Isolated True Surface Wave in a Radiative Band on a Surface of a Stressed Auxetic

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We demonstrate that a surface resonance (pseudosurface wave) may transform into a true surface wave, i.e., acquire an infinite lifetime, at a single isolated point within a bulk band (radiative region) in a model of a stressed auxetic material. In contrast with the secluded supersonic elastic surface waves, the one found here does not belong to a dispersion line of true surface waves. Therefore we propose to call it an isolated true surface wave (ITSW). The ITSW manifests itself by a deltalike peak in the local density of states and by anomalies in reflection coefficients. The phenomenon may be useful in redirecting energy and/or information from the bulk to the surface in devices supporting guided acoustic waves.

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Auxetics are materials showing a negative Poisson ratio. They, thus, increase their transverse dimensions when stretched by a uniaxial tensile stress. The physical mechanisms responsible for this apparently counterintuitive [1,2] property often involve some slanted bonds or other quasi rigid segments that under a tensile stress incline away from the stretch axis [3–6]. The model treated here falls into this category of auxetics although a number of materials are known where a negative Poisson's ratio results from an interplay of occupied and empty electronic states [7].

Although the rigid segments seem to produce new, virtually interesting, dynamical effects due to their rotational degrees of freedom, the literature on the lattice dynamics of auxetics is not abundant. Sparavigna [8] has studied the out-of-plane motions in a structure resembling the generic reentrant honeycomb lattice [6] and found a complete stop band dependent on parameters of her system. The present authors have shown an analogous phononic behavior for in-plane motions in a similar model under external fields: dipolar [3] and quadrupolar [9]. The Poisson's ratio may then take any positive or negative value and, as a consequence, the transverse acoustic wave may become faster than the longitudinal one. The importance of surface effects in many applications of auxetics (e.g., in the arterial prostheses [10]) notwithstanding, the dynamics of surfaces of auxetics remains a practically untouched subject.

In the present Letter we show that the rotational degrees of freedom, essential for the negative Poisson's ratio in many auxetics, may result in a true surface wave (TSW) occurring at a single isolated point within a radiative region of frequency on a surface of a modified reentrant honeycomb structure. The phenomenon should be contradistinguished from the known secluded supersonic elastic surface waves (SSESW) [11,12]. The latter form an entire dispersion line of TSWs, whereas in our case the width of a generally finite-lifetime resonance, also called pseudosurface wave (PSW [13,14]) tends to zero and its height diverges to infinity at exactly one isolated point. Therefore, we propose to call it isolated true surface wave (ITSW). The present authors have already observed such a behavior in a 3D isotropic continuum coated with an intrinsically 2D surface layer [15] satisfying modified equations of motion of thin membranes [2]. The ITSW could then occur, be the underlying continuum auxetic or not, at any desired wave vector, small wave vectors included, provided that the 2D density and the stiffness of the surface layer be properly chosen. The boundary conditions adopted in Ref. [15] do not constitute, however, the vanishing-thickness limit of an elastic continuum [16] so that they should be treated as describing a specific metamaterial. Later on Every [17] reported on a similar effect for an interface consisting of parallel coplanar equidistant crevices in an otherwise homogeneous isotropic continuum. The crevices were geometrically infinitely narrow but at the same time wide enough as to allow the author to apply the free-surface boundary conditions. A surface resonance then narrowed to zero at a crossing point with a dispersion line of secluded supersonic interfacial waves close to the Brillouin zone border. Other true interfacial waves were found in Ref. [17] strictly at the zone center on an optical branch of surface resonances. The group velocities are low or just zero in both cases so that the propagation of these waves seems questionable.

The model treated here is systematically discrete without any need for artificial metamaterial-like assumptions. It is depicted in Fig. 1. The rigid rods of length $2d$, mass m, and moment of inertia τ represented by thick solid lines interact by elastic massless springs with the force constants α and β. The actual lengths l_α, l_β of the springs and the angle θ depend on the external stress. For the sake of specificity we take as a reference the geometry of the reentrant honeycomb structure [6], i.e., the one with the lattice parameters $a_0 = 3d$ and $b_0 = d\sqrt{3}$, so that $l_\alpha = l_\beta = d$ and $\theta = \pi/3$. Then, limiting ourselves to the tensile stresses σ_{11} and σ_{22}, that do not break the symmetry

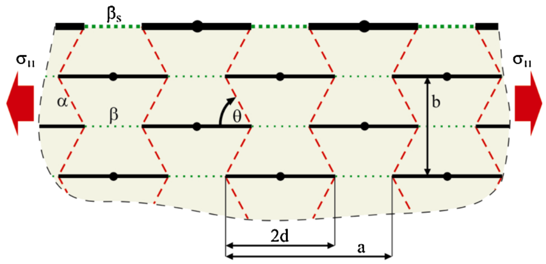

FIG. 1 (color online). Model of 2D auxetic. Rigid rods (thick solid lines) of length $2d$ interact by elastic force constants α (dashed line) and β (dotted line). Masses m_s, moments of inertia τ_s, and force constant β_s in the surface row may differ from their bulk counterparts m, τ and β.

$C2m$ of the system, we define the geometry of the structure by the actual lattice constants a and b or, equivalently, by the relative extensions $\epsilon_{11} = a/a_0 - 1$ and $\epsilon_{22} = b/b_0 - 1$. The actual lengths of the springs then are $l_\alpha = \frac{1}{2}d\sqrt{4 - 6\epsilon_{11} + 9\epsilon_{11}^2 + 6\epsilon_{22} + 3\epsilon_{22}^2}$ and $l_\beta = d(1 + 3\epsilon_{11})$, whereas the stresses σ_{11} and σ_{22} needed to keep the structure in equilibrium read

$$\sigma_{11} = \frac{2\beta l_\alpha(l_\beta - l_{\beta 0}) - \alpha(l_\alpha - l_{\alpha 0})(2d - l_\beta)}{b l_\alpha},$$

$$\sigma_{22} = \frac{b\alpha(l_\alpha - l_{\alpha 0})}{l_\alpha(2d + l_\beta)}.$$

In what follows the free lengths $l_{\alpha 0}$ and $l_{\beta 0}$ of the springs α and β are equal to d and $\alpha/\beta = 1$. The stress-free state then is that of the reentrant honeycomb geometry.

The study of the surface dynamics involves two steps. First the bulk modes are found for the spatially unbound material in the form of the Bloch waves $w_\mu(\boldsymbol{r}, t) = e_\mu(\boldsymbol{k}, \omega)e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{r}}$, where $\boldsymbol{k}$ is a wave vector from the first Brillouin zone, ω is a frequency and e_μ, $\mu = 1, 2, 3$ is the polarization vector. Indices $\mu = 1, 2$ correspond to the translational displacements of the rods and $\mu = 3$ to their angular displacements from the horizontal orientation. The Newton's equations of motion of the lattice then reduce to systems of homogeneous linear equations for the unknowns e_μ for every wave vector $\boldsymbol{k}$ separately. The frequencies $\omega_j(\boldsymbol{k})$, $j = 1, 2, 3$ of the lattice modes are obtained from the condition $\det M(\boldsymbol{k}, \omega) = 0$, where the $\boldsymbol{k}$-dependent 3×3 matrix $M(\boldsymbol{k}, \omega)$ is constructed according to standard textbooks [18]. Projecting the dispersion relations $\omega_j(\boldsymbol{k})$ onto the plane $(k_\parallel, \omega)$, where $k_\parallel$ is the wave vector component parallel to the surface, one obtains the radiative regions (bulk bands) for the corresponding orientation of the surface. Given a wave vector $k_\parallel$ and a frequency ω the equation $\det M(k_\parallel, k_\perp, \omega) = 0$ provides the wave vector component $k_\perp$ perpendicular to the surface. Real solutions for $k_\perp$ correspond to bulk waves occurring in the radiative regions, whereas complex solutions with $\mathrm{Im}k_\perp > 0$ describe near fields, also called evanescent partial waves, that decay exponentially with the distance from the surface into the bulk.

The second step in the analysis of the surface dynamics then is to find such combinations of the aforementioned bulk and evanescent partial waves that satisfy the equations of motion of the surface layer. The equations are generally different from those in the bulk because the surface rods lack neighbors on their vacuum side and can have different masses m_s, moments of inertia τ_s and the force constant β_s. The number of equations of motion of the surface layer equals the total number of the bulk waves and near fields for given $k_\parallel$ and ω. The vanishing of the determinant of these equations for frequencies outside the bulk bands defines the surface waves that manifest themselves by deltalike peaks in the local density of states (LDOS). Peaks of the LDOS occurring in bulk radiative regions are usually broad and finite in height. They correspond to surface resonances (pseudosurface waves, PSW), i.e., excitations with finite lifetimes.

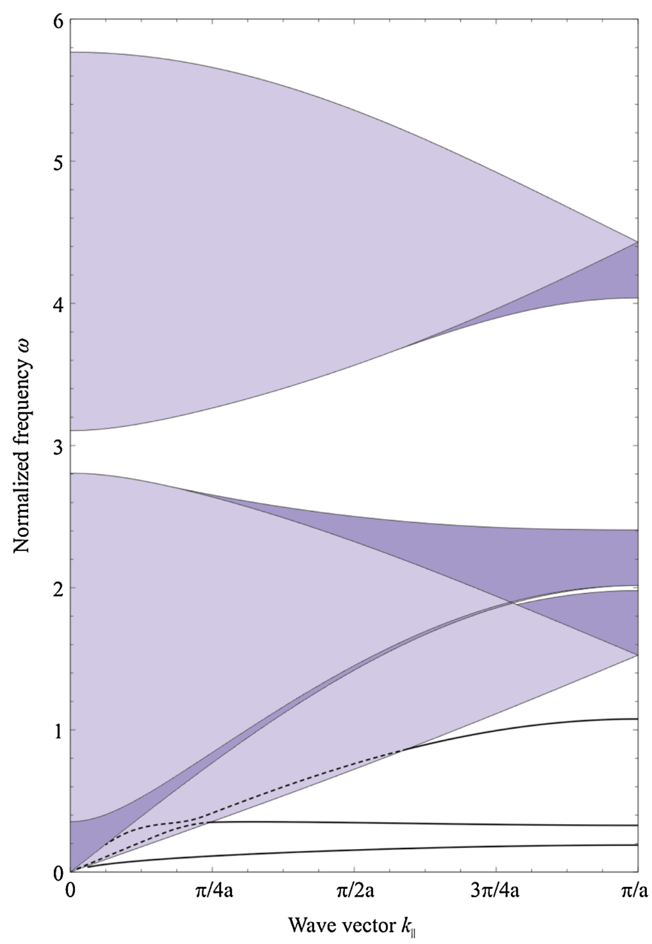

FIG. 2 (color online). Bulk bands, dispersion curves of surface waves (solid lines) and surface resonances (dashed lines) for (01) surface of model with $\alpha/\beta = 1$, $l_{\alpha 0} = l_{\beta 0} = d$, $\epsilon_{11} = \frac{1}{4}$, $\sigma_{22} = 0$, $\beta_s/\beta = 10$, $m_s/m = \tau_s/\tau \approx 34.61$, $\tau/m = d^2/3$. Light shaded areas correspond to single bulk wave and dark shaded areas correspond to two bulk waves. Frequency is normalized to $\sqrt{\beta/m}$.

Figure 2 represents the bulk bands and the dispersion curves of the surface waves on the (01) surface of our model. The frequency in this and in the following figures is normalized to $\sqrt{\beta/m}$. The bulk parameters are $l_{\alpha 0} = l_{\beta 0} = d$, $\epsilon_{11} = \frac{1}{4}$, $\epsilon_{22} = \frac{\sqrt{21}}{4} - 1$ that corresponds to $\frac{\sigma_{11}}{\beta} = \frac{2\sqrt{7}}{7}$ and $\sigma_{22} = 0$. The vanishing of the stress component σ_{22} is not necessary but it produces interesting effects on the wave reflection as discussed below. The dispersion curves of surface waves visible in Fig. 2 have been obtained with the following surface parameters: $\beta_s/\beta = 10$ and $m_s/m = \tau_s/\tau \approx 34.61\ldots$. With this rather specific choice the equations of motion of the surface layer admit a solution involving exclusively evanescent partial waves at a certain wave vector $k_\parallel = k_{\mathrm{ITSW}}$ and a frequency ω_{ITSW} within a bulk band. Figure 3 depicts the long wavelength and low frequency region of Fig. 2. One can notice a dispersion curve of surface waves entering the bulk band. Thus, a deltalike peak in the local density of states transforms into a broad maximum as is shown in Fig. 4. A Lorentzian function has been fitted to the peak at each wave vector $k_\parallel$ and the inverse of its half-width has been used to estimate the lifetime of the resonance. Figure 3 shows that the lifetime goes to infinity at the wave vector $k_\parallel = k_{\mathrm{ITSW}}$. The height of the peak in LDOS in Fig. 4 also tends to infinity at this point. The ITSW occurs at a single point in the reciprocal space in contrast with the secluded supersonic surface waves [11,12] forming an entire dispersion curve.

It is clear that the isolated true surface wave can be easily excited by an external perturbation of the same frequency and wave vector because it is unlikely that such a perturbation be just orthogonal to the displacements involved in this wave. The states of Fig. 4 are polarized in (10) direction so that the LDOS presented there is proportional to the transfer of power into the lattice under a unit external oscillatory force applied parallel to the surface rods. Because the ITSW is by definition decoupled from all the bulk waves, its effect on the reflection of waves coming to the surface from the material is expected for frequencies somewhat detuned from ω_{ITSW}. This is illustrated in Fig. 5. The slower acoustic wave of frequency ω_{ITSW} and unit amplitude arrives at the surface at some incidence angle and produces three reflected partial waves of amplitudes R_1, R_2 and R_3, respectively. The wave R_1 is also slower acoustic so that it satisfies the Snellius reflection law and radiates into the bulk for all incidence angles. The faster reflected wave R_2 becomes an exponentially evanescent near field beyond the point A in Fig. 5 and a damped-sinusoidal near field with period doubling in the (01) direction beyond the point C. For $\sigma_{22} = 0$ the partial wave R_3 behaves in a particular way. It, namely, decreases at an infinite rate so that it is entirely confined to the surface row of rods. Anomalies due to an ITSW occurring when the frequency is slightly detuned are depicted in the inset of Fig. 5 for $\omega = \omega_{\mathrm{ITSW}}(1 + 0.001)$. Both evanescent partial waves R_2 and R_3 show a sharp peak. Both near fields are,

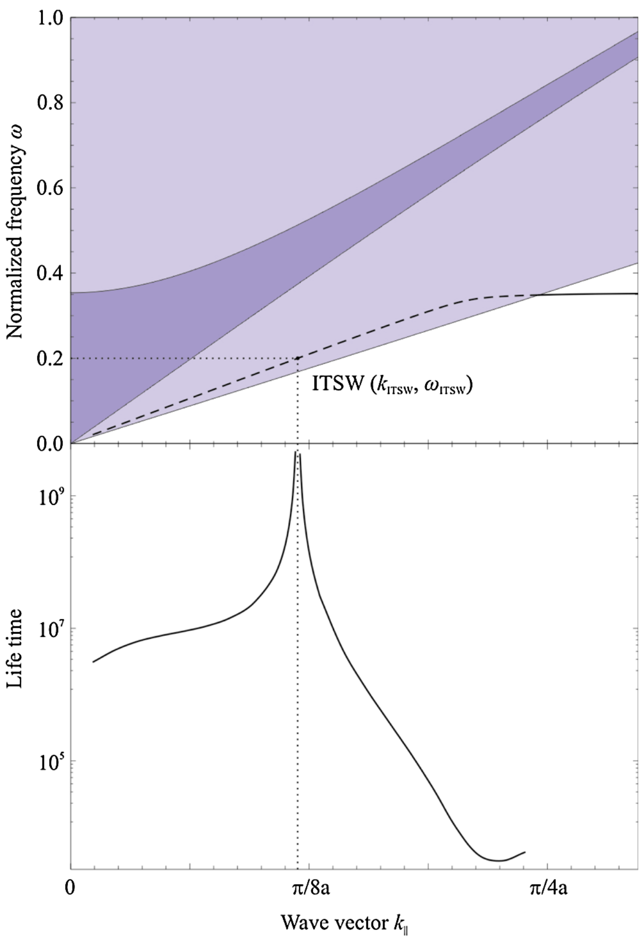

FIG. 3 (color online). Long wavelength and low frequency part of Fig. 2 (upper panel) and lifetime of surface resonance transforming into isolated true surface wave (ITSW, lower panel).

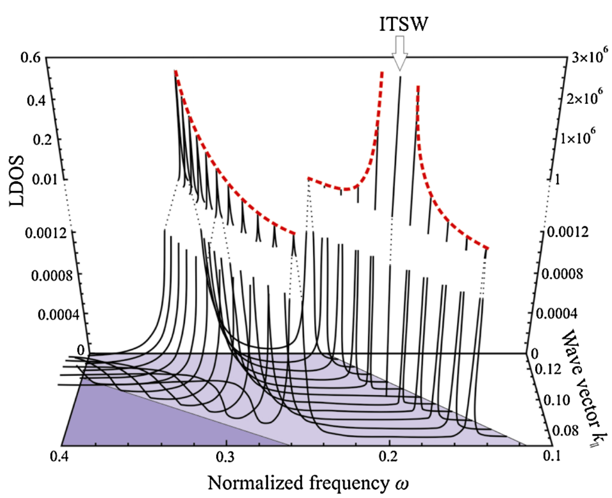

FIG. 4 (color online). Local density of states (LDOS) polarized in (10) direction. Maximum of LDOS tends to infinity and its width tends to zero at wave vector and at frequency corresponding to ITSW. Surface resonance at higher frequency shows no anomaly.

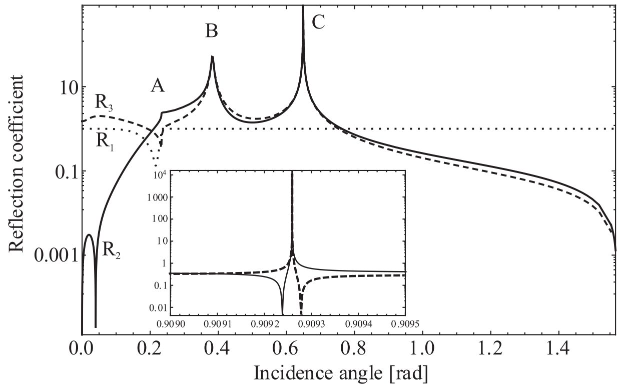

FIG. 5. Amplitudes of three waves: slower acoustic wave (dotted line), faster acoustic wave (dashed line), and near field confined totally in surface layer (solid line) arising from reflection of incident slower acoustic wave of unit amplitude as functions of incidence angle at frequency ω_{ITSW} corresponding to the ITSW. Inset: part of the same graph but for $\omega = \omega_{\mathrm{ITSW}}(1 + 0.001)$.

thus, particularly enhanced at the corresponding incidence. Moreover, the evanescent component R_2. is practically completely eliminated at a slightly lower and the component R_3 at a slightly higher incidence angle. An analogous result, with however, reversed order of the eliminated near fields has been obtained for frequencies lower than ω_{ITSW}.

The described isolated true surface wave occurs for surface parameters specific to the given external stress. Figure 6 shows the ratios m_s/m $(=\tau_s/\tau)$ and β_s/β admitting an ITSW at the same k_{ITSW} and ω_{ITSW} as in Figs. 2–5 as functions of the relative extension ϵ_{11} with respect to the stress-free reentrant honeycomb geometry at $\alpha/\beta = 1$. If $\beta_s/\beta \geq 10.0$ the existence range of the ITSW then is the largest possible and extends from $\epsilon_{11} \approx 0.00075$ i.e. $\theta \approx \pi/3 + 0.0013$ rad to $\epsilon_{11} \approx 0.33245$ i.e. $\theta \approx \pi/2 -$

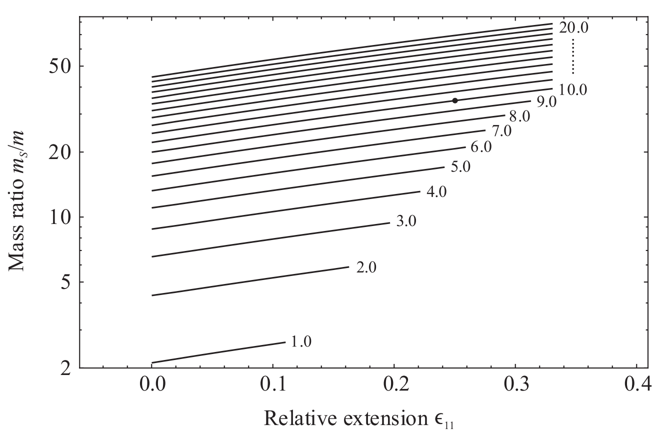

FIG. 6. Range of existence of the ITSW in terms of relative extension ϵ_{11} with respect to reentrant honeycomb structure. Each line gives required mass ratio m_s/m for force constants ratio β_s/β indicated on the right side. Dot corresponds to parameters used in Figs. 2–5.

0.0013 rad. All the ranges lie entirely in the auxetic region of the model, i.e., $\theta < \pi/2$. It is interesting that at weaker force constant β_s the surface mass m_s may be as small as about twice the mass m of the bulk rods. The extent of the existence region of the ITSW then is, however, narrower. ITSWs are not, of course, precluded in the nonauxetic region of the model with a different choice of $l_{\alpha 0}$, $l_{\beta 0}$ and/or α/β.

In summary, we show that a surface resonance can transform into an isolated true surface wave (ITSW), i.e., to acquire an infinite lifetime, at a well defined point within a bulk band on the (01) surface of a 2D auxetic lattice. The stress needed for an ITSW to exist depends on parameters of the surface. Reflection of certain bulk waves may then result in a concentration of the vibrational energy in a well defined evanescent partial wave. The phenomenon of an ITSW may be useful in transferring energy and/or information to a desired output (here along the surface) in some analogy with the acoustic multiplexer [19].

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