

Probing Minicharged Particles with Tests of Coulomb's Law

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Minicharged particles arise in many extensions of the standard model. Their contribution to the vacuum polarization modifies Coulomb's law via the Uehling potential. In this Letter, we argue that tests for electromagnetic fifth forces can therefore be a sensitive probe of minicharged particles. In the low mass range $\lesssim \mu\text{eV}$ existing constraints from Cavendish type experiments provide the best model-independent bounds on minicharged particles.

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1. Introduction.—Coulomb's inverse square law for the force between two charges is one of the central features of classical electrodynamics. Its experimental discovery by Cavendish, Coulomb, and Robison was an important step in the development of classical electrodynamics culminating in Maxwell's equations. Over the last more than 200 years, Coulomb's law has been tested with increasing precision and at various length scales (see, e.g., [1,2]). Indeed, the first detected deviation from it in the form of the Lamb shift was one of the first indications of "new physics" in the form of *quantum* electrodynamics [3–5].

Nowadays, precise tests of Coulomb's law can serve as a powerful probe to search for new physics beyond the standard model. A classical application is the search for a photon mass (see, e.g., [1,2]). Beyond that, tests of Coulomb's law can even be used to search for new particles in so-called hidden sectors, which interact only very weakly with the particles of the standard model and which are accordingly difficult to detect in conventional collider experiments. Because of their feeble interactions, constraints on hidden-sector particles are often quite weak and they may be light with masses in the eV range or even below. Hidden sectors appear in many extensions of the standard model. In fact, it may be exactly those hidden sectors that give us crucial information on how the standard model is embedded into a more fundamental theory as, e.g., string theory. This makes it very desirable to have new, complementary probes of such particles. One example of a hidden-sector particle that can be searched for using tests of Coulomb's law is massive extra U(1) gauge bosons [6,7]. In this Letter, we will argue that they can also be used to search for another interesting class of hidden-sector particles, namely, particles with a small electric charge, so-called minicharged particles (see, [8,9] for a test via the Lamb shift).

Minicharged particles arise naturally and consistently (in theories with kinetic mixing [10] minicharged particles are indeed consistent with the existence of magnetic monopoles [11]) in a wide variety of extensions of the standard model based on field [10] and string theory [12]. Expectations for the size of their charge cover a wide range from 10^{-16} to 10^{-2} [10,12]. The best current laboratory

limits in the sub-eV mass range are of the order of $\sim 10^{-6}$ (cf. Fig. 1). Astrophysical bounds (see, e.g., [8]) are considerably stronger but also somewhat model dependent [13] (see, however, [14,15] which combined give a less model dependent bound $\sim 10^{-9}$ from cosmology). Therefore, it is of particular importance to find new ways to search for minicharged particles in laboratory experiments. In this Letter, we will show that Cavendish type experiments searching for a deviation from Coulomb's law can be used for this purpose.

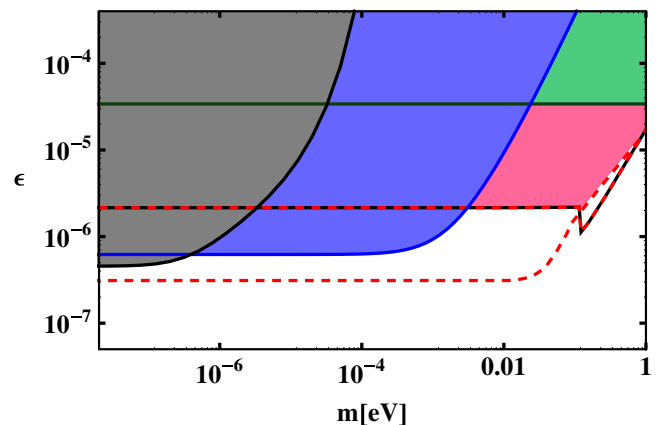


FIG. 1 (color online). Laboratory bounds on minicharged particles. The black line (on the left, bottom solid) corresponds to the exclusion limit obtained in this Letter from the Cavendish type tests of Coulomb's law. The blue bound (on the left, middle solid) arises from constraints on energy losses in high quality accelerator cavities [22]. The dark green curve (on the left, top solid) gives the limit arising from bounds on the invisible decay on orthopositronium [23,24] (a similar bound can be obtained from a reactor experiment [25]). The red-black dashed line denotes the limit [18,26] arising from light-shining-through-a-wall experiments [27,28] and applies only to minicharged particles arising from kinetic mixing (for a way to test pure minicharged particle models in a light-shining-through-a-wall experiment, see [29]) whereas the red dashed curve gives a limit [26,30] from polarization experiments [27,31] (an interesting alternative to polarization experiments is interferometry [32]) and applies for a pure minicharged particle scenario. The shaded areas are excluded in both scenarios.

2. *Deviations from Coulomb's law from minicharged particles.*—Minicharged particles cause a deviation from Coulomb's law via their effect on the vacuum polarization, in particular, via the Uehling potential [16]. The presence of a nonvanishing vacuum polarization modifies the electric potential of a charge Q (see, e.g. [17]),

$$V(\mathbf{x}) = V_{\text{Coulomb}}(x) + \delta V(x) \\ = Q \int \frac{d^3q}{(2\pi)^3} \exp(i\mathbf{q} \cdot \mathbf{x}) \frac{e^2}{|\mathbf{q}|^2 [1 - \hat{\Pi}_\epsilon(q)]}, \quad (2.1)$$

where $\hat{\Pi}_\epsilon(q)$ is an on shell renormalized version of the vacuum polarization, e.g.,

$$\hat{\Pi}_\epsilon(q^2) = -\alpha\epsilon^2 \frac{2}{\pi} \int_0^1 dx x(1-x) \log\left(\frac{m^2}{m^2 - x(1-x)q^2}\right). \quad (2.2)$$

For the vacuum polarization contribution of a particle of charge ϵ to the potential a standard textbook calculation [17] gives,

$$\delta V(r) = \frac{Q\alpha}{r} \left[\frac{2\alpha\epsilon^2}{3\pi} \int_{2m}^\infty dq \frac{\exp(-qr)}{q} \right. \\ \left. \times \sqrt{1 - \frac{4m^2}{q^2} \left(1 + \frac{2m^2}{q^2}\right)} \right]. \quad (2.3)$$

For large distances, $r \gg 1/m$, δV drops off exponentially whereas for small distances, $r \ll 1/m$; its behavior is logarithmic as expected from the running gauge coupling,

$$\delta V(r) \approx \frac{Q\alpha}{r} \left[\frac{\alpha\epsilon^2}{4\sqrt{\pi}} \frac{\exp(-2mr)}{(mr)^{\frac{3}{2}}} \right] \quad \text{for } mr \gg 1, \\ \approx \frac{Q\alpha}{r} \left[-\frac{2\alpha\epsilon^2}{3\pi} \log(2mr) - a \right], \quad \text{for } mr \ll 1. \quad (2.4)$$

$$a \approx \frac{2\alpha\epsilon^2}{3\pi} \gamma$$

So far we have considered the case where the minicharged particles have no further interactions. However, one of the most natural ways in which minicharges arise is via kinetic mixing [10] of the electromagnetic U(1) gauge boson with an extra ‘‘hidden’’ gauge boson under which the standard model particles are uncharged. For such a theory the Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\chi F^{\mu\nu}B_{\mu\nu}, \quad (2.5)$$

where $F_{\mu\nu}$ is the field strength tensor for the ordinary electromagnetic U(1)_{QED} gauge field A^μ , and $B^{\mu\nu}$ is the field strength for the hidden sector U(1)_h field B^μ , i.e., the hidden photon. The first two terms are the standard kinetic terms for the photon and hidden photon fields. Because the field strength itself is gauge invariant for U(1) gauge fields, the third term is also allowed by gauge and Lorentz symmetry. This term corresponds to a nondiagonal kinetic

term, a so-called kinetic mixing. From the viewpoint of the low energy effective theory χ is a completely arbitrary parameter. Typical expectation in extensions of the standard model range from 10^{-16} to 10^{-2} [10,12]. Accordingly, we will treat χ in the following as a small parameter, $\chi \ll 1$.

The kinetic term can be diagonalized by a shift

$$B^\mu \rightarrow \tilde{B}^\mu - \chi A^\mu. \quad (2.6)$$

The only effect on the visible sector field A^μ is a multiplicative change of the gauge coupling,

$$e_0^2 \rightarrow \frac{e_0^2}{1 - \chi^2} \equiv e^2. \quad (2.7)$$

We can now see how a minicharge arises, for example, for a hidden matter fermion h that has charge one under B^μ . Applying the shift (2.6) to the coupling term, we find

$$e_h \bar{h} \not{B} h \rightarrow e_h \bar{h} \not{\tilde{B}} h - \chi e_h \bar{h} \not{A} h, \quad (2.8)$$

where e_h is the hidden-sector gauge coupling. We can read off that the hidden-sector particle now has a small electric charge

$$\epsilon e = -\chi e_h \ll 1 \quad \text{for } \chi \ll 1 \quad (2.9)$$

under the visible electromagnetic gauge field A^μ which has gauge coupling e .

The interesting question is now what is the effect of a vacuum polarization caused by the hidden-sector particle h , i.e., the minicharged particle. The vacuum polarization can be most easily calculated in the unshifted basis where h couples only to the hidden-sector field B^μ . Accordingly its only effect is that the B^μ field has to be renormalized by a factor,

$$\sqrt{Z_h} = \sqrt{1 - \hat{\Pi}_h(q^2)}, \quad (2.10)$$

where $\hat{\Pi}_h$ is given exactly by the same expression, Eq. (2.2), as for $\hat{\Pi}_\epsilon$ only with $\alpha\epsilon^2$ replaced by the full hidden-sector gauge coupling α_h . This renormalization entails that χ , too, has to be renormalized,

$$\chi \rightarrow \frac{\chi}{\sqrt{Z_h}}. \quad (2.11)$$

In turn, this then affects the renormalization Eq. (2.7) of the ordinary electromagnetic gauge coupling after the diagonalization Eq. (2.6),

$$e_0^2 \rightarrow \frac{e_0^2}{1 - \frac{\chi^2}{Z_h}} = \frac{e_0^2}{1 - \frac{\chi^2}{1 - \hat{\Pi}_h(q^2)}} \\ \approx \frac{e_0^2}{(1 - \chi^2)[1 - \chi^2 \hat{\Pi}_h(q^2)]} + \mathcal{O}(\chi^2 e^2 \alpha_h^2) \\ = \frac{e^2}{1 - \hat{\Pi}_\epsilon(q^2)}, \quad (2.12)$$

where we have treated $\hat{\Pi}_h$ as a small parameter and have used the relations Eqs. (2.7) and (2.9) to obtain the last equality. The right-hand side is exactly the expression that

appears in Eq. (2.1). Accordingly, the vacuum polarization effects in theories with kinetic mixing cause the same (up to higher power in the hidden-sector gauge coupling α_h) deviations from Coulomb's law as in models with only minicharged particles. The bounds derived in the following are therefore independent of the specific way in which the minicharge arises.

Finally, in the above setting with kinetic mixing one could imagine that the hidden U(1) gauge boson has a (small) mass $m_{\gamma'}$. At large distances $r \gg 1/m_{\gamma'}$ the behavior is then dominated by the (renormalized) mass of the hidden photon and one gets the same Yukawa type modification of Coulomb's law as discussed in [6,7] in the absence of a minicharged particle [this is in alignment with the fact that at momenta $\ll m_{\gamma'}$ the effective minicharge vanishes (see, [18])]. For small distances $r \ll 1/m_{\gamma'}$ the mass of the hidden photon plays no role and one obtains the modification of Coulomb's law discussed in this Letter.

3. *Bounds from Cavendish type experiments.*—Having identified the modification of the potential, we can now turn to how it can be detected in experiments. Currently, one of the most sensitive tests of the Coulomb potential is a variation of the concentric sphere setup used by Cavendish. The idea behind this experiment is that if and only if the potential has a $1/r$ form, the inside of a charged sphere is field free and the potential accordingly constant. In other words, the potential difference between a charged outer sphere and an uncharged inner sphere is zero if and only if the electric potential has the Coulomb form. Deviations

from this form would lead to a nonvanishing potential difference that can be measured.

In general, the potential of a sphere with a charge Q and a radius c at the distance r from the center of the sphere is

$$U(Q, r, c) = \frac{Q}{2cr} [f(r+c) - f(|r-c|)], \quad (3.1)$$

where

$$f(r) = \int_0^r ds sV(s, Q=1). \quad (3.2)$$

It is straightforward to check that for a Coulomb potential $V_{\text{Coulomb}}(r) = \alpha/r$, the potential inside the sphere, i.e., for $r < c$, is constant. Explicitly one finds $f(r) = \alpha r$ and $U(Q, r, c) = Q\alpha/c = \text{const}$.

In the simplest version of the Cavendish experiment, one has an outer sphere of radius b , charged to a certain voltage, and then measures the relative voltage difference to the uncharged inner sphere of radius $a < b$,

$$\begin{aligned} \gamma_{ab} &= \left| \frac{\mathcal{V}_b - \mathcal{V}_a}{\mathcal{V}_b} \right| = \left| \frac{U(Q, b, b) - U(Q, a, b)}{U(Q, b, b)} \right| \\ &= \left| \frac{b}{\alpha Q} \frac{\delta U(Q, b, b) - \delta U(Q, a, b)}{1 + \delta U(Q, b, b)b/(\alpha Q)} \right| \\ &= \left| \frac{b}{\alpha} \frac{\delta U(1, b, b) - \delta U(1, a, b)}{1 + \delta U(1, b, b)b/\alpha} \right|. \end{aligned} \quad (3.3)$$

On the right-hand side it is understood that δU is the part arising from the non-Coulomb part δV . Using Eq. (2.3) for δV we can easily find,

$$\delta f(r) = \alpha^2 \epsilon^2 \frac{2}{3\pi} r \int_{2mr}^{\infty} dx \frac{1 - \exp(-x)}{x^2} \sqrt{1 - \frac{4(mr)^2}{x^2} \left(1 + \frac{2(mr)^2}{x^2}\right)} \equiv \alpha^2 \epsilon^2 r g(mr). \quad (3.4)$$

Inserting into (3.3) we find,

$$\gamma_{ab} = \frac{\alpha \epsilon^2}{2} \frac{2g(2bm) - (1+b/a)g[(a+b)m] + (b/a-1)g[(b-a)m]}{1 + \alpha \epsilon^2 g(2bm)}. \quad (3.5)$$

We expect the best bounds for small masses $m \ll 1/a, 1/b$. In this limit we have,

$$g(mr) \approx -\frac{2}{3\pi} \log(2mr), \quad \text{for } mr \ll 1. \quad (3.6)$$

In this regime we therefore have,

$$\gamma_{ab} = \frac{\alpha \epsilon^2}{3\pi} \left| \log\left(\frac{2b}{a+b}\right) + \log\left(\frac{2b}{b-a}\right) + \frac{b}{a} \log\left(\frac{b-a}{b+a}\right) \right| + \mathcal{O}[(\alpha \epsilon^2)^2] \sim 0.05 \alpha \epsilon^2 \quad ma, mb \ll 1, \quad (3.7)$$

where we have assumed $b/a \sim 2$ in the second line.

In such a setup, Plimpton and Lawton [19] achieved already in 1936 a sensitivity of the order $|\gamma_{ab}| \lesssim 3 \times 10^{-10}$. This corresponds to a bound of the order of $\epsilon \lesssim 9 \times 10^{-4}$. Later experiments used a somewhat more complicated setup with several spheres. The latest and most precise [20,21] uses four spheres (to be precise they used icosahedrons but we will approximate those as spheres) with radii $d > c > b > a$. A very high voltage is applied between the outer two and the voltage difference is mea-

sured between the innermost pair. One can easily derive the appropriate expression for this case,

$$\begin{aligned} \gamma_{abcd} &= \left| \frac{\mathcal{V}_b - \mathcal{V}_a}{\mathcal{V}_d - \mathcal{V}_c} \right| \\ &= \left| \frac{C[\delta U(b, c) - \delta U(a, c) - \delta U(b, d) + \delta U(a, d)]}{1 + C[\delta U(c, c) + \delta U(d, d) - 2\delta U(c, d)]} \right|, \end{aligned} \quad (3.8)$$

where $C^{-1} = \alpha(1/c - 1/d)$. For this quantity the experiment achieves a precision [20,21],

$$\gamma_{abcd} \lesssim 2 \times 10^{-16}, \quad \text{with } a = 60 \text{ cm}, \quad b = 94 \text{ cm}, \\ c = 94.7 \text{ cm}, \quad d = 127 \text{ cm.} \quad (3.9)$$

From our earlier discussion we expect $\epsilon \lesssim \text{few} \times 10^{-7}$ for small masses. For larger masses $m \gg 1/a$, we expect a rapid weakening of the bound due to the exponential decay of the Uehling potential [cf. Eq. (2.4)] in this regime. The bound obtained from a numerical evaluation of Eq. (3.8) is shown in Fig. 1 as the black area. We can see that for small masses this is the best laboratory bound that applies for minicharged particles with and without an additional massless hidden photon.

4. Conclusions.—In this Letter, we have shown that tests of Coulomb's law can be used to search for minicharged particles. Minicharged particles leave their imprint on the potential between charges via their contribution to the vacuum polarization resulting in the Uehling contribution to the potential. Very precise laboratory limits on deviations from the Coulomb potential arise from Cavendish type experiments. Using these we find a limit of $\epsilon \lesssim 5 \times 10^{-7}$ in the mass range below $0.1 \mu\text{eV}$ (cf., also, Fig. 1). In this mass range this is the best laboratory bound for minicharged particles in theories where the minicharge arises from kinetic mixing.

As a final note, we would like to point out that the Cavendish experiments [20] we used to derive our bounds were performed nearly 40 years ago. One could hope that with current technology significant improvements are possible. Therefore, precision tests of Coulomb's law may yet again help us to explore the frontier of new physics.

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