Optical Kagome Lattice for Ultracold Atoms with Nearest Neighbor Interactions

J. Ruostekoski

School of Mathematics, University of Southampton, Southampton, SO17 1BJ, United Kingdom

(Received 9 June 2009; published 21 August 2009)

We propose a scheme to implement an optical kagome lattice for ultracold atoms with controllable *s*-wave interactions between nearest neighbor sites and a gauge potential. The atoms occupy three different internal atomic levels with electromagnetically induced coupling between the levels. We show that by appropriately shifting the triangular lattice potentials, experienced by atoms in different levels, the kagome lattice can be realized using only two standing waves, generating a highly frustrated quantum system for the atoms.

DOI: 10.1103/PhysRevLett.103.080406 PACS numbers: 03.75.Lm, 03.75.Hh, 03.75.Mn, 03.75.Ss

Ultracold atomic gases in periodic optical lattice potentials can form a highly controllable quantum many-particle system that has demonstrated interesting analogies to crystal lattice systems of strongly interacting electrons. Among such experimental developments are the Mott insulator states of both bosonic [1] and fermionic atoms [2,3], fermionic superfluidity [4], atom transport [5–7], and superexchange correlations in a collection of double wells [8]. The rapid progress has generated interest in engineering ultracold atomic lattice systems that could provide clean realizations of Hubbard models of strongly correlated crystal lattice systems with the potential use of atoms as quantum simulators of some unresolved models, e.g., in high-T_c superconductivity [9].

One of the challenging problems in the theory of strongly correlated systems has been to characterize the phase diagrams of two-dimensional (2D) and 3D Hubbard models of geometrically frustrated lattices, with competing interactions resulting in highly degenerate ground states. Of particular interest are corner-sharing networks of complete graphs, such as kagome and pyrochlore or checkerboard lattices. It was recently proposed that frustration in pyrochlore and diamond lattices could generate fractional charges in the presence of nearest neighbor (NN) repulsion [10], with similar excitations potentially existing in a kagome lattice. Moreover, spin-1/2 NN Heisenberg antiferromagnet on a kagome lattice provides arguably the most promising candidate for a quantum spin liquid, with the kagome lattice exhibiting a higher degree of frustration than, e.g., a triangular lattice [11], and it has recently attracted considerable interest in volborthite and herbertsmithite compounds. Despite extensive theoretical effort the true nature of the ground state of the kagome system has been evasive [12]. In addition, kagome systems can exhibit, e.g., kinetic ferromagnetism [13], and production of trimerized and ideal kagome lattices for ultracold atoms has also started attracting theoretical interest [14].

In this Letter we propose a constructed 2D optical kagome lattice system of neutral atoms with collisional interactions between NN sites. The hopping between the NN sites is induced by electromagnetic (EM) transitions between three internal atomic states. Because of the spin-dependent lattice system, the kagome lattice can be prepared with only two optical standing waves (SWs), as compared to six SWs in previous proposals [14], with the additional advantage of controllable nonlocal two-body interactions between different sites and the possibility for the creation of Abelian and non-Abelian gauge potentials. The strength of the EM-driven hopping can be tuned over a wide range of values with respect to tunneling between more distant sites and the NN collisions. Spin-dependent lattices where the different atomic spin components were moved around independently were experimentally created using ^{87}Rb atoms [15]. Alkaline-earth-metal atoms and rare-earth metals with narrow optical resonances (e.g., Sr, Yb) are particularly suitable for realizing spin-dependent optical lattices, because of slow loss rates due to spontaneous emission [16].

We consider ultracold (bosonic or fermionic) atoms occupying three internal sublevels $|j\rangle$ ($j = 1, 2, 3$) of the same atom that are coupled by EM transitions. The atom dynamics is assumed to be restricted to 2D on a yz plane due to a tight magnetic or optical confinement [17,18]. On the yz plane each species experiences a triangular optical lattice potential, generated by two SWs with wave vectors $\mathbf{k}_{\alpha,\beta} = k(\hat{\mathbf{e}}_y \pm \sqrt{3}\hat{\mathbf{e}}_z)/2$. The atom-lattice model is based on an atomic (single-band) Hubbard Hamiltonian [19] where the atoms occupy the lowest mode of each lattice site. We show that by appropriately tuning the frequencies of the lattice lasers, the lattice potentials of the three species can be shifted in a triangular shape to form a kagome lattice pattern. Then the four NN sites are occupied by the atoms in the other two sublevels and the hopping of the atoms between adjacent sites, with amplitude κ_{jk}, only occurs as a result of driving by coherent EM fields that change the internal atomic level. The hopping to the nearest sites occupied by the atoms in the same sublevel results from tunneling between the sites and, in sufficiently deep lattices, it may be suppressed. Moreover, the specialty of the proposed scheme is that by adjusting the overlap of the NN lattice site wave functions (Wannier functions) we can prepare a lattice system with a non-negligible, control-

lable s-wave interaction between adjacent sites U_{jk}^{NN} ($j \neq k$), with both limits $U_{jk}^{\mathrm{NN}} \gg \kappa_{jk}$ and $U_{jk}^{\mathrm{NN}} \ll \kappa_{jk}$ achievable, providing a frustrated quantum system with NN interactions.

As a realization of a frustrated kagome lattice with desired interactions, we consider a tripod four-level scheme; Fig. 1. The atoms in the three electronic ground states are coupled to a common electronically excited level $|e\rangle$ by different resonant transition frequencies ω_j ($\omega_3 > \omega_2 > \omega_1$). The lattice laser α, with the frequency ω_α, is blue detuned with respect to two of the transitions $\delta_{\alpha,j} = \omega_j - \omega_\alpha < 0$, for $j = 1, 2$, and red detuned with respect to one $\delta_{\alpha,3} > 0$. Then the atoms in the states $|1\rangle$, $|2\rangle$ ($|3\rangle$) are attracted towards low-intensity (high-intensity) regions of α. The lattice laser β, with the frequency ω_β, is blue detuned from one of the transitions $\delta_{\beta,1} < 0$ and red detuned from the others. For simplicity, we assume $|e\rangle = |e, m\rangle$, $|1\rangle = |1, m-1\rangle$, $|2\rangle = |2, m\rangle$, and $|3\rangle = |3, m+1\rangle$, so that the transitions $|1\rangle \rightarrow |e\rangle$, $|3\rangle \rightarrow |e\rangle$, and $|2\rangle \rightarrow |e\rangle$ have dipole matrix elements $\mathbf{d}_{e1} = \langle e|\mathbf{d}|1\rangle = \mathfrak{D}\langle e|1, 1\rangle\hat{\mathbf{e}}_{+1}^*$, $\mathbf{d}_{e3} = \mathfrak{D}\langle e|-1, 3\rangle\hat{\mathbf{e}}_{-1}^*$, and $\mathbf{d}_{e2} = \mathfrak{D}\langle e|0, 2\rangle\hat{\mathbf{e}}_0^*$, coupling to the light with polarizations σ^+, σ^-, and $\hat{\mathbf{e}}_0$, respectively. Here $\mathfrak{D}$ is the reduced dipole matrix element, $\langle e|\sigma g\rangle$ are the Clebsch-Gordan coefficients, and we use $\hat{\mathbf{e}}_{+1} = -(\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y)/\sqrt{2}$, $\hat{\mathbf{e}}_{-1} = (\hat{\mathbf{e}}_x - i\hat{\mathbf{e}}_y)/\sqrt{2}$, and $\hat{\mathbf{e}}_0 = \hat{\mathbf{e}}_z$. We assume the polarizations of the two SWs to be orthogonal $\hat{\mathbf{e}}_\alpha \cdot \hat{\mathbf{e}}_\beta^* = 0$, so that the lattice potential reads $V_j = V_j^\alpha + V_j^\beta$ with

$$V_j^\eta = s_{j\eta} E_r \sin^2[k(y \pm \sqrt{3}z)/2 + \varphi_{\eta j}], \qquad (1)$$

where $\eta = \alpha, \beta$ and $+(-)$ refers to α (β). Here $\varphi_{1\eta} =$

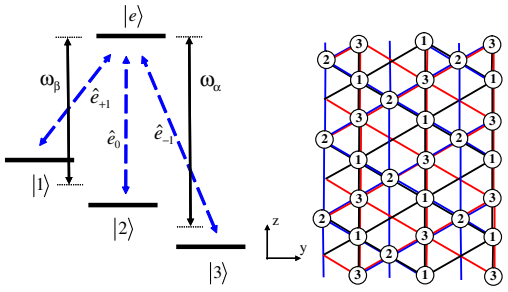

FIG. 1 (color online). Left: In the kagome lattice the atoms occupy three electronic ground states with a common excited state. The frequencies of the two lattice lasers are tuned between the atomic resonance frequencies in such a way that $\omega_3 > \omega_\alpha > \omega_2 > \omega_\beta > \omega_1$. The polarization components of the lattice lasers that couple to each individual transition are indicated. Right: Atoms in each of the three internal levels $|j\rangle$ ($j = 1, 2, 3$) experience an equilateral triangular lattice potential. These lattices are shifted with respect to each other, so that the lattice site occupied by the atoms in $|2\rangle$ ($|3\rangle$) is at the midpoint of the triangle side of $|1\rangle$ defined by the unit vector $\mathbf{k}_\beta/k = (\hat{\mathbf{e}}_y - \sqrt{3}\hat{\mathbf{e}}_z)/2$ ($\hat{\mathbf{e}}_z$). Each site has four NN sites, so that, e.g., the site of $|1\rangle$ has two NN sites of $|2\rangle$ and two of $|3\rangle$.

$\varphi_{3\alpha} = 0$, $\varphi_{2\eta} = \varphi_{3\beta} = \pi/2$, and $s_{j\eta} \propto |(\mathbf{d}_{ej} \cdot \hat{\mathbf{e}}_\eta)^2/\delta_{\eta,j}|$ denotes the lattice strength in the lattice photon recoil energy units $E_r = \hbar^2 k^2/2m$, depending on the light polarization, atomic sublevel, and the detuning. To produce three triangular lattices the lasers need to couple simultaneously to all the three transitions, so that $s_{j\alpha}$ and $s_{j\beta}$ are nonvanishing for all j. This can be obtained by choosing the SW polarizations so that $\hat{\mathbf{e}}_\eta \cdot \hat{\mathbf{e}}_{\pm 1,0}^* \neq 0$ together with $\hat{\mathbf{e}}_\eta \cdot \mathbf{k}_\eta = 0$ ($\eta = \alpha, \beta$) and $\hat{\mathbf{e}}_\alpha \cdot \hat{\mathbf{e}}_\beta^* = 0$.

We will next demonstrate an example to show that such a solution can be found for $\hat{\mathbf{e}}_{\alpha,\beta}$ and can also be used to control the relative strengths of the three triangular lattices. This can be especially useful if the absolute values of the detunings are very different. We consider the level scheme shown in Fig. 1 and, for simplicity, assume that $|\delta_{\alpha,3}| = |\delta_{\alpha,2}| = 3|\delta_{\alpha,1}| = 3|\delta_{\beta,3}| = |\delta_{\beta,2}| = |\delta_{\beta,1}|$ (indicating that $\omega_1 - \omega_2 = \omega_2 - \omega_3$ and that the lasers are detuned exactly at the midpoint between the nearest transitions) and that the Clebsch-Gordan coefficients are equal. The polarizations of the SWs (orthogonal to the corresponding wave vectors $\mathbf{k}_j$) are $\hat{\mathbf{e}}_{\alpha,\beta} = \mathfrak{a}_{\alpha,\beta}(\mp\sqrt{3}\hat{\mathbf{e}}_y + \hat{\mathbf{e}}_z)/2 + \mathfrak{b}_{\alpha,\beta}\hat{\mathbf{e}}_x$. We choose the complex coefficients $\mathfrak{a}_{\alpha,\beta}$ and $\mathfrak{b}_{\alpha,\beta}$ in such a way that $\hat{\mathbf{e}}_\alpha \cdot \hat{\mathbf{e}}_\beta^* = 0$, $|\hat{\mathbf{e}}_{-1}^* \cdot \hat{\mathbf{e}}_\alpha|^2 = 3|\hat{\mathbf{e}}_{+1}^* \cdot \hat{\mathbf{e}}_\alpha|^2$, and $|\hat{\mathbf{e}}_{+1}^* \cdot \hat{\mathbf{e}}_\beta|^2 = 3|\hat{\mathbf{e}}_{-1}^* \cdot \hat{\mathbf{e}}_\beta|^2$. The last two conditions ensure that we have $s_{1\eta} = s_{3\eta}$, compensating for the different values of the detunings. A straightforward algebra yields solutions for $\mathfrak{a}_{\alpha,\beta}$ and $\mathfrak{b}_{\alpha,\beta}$ with $s_{2\eta} = 4s_{3\eta}/5$ ($\eta = \alpha, \beta$), resulting in only very small differences between the lattice strengths. We also find that the two SWs in Eq. (1) have equal amplitudes for each level, i.e., $s_{j\alpha} = s_{j\beta}$ for all j.

For each species the lattice potential V_j is an equilateral triangle with the side $2d$, where $d = \pi/\sqrt{3}k$ denotes the NN separation; Fig. 1. The minima of the potential are at the triangle corners which for $|1\rangle$ are at $(y, z) = [(n+m)\pi/k, (n-m)d]$, for $|2\rangle$ at $[(n+m+1/2)\pi/k, (n-m-1/2)d]$, and for $|3\rangle$ at $[(n+m+1)\pi/k, (n-m)d]$, with n, m integers. The triangular lattices of $|2\rangle$ and $|3\rangle$ are shifted to coincide with the side midpoints of the triangular lattice of $|1\rangle$. The combined system of interlaced triangular lattices forms a kagome lattice.

In order to estimate the relative strengths of the different terms in the lattice Hamiltonian we evaluate the corresponding integral representations. The direct tunneling amplitude, where atoms remain in the same hyperfine level during the hopping process, reads $J_b^{\mathbf{p}} \simeq -\int dydz(-\frac{\hbar^2}{2m}\phi_{bn}^*\nabla_\parallel^2\phi_{bn'} + \phi_{bn}^* V_b \phi_{bn'}) > 0$, where $\nabla_\parallel^2 = \partial_y^2 + \partial_z^2$ and V_b is given by Eq. (1). The Wannier functions for atoms in $|b\rangle$ at site n are $\phi_{bn}(y, z)$ and may be approximated by the ground state harmonic oscillator wave function with the trap frequencies $\omega_y \simeq \sqrt{2s}E_r/\hbar$ and $\omega_z \simeq \sqrt{6s}E_r/\hbar$ at the lattice site minimum [19]. The site n' here refers to the nearest site to n occupied by the same atomic hyperfine level along the direction of $\mathbf{p}$, where $\mathbf{p}$ takes the

values of the triangle sides: $\mathbf{k}_{\alpha,\beta}, \hat{\mathbf{z}}$. Here $J_b^{\mathbf{k}_\alpha} = J_b^{\mathbf{k}_\beta}$ by symmetry. For $J_b^{\mathbf{k}_\alpha}$ and $J_b^{\hat{\mathbf{z}}}$ we have $\phi_{b,n'}(y,z) = \phi_{b,n}(y+\sqrt{3}d, z+d)$ and $\phi_{b,n'}(y,z) = \phi_{b,n}(y, z+2d)$, respectively. Because of the anisotropy of the individual lattice site wave functions, the hopping amplitudes along the z direction differ slightly from those along the direction of the two SWs. Although a more rigorous calculation of the hopping amplitudes would involve a full band-structure calculation, here it is sufficient to provide order-of-magnitude analytic estimates using the Gaussian approximations to ϕ_{bn}.

The EM field changes the internal level of the atom. Because the lattice site minima of the different sublevels are shifted with respect to each other, the atoms simultaneously also undergo spatial hopping along the lattice. The hopping amplitude reads

$$\kappa_{bc} = \int d^3 r \phi_{bn}^* \hbar \mathcal{R}_{bc} \phi_{cn'}, \tag{2}$$

where $\mathcal{R}_{bc}$ denotes the effective Rabi frequency for the transition between the levels $|b\rangle$ and $|c\rangle$ and n' refers to the NN site of species $|c\rangle$ to the site n occupied by the species $|b\rangle$, with κ_{bc} proportional to the spatial overlap between the atoms in the NN sites. As shown in Fig. 1, κ_{12}, κ_{23}, and κ_{13} represent hopping along the directions of $\mathbf{k}_\beta$, $\mathbf{k}_\alpha$, and $\hat{\mathbf{z}}$, respectively. As for the direct tunneling we then have $\kappa_{12} = \kappa_{23}$, but κ_{13} is not exactly equal. Note that κ_{bc} in Eq. (2), unlike the direct tunneling, can take positive, negative, or even complex values.

The explicit expression for the Rabi frequency $\mathcal{R}_{bc}$ depends on the particular form of the EM coupling between the internal levels which can be a one-photon or a multiphoton transition. For a two-photon transition via an off-resonant intermediate level $|e'\rangle$ (which for a laser can be electronically excited and for a rf or microwave an electronic ground state) we may adiabatically eliminate $|e'\rangle$ [20]. We then obtain $\mathcal{R}_{bc} = \mathcal{E}_b \mathcal{E}_c^* d_{e'b} d_{e'c}^*/(2\hbar^2\Delta)$ and a contribution to the EM-induced level shifts $-|\mathcal{E}_j|^2 |d_{e'j}|^2/(2\hbar^2\Delta)$ for $|j\rangle$. Here $d_{e'j} \equiv \mathbf{d}_{e'j} \cdot \hat{\mathbf{e}}_j$ and we assumed that $|b\rangle$ is coupled to $|e'\rangle$ by the EM field with the positive frequency component $\mathbf{E}_b^+ = \frac{1}{2}\hat{\mathbf{e}}_b \mathcal{E}_b e^{i\mathbf{k}_b\cdot\mathbf{r}} e^{i\Omega_b t}$ and detuning Δ. Using the Gaussian approximation to ϕ_{bn}, we obtain $\kappa_{jk} \simeq \hbar \mathcal{R}_{jk} \epsilon_{jk} \exp[-(3+\sqrt{3})\pi^2\sqrt{\bar{s}_{jk}}/48\sqrt{2}]$, for $(j,k) = (1,2), (2,3)$ and $\kappa_{13} \simeq \hbar \mathcal{R}_{13}\epsilon_{13} \times \exp(-\pi^2\sqrt{\bar{s}_{13}}/4\sqrt{6})$, where $\bar{s}_{jk} \equiv 4 s_j s_k/(\sqrt{s_j}+\sqrt{s_k})^2$ and $\epsilon_{jk} \equiv [\bar{s}_{jk}^2/(s_j s_k)]^{1/4}$. Here we assumed that the lattice potentials by the lasers α and β in Eq. (1) have the same amplitude in each level $|j\rangle$, i.e., $s_{j\alpha} = s_{j\beta}$, and we suppressed in the notation the index referring to the particular laser.

The on site interaction term may also be obtained from the Wannier functions $U_{jj} \simeq g_{2D}^{(jj)} \int dydz |\phi_{jn}|^4$. The 2D nonlinearity $g_{2D}^{(jk)} \simeq 2\pi\hbar^2 a_{jk}/m l_x \sqrt{2\pi}$ is given in terms of the scattering length a_{jk} and the 2D trap confinement $l_x = \sqrt{\hbar/m\omega_x}$ where the oscillator frequency perpendicular to the lattice is ω_x. The additional density-dependent contribution to the scattering length in 2D is negligible when $\sqrt{2\pi} l_x/a_{jj} \gg \ln(8\pi^{3/2} l_x n_{2D} a_{jj})$ [21], where n_{2D} denotes the 2D atom density. For fermionic atoms $a_{jj} = 0$ and the on site interaction term vanishes for a single-species gas if the atoms only occupy the lowest mode of each lattice site, but is nonvanishing for a two-species gas trapped in each lattice site. The NN interaction due to the s-wave scattering between the atoms in different sublevels is always nonvanishing for both bosonic and fermionic atoms. It is generated in the spatial overlapping area of the adjacent sites and may be estimated by $U_{jk}^{\mathrm{NN}} \simeq 2 g_{2D}^{(jk)} \int dydz |\phi_{jn}|^2 |\phi_{kn'}|^2$. We obtain $U_{jj} \simeq a_{jj} 3^{1/4} E_r \sqrt{s_j}/l_x\sqrt{\pi}$, $U_{13}^{\mathrm{NN}} \simeq 2a_{13} 3^{1/4} E_r \sqrt{\bar{s}_{13}} e^{-\pi^2\sqrt{\bar{s}_{13}}/2\sqrt{6}}/l_x\sqrt{\pi}$, and for other states $U_{jk}^{\mathrm{NN}} \simeq 2a_{jk} 3^{1/4} E_r \sqrt{\bar{s}_{jk}} e^{-(3+\sqrt{3})\pi^2\sqrt{\bar{s}_{jk}}/24\sqrt{2}}/l_x\sqrt{\pi}$.

The Hamiltonian for the atomic system then reads

$$H = \sum_k [\epsilon_k c_k^\dagger c_k + U_{kk} c_k^\dagger c_k^\dagger c_k c_k] - \sum_{\langle jk\rangle'} (J_b^{\mathbf{p}} c_j^\dagger c_k + \mathrm{H.c.}) - \sum_{\langle jk\rangle} [(\kappa_{bc} c_j^\dagger c_k + \mathrm{H.c.}) + U_{bc}^{\mathrm{NN}} c_j^\dagger c_j c_k^\dagger c_k], \tag{3}$$

where $\langle jk\rangle$ denotes the summation over adjacent lattice sites and $\langle jk\rangle'$ over the nearest sites that are occupied by the same atomic species. The level shifts and the detunings due to EM-induced hoppings are included in ϵ_k. Here $J_b^{\mathbf{p}} > 0$, but the other coefficients can also take negative values, and κ_{bc} can be complex.

We may compare the different terms in the Hamiltonian. For typical experimental parameters for bosonic atoms we have $U_{jj} \gg U_{jk}^{\mathrm{NN}}$ $(j \neq k)$. For simplicity, assuming $s_1 = s_2$, we obtain for lattice heights $s = 25$ and 40, $U_{12}^{\mathrm{NN}}/U_{jj} \simeq 10^{-3} a_{12}/a_{jj}$ and $10^{-4} a_{12}/a_{jj}$. At the same lattice heights the EM-driven hopping amplitudes in terms of the direct tunneling and the NN interactions are given by $\kappa_{12}/J_1^{\mathbf{k}_\alpha} \simeq 540(\hbar\mathcal{R}_{12}/E_r)$ and $5100(\hbar\mathcal{R}_{12}/E_r)$, and $\kappa_{12}/U_{12}^{\mathrm{NN}} \simeq 4.2(l_x/a_{12})(\hbar\mathcal{R}_{12}/E_r)$ and $8.3(l_x/a_{12})(\hbar\mathcal{R}_{12}/E_r)$. In shallow lattices, with weak EM coupling between the sublevels $\hbar\mathcal{R}_{12}/E_r \ll 1$, the NN hopping κ_{bc} and the direct tunneling $J_b^{\mathbf{p}}$ between more distant sites may be comparable. In deeper lattices and for stronger κ_{bc} the direct tunneling terms may be ignored. If the transverse confinement of the 2D lattice is weak (l_x large) and the Rabi field sufficiently strong $\hbar\mathcal{R}_{12} l_x/(a_{12}E_r) \gg 1$, we may also neglect U_{bc}^{NN} with $\kappa_{bc} \gg U_{bc}^{\mathrm{NN}}$, and in Eq. (3) we only keep terms proportional to ϵ_k, U_{kk}, and κ_{bc}. An especially interesting property of the proposed scheme is that we may also find a wide range of parameter values for which $U_{bc}^{\mathrm{NN}} \gtrsim \kappa_{bc}$. This limit can always be achieved with sufficiently weak EM coupling. If we also simultaneously require that $\kappa_{bc} \gg J_b^{\mathbf{p}}$, we may, e.g., at $s = 50$ select $\hbar\mathcal{R}_{12}/E_r \simeq 5 \times 10^{-4}$ and $l_x \simeq 17 a_{12}$, resulting in $U_{bc}^{\mathrm{NN}} \simeq 10\kappa_{bc}$. For instance, for ^{87}Rb the s-wave scattering length between $|F=1, M_F=-1\rangle$ and $|2,+1\rangle$ hyperfine states is about 5.191 nm [22], corre-

sponding to the transverse trap frequency of $\omega_x \simeq 2\pi \times 15$ kHz, achievable, e.g., by an optical lattice. The interspecies scattering length could be increased by Feshbach resonances, further enhancing the effect of the NN interactions U_{bc}^{NN}, so that one could reach $U_{jj} \gg U_{bc}^{\mathrm{NN}} \gg \kappa_{bc} \gg J_b^{\mathbf{p}}$ also in shallow lattices. Interspecies Feshbach resonances were observed also in ^{87}Rb between different hyperfine states [23], and in optical lattices at low occupation numbers the harmful three-body losses are suppressed. Moreover, alkaline-earth-metal and rare-earth-metal atoms with very narrow optical resonances [16] may allow the lattice lasers to be tuned close to the atomic resonance making it easier to produce deep lattices.

The simplest situation is to select the phases of the EM fields in Eq. (2) so that all the hopping amplitudes κ_{bc} are real and positive. It is, however, also possible to engineer a nonuniform phase profile for the hopping amplitudes which was in Ref. [24] proposed as a mechanism to construct topologically nontrivial ground states with fractional fermion numbers in 1D. In a 2D lattice the technique can be used to create an effective magnetic field for neutral atoms [25]. Here we may similarly induce a phase for atoms hopping around a closed path in the lattice, mimicking a magnetic flux experienced by charged particles. We write $\kappa_{bc} = |\kappa_{bc}|e^{i\nu_{bc}}$ where the phases $\nu_{bc}(\mathbf{r})$ may be constant or spatially varying [24,25]. The hopping around one unit triangle then generates the phase $\Delta\nu = \nu_{12} + \nu_{23} - \nu_{13}$ for the atoms, corresponding to a magnetic flux $\Phi \propto \Delta\nu$ through the area enclosed by the triangle. For instance, for spatially constant ν_{bc} with $\Delta\nu \neq 0$, the entire lattice area may be divided into side-sharing triangles where each adjacent triangle experiences the flux with the opposite sign. In the case of atoms occupying more than one sublevel in each lattice site, we may also generate non-Abelian vector potentials similarly to Ref. [26]; see also [27,28].

For strong NN interactions with $1/3$ filling, even without a vector potential, our model produces a frustrated ground state where one atom in each triangle of sites is strongly influenced by the atoms in other corner-sharing triangles. It is helpful to consider a honeycomb lattice, formed by connecting the centers of triangles of the kagome lattice, where the sites are coupled by ring-hopping processes [29]. A lattice system described by an analogous quantum dimer model on a pyrochlore or checkerboard lattice and on a 3D diamond lattice was recently shown to support fractional charges [10]. Similar fractional excitations are expected to exist in the kagome lattice system, where they can act as independent, deconfined particles over finite distances at temperatures above the ordering transition driven by quantum fluctuations—in this case by the ring-exchange processes [30]. Atomic states in the prepared lattice system could potentially be detected optically [31].

Our formalism considers one atomic species per lattice site. It is straightforward to generalize it to the situation where a two-species gas is trapped in each site with EM field inducing hopping for both species. Then the on site interaction and the hopping terms can be expressed as an effective Heisenberg spin-1/2 Hamiltonian $H_{\mathrm{eff}} \simeq \sum_{\langle i,j\rangle}[t_z S_i^z S_j^z \pm t_\perp (S_i^x S_j^x + S_i^y S_j^y)]$ where the $+$ $(-)$ sign refers to fermionic (bosonic) atoms and S_i^k denote the spin matrices [32]. The fermionic version has been extensively studied in 2D kagome lattices where the ground state of the SU(2) symmetric case ($t_z = t_\perp$) still has unsettled questions, e.g., in the existence of spontaneously broken symmetries and finite energy gaps [12]. The SU(2) symmetric case can also be realized with fermionic atoms as t_z, $t_\perp$ can be independently varied [32] by controlling the EM-induced hopping of the two species.

We are grateful to N. Shannon for explanations of the importance of kagome lattices and EPSRC for funding.

[1] M. Greiner *et al.*, Nature (London) **415**, 39 (2002).
[2] R. Jördens *et al.*, Nature (London) **455**, 204 (2008).
[3] U. Schneider *et al.*, Science **322**, 1520 (2008).
[4] J. K. Chin *et al.*, Nature (London) **443**, 961 (2006).
[5] C. D. Fertig *et al.*, Phys. Rev. Lett. **94**, 120403 (2005).
[6] T. Kinoshita *et al.*, Nature (London) **440**, 900 (2006).
[7] J. Mun *et al.*, Phys. Rev. Lett. **99**, 150604 (2007).
[8] S. Trotzky *et al.*, Science **319**, 295 (2008).
[9] A. F. Ho *et al.*, Phys. Rev. A **79**, 033620 (2009).
[10] P. Fulde *et al.*, Ann. Phys. (Leipzig) **11**, 892 (2002); O. Sikora *et al.*, arXiv:0901.1057.
[11] B. G. Levi, Phys. Today **60**, No. 2, 16 (2007).
[12] For recent work, see, e.g., A. M. Läuchli and C. Lhuillier, arXiv:0901.1065; Y. Ran *et al.*, Phys. Rev. Lett. **98**, 117205 (2007); S. Ryu *et al.*, Phys. Rev. B **75**, 184406 (2007); P. Sindzingre, arXiv:0707.4264.
[13] F. Pollmann *et al.*, Phys. Rev. Lett. **100**, 136404 (2008).
[14] L. Santos *et al.*, Phys. Rev. Lett. **93**, 030601 (2004); B. Damski *et al.*, *ibid.* **95**, 060403 (2005); Phys. Rev. A **72**, 053612 (2005).
[15] O. Mandel *et al.*, Nature (London) **425**, 937 (2003).
[16] T. Fukuhara *et al.*, Phys. Rev. A **79**, 041604(R) (2009).
[17] Z. Hadzibabic *et al.*, Nature (London) **441**, 1118 (2006).
[18] N. Smith *et al.*, J. Phys. B **38**, 223 (2005).
[19] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998).
[20] J. Javanainen and J. Ruostekoski, Phys. Rev. A **52**, 3033 (1995).
[21] D. S. Petrov *et al.*, Phys. Rev. Lett. **84**, 2551 (2000).
[22] D. M. Harber *et al.*, Phys. Rev. A **66**, 053616 (2002).
[23] A. Marte *et al.*, Phys. Rev. Lett. **89**, 283202 (2002).
[24] J. Ruostekoski *et al.*, Phys. Rev. Lett. **88**, 180401 (2002); Phys. Rev. A **77**, 013603 (2008).
[25] D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003).
[26] K. Osterloh *et al.*, Phys. Rev. Lett. **95**, 010403 (2005).
[27] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).
[28] J. Ruseckas *et al.*, Phys. Rev. Lett. **95**, 010404 (2005).
[29] F. Pollmann *et al.*, Phys. Rev. B **73**, 174417 (2006).
[30] N. Shannon (private communication).
[31] J. Ruostekoski *et al.* (unpublished); J. Javanainen and J. Ruostekoski, Phys. Rev. Lett. **91**, 150404 (2003).
[32] L.-M. Duan *et al.*, Phys. Rev. Lett. **91**, 090402 (2003).