5–100 GeV cannot be due to propagation effects [3]; rather it requires a local primary source of cosmic ray electrons and positrons, e.g., nearby pulsars [4–7]. More excitingly, this could be the long sought for signature of the annihilation [8–11] or decay [12,13] of dark matter particles in the Galaxy.

Alternatively the observed rise in the positron fraction could be due to the acceleration of positrons produced by the decay of charged pions, which are created through hadronic interactions of cosmic ray protons undergoing acceleration in a nearby source [14]. That the secondary-to-primary ratio should *increase* with energy if secondaries are accelerated in the same spatial region as the primaries had been noted quite some time ago in the context of cosmic ray acceleration in the interstellar medium [15–17]. This model is conservative since it invokes only processes that are expected to occur in candidate cosmic ray sources, in particular, supernova remnants (SNRs). One way to distinguish it from the other models is to, e.g., compute the expected anti-proton-to-proton ratio, which is experimentally observed to be *consistent* with the standard background [18]. This is in fact in accord with the above model which predicts a rise in the $\bar{p}$ fraction only at energies above ~100 GeV [19] (see also Ref. [20]). This prediction cannot however be tested presently but must await data from the forthcoming AMS-2 mission [21] as well as PAMELA.

Although dark matter annihilation or decay as the explanation of the positron signal would appear to be disfavored by the absence of a corresponding antiproton signal, this can in principle be accommodated in models with large dark matter particle masses or preferential leptonic annihilation or decay modes [22–25]. Nearby pulsars as the source of the positrons are of course quite consistent with the absence of antiprotons. To differentiate between these possibilities and the model [14] in which secondary positrons from *hadronic* interactions are accelerated in the same region, we consider secondary nuclei in cosmic rays which are produced by the spallation of the primaries. An increasing secondary-to-primary ratio (e.g., boron-to-carbon or titanium-to-iron) in the *same* energy region would confirm that there is indeed a nearby cosmic ray source where nuclei are being accelerated stochastically along with protons.

An issue with this model [14] is that a crucial parameter is not known *a priori* but needs to be obtained from observations. This is the diffusion coefficient of relativistic particles near the accelerating SNR shock which determines the importance of a flatter spectral component over the usual Fermi spectrum and leads to the rise in secondary-to-primary ratios. Its absolute value cannot presently be reliably calculated. Observations of SNRs indicate that the magnetic field is quite turbulent so that relativistic electrons diffuse close to the "Bohm limit" with diffusion coefficient: $D^{\mathrm{Bohm}} = r_\ell c/3$, where the Larmor radius r_ℓ of the nucleus is proportional to the rigidity E/Z [26]. We need to determine the actual diffusion coefficient of ions in SNRs in ratio to the Bohm value by fitting to data. A measurement of one nuclear secondary-to-primary ratio therefore allows us to make predictions for other ratios in the framework of this model.

Very recently, data on the titanium-to-iron ratio (Ti/Fe) from the ATIC-2 experiment have been announced [27] that indeed show a rise above ~100 GeV. We use this data as a calibration to determine the diffusion coefficient and then, extrapolating it according to its rigidity dependence, we predict the boron-to-carbon ratio (B/C) that should soon be measured by PAMELA [28].

Galactic cosmic rays with energies up to the "knee" in the spectrum at $\sim 3 \times 10^{15}$ eV are believed to be accelerated by SNRs. The strong shocks present in these environments allow for efficient diffusive shock acceleration (DSA) by the 1st-order Fermi process [29]. In the simple test-particle approximation, which is adequate for the level of accuracy of the present discussion [30], protons and nuclei that are injected upstream are accelerated to form a nonthermal power-law spectrum whose index depends only on the parameters of the shock front, in particular, the compression ratio r. For a supersonic shock with $r = 4$ the steady state energy spectrum for protons and nuclei is $NdE \propto E^{-\gamma+2}dE$ where $\gamma = 3r/(r-1)$. In the standard model of galactic cosmic ray origin, the accelerated primary nuclei produce secondaries by spallation on hydrogen and helium nuclei in the interstellar medium (ISM). In the simple "leaky box model" [31] an energy-dependent escape of the cosmic rays out of the Galaxy is invoked to obtain a secondary-to-primary ratio that decreases with energy as observed to date in the region ~1–100 GeV. This is also obtained by using the GALPROP code [2] which solves the full transport equation in 3 dimensions, and can yield both the time-independent as well as equilibrium solution.

However, as the acceleration time for the highest energy particles is of the same order as the time scale for spallation, the production of secondaries inside the sources must be taken into account. In any *stochastic* acceleration process one then expects the secondary-to-primary ratio to increase with energy since particles with higher energy have spent more time in the acceleration region and have therefore produced more secondaries [15–17]. This general argument can be quantified for the case of DSA by including the production of secondaries due to spallation and decay as a source term,

$$Q_i(\varepsilon_k)d\varepsilon_k = \sum_j N_j(\varepsilon_k)\left[\sigma_{j\to i}^{\rm spall}\beta c n_{\rm gas} + \frac{1}{\varepsilon_k \tau_{j\to i}^{\rm dec}}\right]d\varepsilon_k, \quad (1)$$

where ε_k is the K.E./nucleon (in GeV), and a loss term,

$$\Gamma_i N_i(\varepsilon_k)d\varepsilon_k = N_i(\varepsilon_k)\left[\sigma_i^{\rm spall}\beta c n_{\rm gas} + \frac{1}{\varepsilon_k \tau_i^{\rm dec}}\right]d\varepsilon_k, \quad (2)$$

where $\sigma_{j\to i}^{\rm spall}$ ($\sigma_i^{\rm spall}$) and $\tau_{j\to i}^{\rm dec}$ ($\tau_i^{\rm dec}$) are the partial (total) cross sections and decay times, respectively. The transport equation for any nuclear species i then reads

$$u\frac{\partial f_i}{\partial x} = D_i \frac{\partial^2 f_i}{\partial x^2} + \frac{1}{3}\frac{du}{dx} p \frac{\partial f_i}{\partial p} - \Gamma_i f_i + q_i, \quad (3)$$

where f_i is the phase space density and the different terms from left to right describe convection, spatial diffusion, adiabatic energy losses as well as losses and injection of particles from spallation or decay. We consider the acceleration of *all* species in the usual setup: in the frame of the shock front the plasma upstream ($x < 0$) and downstream ($x > 0$) is moving with velocity u_- and u_+, respectively. We solve Eq. (3) analytically for relativistic energies ε_k greater than a few GeV/nucleon such that $p \approx E$, $\beta \approx 1$ and $N_i dE \approx 4\pi p^2 f_i dp$. At these energies ionization losses can be neglected and the spallation cross sections become energy independent.

There are three relevant time scales in the problem: (1) Acceleration time $\tau_{\rm acc}$:

$$\tau_{\rm acc} = \frac{3}{u_- - u_+}\int_0^p \left(\frac{D_i^+}{u_+} + \frac{D_i^-}{u_-}\right)\frac{dp'}{p'} \simeq 8.8 E_{\rm GeV} Z^{-1} B_{\mu \rm G}\ {\rm yr} \quad (4)$$

for Bohm diffusion and the parameter values mentioned later. (2) Spallation and decay time τ_i.

$$\tau_i^{\rm spall} \equiv 1/\Gamma_i^{\rm spall} \sim 1.2 \times 10^7 \left(\frac{n_{\rm gas}}{\rm cm^{-3}}\right)^{-1}\ {\rm yr}. \quad (5)$$

where an average σ_i of $\mathcal{O}(100)$ mb has been assumed. The rest lifetime $\tau_i^{\rm dec}$ of the isotopes considered ranges between 4×10^{-2} yr and 10^{17} yr. (3) Age of the SNRs under consideration [14,19]

$$\tau_{\rm SNR} = x_{\rm max}/u_+ \sim 2 \times 10^4\ {\rm yr}. \quad (6)$$

There are two essential requirements for SNRs to efficiently accelerate nuclei by the DSA mechanism: (a) $\tau_{\rm acc} \ll \tau_i^{\rm spall}$, which is equivalent to

$$20\frac{\Gamma_i^- D_i}{u_-^2} \ll 1 \Rightarrow \epsilon_k \ll 6.4 \times 10^5 \frac{Z_i}{A_i} B_{\mu \rm G}\ {\rm GeV}. \quad (7)$$

(b) $\tau_{\rm SNR} \ll \tau_i$ which implies,

$$\frac{x_{\rm max}}{u_+} \ll \frac{1}{\Gamma_i} \Rightarrow x\frac{\Gamma_i}{u_+} \ll 1. \quad (8)$$

The isotopes for which condition (b) is not satisfied at the lowest energy considered viz. ^{56}Ni, ^{57}Co, ^{55}Fe, ^{54}Mn, ^{51}Cr, ^{49}V, ^{44}Ti and ^{7}Be do *not* contribute significantly, so their decays in the source region are neglected.

We find that the general solution to Eq. (3) for $x \neq 0$ is

$$f_i^\pm = \sum_{j\le i}(E_{ji}^\pm e^{\lambda_j^\pm x/2} + F_{ji}^\pm e^{\kappa_j^\pm x/2}) + G_i^\pm,$$

$$\text{where } \lambda_i^\pm = \frac{u_\pm}{D_i^\pm}\left(1 - \sqrt{1 + 4D_i^\pm \Gamma_i^\pm/u_\pm^2}\right), \quad (9)$$

$$\kappa_i^\pm = \frac{u_\pm}{D_i^\pm}\left(1 + \sqrt{1 + 4D_i^\pm \Gamma_i^\pm/u_\pm^2}\right),$$

where $G_i^\pm$ is the asymptotic value and $E_{ji}^\pm$ and $F_{ji}^\pm$ are determined by the recursive relations:

$$E_{ji}^\pm = \frac{-4\sum_{m\ge j} E_{mj}^\pm \Gamma_{j\to i}^\pm}{D_i^\pm \lambda_j^{\pm 2} - 2u\lambda_j^\pm - 4\Gamma_i^\pm}, \quad (10)$$

$$F_{ji}^\pm = \frac{-4\sum_{m\ge j} F_{mj}^\pm \Gamma_{j\to i}^\pm}{D_i^\pm \kappa_j^{\pm 2} - 2u\kappa_j^\pm - 4\Gamma_i^\pm}. \quad (11)$$

We require that the phase space distribution function converges to the adopted primary composition Y_i (at the injection energy p_0) far upstream of the SNR shock:

$$f_i(x,p) \overset{x\to-\infty}{\longrightarrow} Y_i\delta(p-p_0), \qquad \partial f_i/\partial p(x,p) \overset{x\to-\infty}{\longrightarrow} 0. \tag{12}$$

We also require the solution to remain finite far downstream. As the phase space density is continuous at the shock front, we connect the solutions in both half planes to $f_i^0 = f_i(x=0,p)$ and find them to be:

$$f_i^- = f_i^0 e^{\kappa_i^- x/2} + \sum_{j<i} F_{ji}^-(e^{\kappa_j^- x/2} - e^{\kappa_i^- x/2}) + Y_i\delta(p-p_0)(1-e^{\kappa_i^- x/2}), \tag{13}$$

$$f_i^+ = f_i^0 e^{\lambda_i^+ x/2} + \sum_{j<i} E_{ji}^+(e^{\lambda_j^+ x/2} - e^{\lambda_i^+ x/2}) + G_i^+(1-e^{\lambda_i^+ x/2}). \tag{14}$$

Using Eqs. (7) and (8), we can linearly expand λ_i^+ and κ_i^- in Eq. (10) and the exponentials in Eqs. (13) and (14)

$$e^{\lambda_i^+ x/2} \simeq 1 - \frac{\Gamma_i^+}{u_+}x, \qquad e^{\kappa_i^- x/2} \simeq \left(1 + \frac{\Gamma_i^-}{u_-}x\right)e^{u_- x/D_i} \tag{15}$$

to obtain:

$$f_i^+ = f_i^0 + \frac{q_i^+(x=0) - \Gamma_i^+ f_i^0}{u_+}x, \tag{16}$$

where $q_i^\pm$ denotes the downstream or upstream source term: $q_i^\pm = \sum_{j<i} f_j \Gamma_{i\to j}^\pm$.

Finally we integrate the transport equation over an infinitesimal interval around the shock, assuming that $q_i^+/q_i^- = \Gamma_i^+/\Gamma_i^- = n_{\rm gas}^+/n_{\rm gas}^- = r$ and that $D_i^+ \simeq D_i^-$:

$$p\frac{\partial f_i}{\partial p} = -\gamma f_i^0 - \gamma(1+r^2)\frac{\Gamma_i^- D_i^-}{u_-^2}f_i^0 + \gamma\left[(1+r^2)\frac{q_i^-(x=0)D_i^-}{u_-^2} + Y_i\delta(p-p_0)\right], \tag{17}$$

which is readily solved by

$$f_i^0(p) = \int_0^p \frac{dp'}{p'}\left(\frac{p'}{p}\right)^\gamma e^{-\gamma(1+r^2)(D_i^-(p)-D_i^-(p'))\Gamma_i^-/u_-^2} \times \gamma\left[(1+r^2)\frac{q_i^-(x=0)D_i^-(p')}{u_-^2} + Y_i\delta(p'-p_0)\right]. \tag{18}$$

Our Eqs. (16)–(18) should be compared to Eqs. (4–6) of Ref. [14] where the loss terms $\Gamma_i f_i$ were not taken into account. The exponential in our Eq. (18) leads to a natural cutoff in both the primary and secondary spectra above the energy predicted by Eq. (7). However, due to the approximations we have made, the secondary-to-primary ratios cannot be predicted reliably for $4\Gamma_i D_i/u^2 \gtrsim 0.1$, i.e., much beyond ~1 TeV.

Starting from the heaviest isotope, Eqs. (16) and (18) can be solved iteratively to obtain the injection spectrum after integrating over the SNR volume,

$$N_i(E) = 4\pi \int_0^{u_+\tau_{\rm SN}} dx p^2 f_i(p) 4\pi x^2. \tag{19}$$

To account for the subsequent propagation of the nuclei through the ISM we solve the transport equation in the leaky box model [31] which reproduces the observed decrease of secondary-to-primary ratios with energy in the range ~1–100 GeV by assuming an energy-dependent lifetime for escape from the Galaxy. The steady state cosmic ray densities $\mathcal{N}_i$ observed at Earth are then given by recursion, starting from the heaviest isotope,

$$\mathcal{N}_i = \frac{\sum_{j<i}(\Gamma_{i\to j}^{\rm spall} + 1/\varepsilon_k\tau_{i\to j})\mathcal{N}_j + \mathcal{R}_{\rm SN}N_i}{1/\tau_{{\rm esc},i} + \Gamma_i}, \tag{20}$$

where $\mathcal{R}_{\rm SN} \sim 0.03\ {\rm yr}^{-1}$ is the Galactic supernova rate.

We calculate the source densities N_i and ambient densities $\mathcal{N}_i$, taking into account all stable and metastable isotopes from ^{64}Ni down to ^{46}Cr/^{46}Ca for the Ti/Fe ratio, and from ^{18}O down to ^{10}Be for the B/C ratio. Short lived isotopes that $\beta^\pm$ decay immediately into (meta)stable elements are accounted for in the cross sections. The primary source abundances are taken from Ref. [32] and we have adopted an injection energy of 1 GeV independent of the species. The partial spallation cross sections are from semianalytical tabulations and the total inelastic cross sections are obtained from an empirical formula [33]. The escape time is modeled according to the usual relation:

$$\tau_{{\rm esc},i} = \rho c x_{{\rm esc},i} = \rho c x_{{\rm esc},i}^0 (E/Z_i)^{-\mu}, \tag{21}$$

where x is the column density traversed in the ISM and

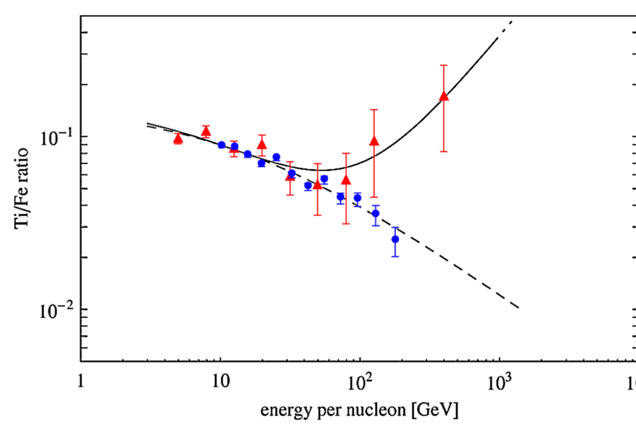

FIG. 1 (color online). The Ti/Fe ratio in cosmic rays along with model predictions—the leaky box model with production of secondaries during propagation only (dashed line), and including production and acceleration of secondaries in a nearby source (solid line—dotted beyond the validity of our calculation). The data points are from ATIC-2 (triangles) [27] and HEAO-3-C3 (circles) [34].

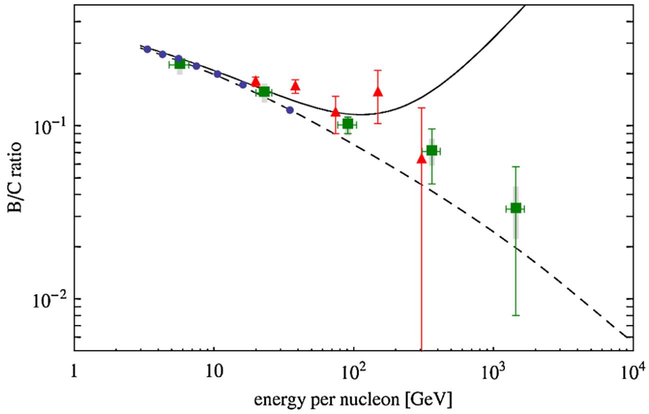

FIG. 2 (color online). The B/C ratio in cosmic rays along with model predictions—the leaky box model with production of secondaries during propagation only (dashed line), and including production and acceleration of secondaries in a nearby source (solid line). The data points are from HEAO-3-C2 (circles) [32], ATIC-2 (triangles) [38] and CREAM (squares) [35].

$\rho = 0.02$ atom cm^{-3} is the typical mass density of hydrogen in the ISM. We have neglected spallation on helium at this level of precision as its inclusion will have an effect $<10\%$. The fit parameters are sensitive to the adopted partial spallation cross sections, for example $\mu \simeq 0.7$ for the Ti/Fe ratio but $\sim$0.6 for the B/C ratio.

Following Ref. [19], the parameters are chosen to be: $r = 4$, $u_- = 0.5 \times 10^8$ cm s^{-1}, $n_{\rm gas}^- = 2$ cm^{-3} and $B = 1\ \mu$G. The diffusion coefficient in the SNR is

$$D_i(E) = 3.3 \times 10^{22} \mathcal{F}^{-1} B_\mu^{-1} E_{\rm GeV} Z_i^{-1}\ {\rm cm}^2\,{\rm s}^{-1} \qquad (22)$$

where the fudge factor $\mathcal{F}^{-1}$ is the ratio of the diffusion coefficient to the Bohm value and is determined by fitting to the measured Ti/Fe ratio.

The calculated Ti/Fe ratio together with the relevant experimental data are shown in Fig. 1. The dashed line corresponds to the leaky box model with production of secondaries during propagation only and is a good fit to the (reanalysed) HEAO-3-C3 data [34]. The solid line includes production and acceleration of secondaries inside the source regions which results in an *increasing* ratio for energies above $\sim$50 GeV/n and reproduces well the ATIC-2 data [27] taking $\mathcal{F}^{-1} \simeq 40$. This is similar to the value reported in Refs. [14,19], thus ensuring consistency with the e^+ as well as $\bar{p}$ fraction measured by PAMELA.

Clearly the experimental situation is inconclusive so a new test is called for. Figure 2 shows the corresponding expectation for the B/C ratio with the diffusion coefficient scaled proportional to rigidity according to Eq. (22). The CREAM data [35] do show a downward trend as has been emphasized recently [36], but the uncertainties are still large so we await more precise measurements by PAMELA which has been directly calibrated in a test beam [37]. Agreement with our prediction would confirm the astrophysical origin of the positron excess as proposed in Ref. [14] and thus establish the existence of an accelerator of hadronic cosmic rays within a few kpc.

P. M. is grateful to Markus Ahlers for many helpful and inspiring discussions and S. S. thanks Ramanath Cowsik for a prescient remark as well as Pasquale Blasi and Dan Hooper for helpful comments. P. M. acknowledges support by the EU Marie Curie Network UniverseNet (HPRN-CT-2006-035863) and STFC, UK.

[1] O. Adriani *et al.*, Nature (London) **458**, 607 (2009).
[2] I. V. Moskalenko and A. W. Strong, Astrophys. J. **493**, 694 (1998).
[3] P. D. Serpico, Phys. Rev. D **79**, 021302 (2009).
[4] A. M. Atoian *et al.*, Phys. Rev. D **52**, 3265 (1995).
[5] D. Hooper *et al.*, J. Cosmol. Astropart. Phys. 01 (2009) 025.
[6] H. Yuksel *et al.*, Phys. Rev. Lett. **103**, 051101 (2009).
[7] S. Profumo, arXiv:0812.4457.
[8] L. Bergstrom *et al.*, Phys. Rev. D **78**, 103520 (2008).
[9] M. Cirelli and A. Strumia, arXiv:0808.3867.
[10] V. Barger *et al.*, Phys. Lett. B **672**, 141 (2009).
[11] I. Cholis *et al.*, arXiv:0809.1683.
[12] E. Nardi *et al.*, J. Cosmol. Astropart. Phys. 01 (2009) 043.
[13] A. Arvanitaki *et al.*, arXiv:0904.2789.
[14] P. Blasi, Phys. Rev. Lett. **103**, 051104 (2009).
[15] D. Eichler, Astrophys. J. **237**, 809 (1980).
[16] R. Cowsik, Astrophys. J. **241**, 1195 (1980).
[17] C. Fransson and R. Epstein, Astrophys. J. **242**, 411 (1980).
[18] O. Adriani *et al.*, Phys. Rev. Lett. **102**, 051101 (2009).
[19] P. Blasi and P. D. Serpico, arXiv:0904.0871 [Phys. Rev. Lett. (to be published)].
[20] Y. Fujita *et al.*, arXiv:0903.5298.
[21] http://ams.cern.ch/.
[22] M. Cirelli *et al.*, Nucl. Phys. B **813**, 1 (2009).
[23] I. Cholis *et al.*, arXiv:0810.5344.
[24] P. f. Yin *et al.*, Phys. Rev. D **79**, 023512 (2009).
[25] P. J. Fox and E. Poppitz, Phys. Rev. D **79**, 083528 (2009).
[26] M. D. Stage *et al.*, Nature Phys. **2**, 614 (2006).
[27] V. I. Zatsepin *et al.*, Astron. Lett. **35**, 338 (2009).
[28] P. Picozza (private communication).
[29] R. Blandford and D. Eichler, Phys. Rep. **154**, 1 (1987).
[30] M. Malkov and L. Drury, Rep. Prog. Phys. **64**, 429 (2001).
[31] R. Cowsik *et al.*, Phys. Rev. **158**, 1238 (1967).
[32] J. Engelmann *et al.*, Astron. Astrophys. **233**, 96 (1990).
[33] R. Silberberg and C. Tsao, Astrophys. J. Suppl. Ser. **25**, 315 (1973); **35**, 137 (1977); Phys. Rep. **191**, 351 (1990).
[34] V. Vylet *et al.*, in Proc. 21st Intern. Cosmic Ray Conf., Adeliade, 1989, edited by (Graphic Services, North Field, South Australia, 1990), Vol. 3, p.19; W. R. Binns *et al.*, Astrophys. J. **324**, 1106 (1988).
[35] H. S. Ahn *et al.*, Astropart. Phys. **30**, 133 (2008).
[36] M. Simet and D. Hooper, arXiv:0904.2398.
[37] D. Campana *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **598**, 696 (2009).
[38] A. D. Panov *et al.*, arXiv:0707.4415.