Erratum: Sixth-Order Vacuum-Polarization Contribution to the Lamb Shift of Muonic Hydrogen [Phys. Rev. Lett. 82, 3240 (1999)]

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In Eq. (25) of the Letter [1] we reported the contribution to the $2P_{1/2} - 2S_{1/2}$ Lamb shift of the muonic hydrogen due to exchange of 3 second-order electron-vacuum-polarization functions:

$$\Delta E(\text{Fig. 5c: Ref. [1]}) = 0.002\,535(1) \times m_r(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^3. \tag{1}$$

This term comes from the third-order perturbation theory of the nonrelativistic Coulomb-Schrödinger theory

$$\delta E_1 = \langle\phi_n|H_I G_n' H_I G_n' H_I|\phi_n\rangle, \qquad G_n' \equiv \sum_{k\neq n}\frac{|\phi_k\rangle\langle\phi_k|}{E_n - H_0}, \tag{2}$$

where H_0 and H_I are the nonperturbative and perturbative parts of the Hamiltonian, respectively. In the case of Fig. 5c, H_I is the Uehling potential.

Recently, Ivanov, Korzinin, and Karshenboim have pointed out that another type of third-order correction

$$\delta E_2 = \langle\phi_n|H_I|\phi_n\rangle\langle\phi_n|H_I G_n' G_n' H_I|\phi_n\rangle \tag{3}$$

should also be taken into account [2]. We agree with them that we overlooked this term, and evaluated it by ourselves, too. Our result is

$$\Delta E(\text{Fig. 5c: new}) = -0.001\,365\,8(18) \times m_r(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^3, \tag{4}$$

which confirms the value $-0.001\,3657$ given in Eq. (9) of Ref. [2]. [The same value is also given in Eq. (3) of Ref. [3].]

Combined with the previous result Eq. (1), the total contribution from Fig. 5c must be modified to

$$\Delta E(\text{Fig. 5c}) = 0.001\,169\,3(21) \times m_r(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^3. \tag{5}$$

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[1] T. Kinoshita and M. Nio, Phys. Rev. Lett. **82**, 3240 (1999).
[2] V. G. Ivanov, E. Y. Korzinin, and S. G. Karshenboim, Phys. Rev. D **80**, 027702 (2009).
[3] V. G. Ivanov, E. Y. Korzinin, and S. G. Karshenboim, arXiv:0905.4471.

