Testing New Indirect *CP* Violation

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If new *CP* violating physics contributes to neutral meson mixing, but its contribution to *CP* violation in decay amplitudes is negligible, then there is a model independent relation between four (generally independent) observables related to the mixing: the mass splitting (x), the width splitting (y), the *CP* violation in mixing ($1-|q/p|$), and the *CP* violation in the interference of decays with and without mixing (ϕ). For the four neutral meson systems, this relation can be written in a simple approximate form: $y\tan\phi \approx x(1-|q/p|)$. In the K system, all four observables have been measured and obey the relation to excellent accuracy. For the B_s and D systems, new predictions are provided. The success or failure of these relations will probe the physics that is responsible for the *CP* violation.

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Introduction.—The fact that the standard model depends on a single *CP* violating phase gives it a strong predictive power concerning *CP* asymmetries. The fact that *CP* is a good symmetry of the strong interactions makes the theoretical analysis of *CP* asymmetries often impressively clean. These theoretical advantages, combined with the huge experimental progress in the measurements of *CP* violation in B decays and in the search for *CP* violation in B_s and D decays, provide a powerful probe of new physics. Observing deviations from the standard model predictions will not only imply the existence of new physics, but also give detailed information about features of the required new physics.

CP violation in meson decays can be classified to indirect and direct *CP* violation. Indirect *CP* violation can be completely described by phases in the dispersive part of the neutral meson mixing amplitude (M_{12}). In contrast, direct *CP* violation requires that there are some phases in the decay amplitudes (A_f). Within the standard model, many *CP* asymmetries require—to an excellent approximation—only indirect *CP* violation. Examples include $K\to\pi\pi$, $B\to\psi K_S$, and $B_s\to\psi\phi$. This situation persists in many—though not all—extensions of the standard model.

Indirect *CP* violation can manifest itself in two ways: *CP* violation in mixing, which is the source of *CP* asymmetries in semileptonic decays, and *CP* violation in the interference of decays with and without mixing, which is often the dominant effect in decays into final *CP* eigenstates. When there is no direct *CP* violation, these two manifestations are not independent of each other. They are correlated in a way that depends on the mass and width splittings between the two neutral meson mass eigenstates. In this work, we derive this model independent relation, and analyze its applicability and implications in each of the four neutral meson systems (K, D, B, B_s).

The relation that we derive is rather unique in the sense that it involves only experimental observables. In particular, it does not depend on any hadronic parameters. This is in contrast to, for example, the use of ϵ that depends on hadronic parameters, or even the $S_{\psi K}=\sin 2\beta$ relation, where the constraints on $\sin 2\beta$ involve, again, hadronic uncertainties. It will clearly distinguish between two qualitatively different classes of models: those that give only indirect *CP* violation and those that affect also the *CP* violation in decay amplitudes.

The experimental parameters.—We refer here explicitly to the neural D system, but our formalism applies equally well to all four neutral meson systems. The two neutral D-meson mass eigenstates, $|D_1\rangle$ of mass m_1 and width Γ_1 and $|D_2\rangle$ of mass m_2 and width Γ_2, are linear combinations of the interaction eigenstates $|D^0\rangle$ (with quark content $c\bar{u}$) and $|\bar{D}^0\rangle$ (with quark content $\bar{c}u$):

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle. \tag{1}$$

The average and the difference in mass and width are given by

$$m\equiv\frac{m_1+m_2}{2}, \qquad \Gamma\equiv\frac{\Gamma_1+\Gamma_2}{2}, \qquad x\equiv\frac{m_2-m_1}{\Gamma}, \qquad y\equiv\frac{\Gamma_2-\Gamma_1}{2\Gamma}. \tag{2}$$

The decay amplitudes into a final state f are defined as $A_f=\langle f|\mathcal{H}|D^0\rangle$ and $\bar{A}_f=\langle f|\mathcal{H}|\bar{D}^0\rangle$. We define a complex dimensionless parameter λ_f:

$$\lambda_f=(q/p)(\bar{A}_f/A_f). \tag{3}$$

As concrete examples, consider the doubly-Cabibbo-suppressed decay $D^0\to K^+\pi^-$, the singly-Cabibbo-suppressed decay $D^0\to K^+K^-$, and the Cabibbo-favored decay $D^0\to K^-\pi^+$. Let us assume that effects of direct *CP* violation are negligibly small even in the presence of new physics. On the other hand, new physics could easily generate indirect *CP* violation. The effects of indirect

CP violation can be parametrized in the following way:

$$\begin{aligned} \lambda^{-1}_{K^+\pi^-} &= r_d|p/q|e^{-i(\delta_{K\pi}+\phi)}, \\ \lambda_{K^-\pi^+} &= r_d|q/p|e^{-i(\delta_{K\pi}-\phi)}, \\ \lambda_{K^+K^-} &= -|q/p|e^{i\phi}, \end{aligned} \tag{4}$$

where $r_d = |\bar{A}_{K^-\pi^+}/A_{K^-\pi^+}|$, $\delta_{K\pi}$ is a strong (CP conserving) phase, and ϕ is a weak (CP violating) universal phase.

Explicit expressions for the time-dependent decay rates can be found, for example, in Refs. [1,2]. The four parameters that are related to $D^0 - \bar{D}^0$ mixing—the two CP conserving parameters x and y and the two CP violating parameters $(1 - |q/p|)$ and ϕ—can be extracted by fitting to the experimentally measured time-dependent decay rates. We thus call them "experimental parameters."

The Theoretical Parameters.—The $\bar{D}^0 - D^0$ transition amplitudes are defined as follows:

$$\langle D^0|\mathcal{H}|\bar{D}^0\rangle = M_{12} - \frac{i}{2}\Gamma_{12}, \quad \langle \bar{D}^0|\mathcal{H}|D^0\rangle = M^*_{12} - \frac{i}{2}\Gamma^*_{12}. \tag{5}$$

The overall phase of the mixing amplitude is not a physical quantity. It can be changed by the choice of phase convention for the up and charm quarks. The relative phase between M_{12} and Γ_{12} is, however, phase convention independent and has physics consequences. The three physical quantities related to the mixing can be defined as

$$y_{12} \equiv |\Gamma_{12}|/\Gamma, \quad x_{12} \equiv 2|M_{12}|/\Gamma, \quad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12}). \tag{6}$$

Given a particle physics model, one can calculate the three parameters y_{12}, x_{12}, and ϕ_{12} as a function of the model parameters. We thus call them "theoretical parameters." Note that y_{12} is generated by final states that are common to D^0 and $\bar{D}^0$ decays. Thus it is very likely that it is described to a very good approximation by standard model physics (see, however, [3]). On the other hand, x_{12} and ϕ_{12} can be affected by new physics parameters.

From Theory to Experiment.—The following expressions give the experimental parameters in terms of the theoretical ones:

$$\begin{gathered} xy = x_{12}y_{12}\cos\phi_{12}, \qquad x^2 - y^2 = x^2_{12} - y^2_{12}, \\ (x^2 + y^2)|q/p|^2 = x^2_{12} + y^2_{12} + 2x_{12}y_{12}\sin\phi_{12}, \\ x^2\cos^2\phi - y^2\sin^2\phi = x^2_{12}\cos^2\phi_{12}. \end{gathered} \tag{7}$$

To obtain the last relation, we took into account the fact that, in the absence of direct CP violation, we have for final CP eigenstates

$$\mathrm{Im}\,(\Gamma^*_{12}\bar{A}_f/A_f) = 0, \qquad |\bar{A}_f/A_f| = 1. \tag{8}$$

The relations that we derive below depend crucially on this condition. Even if, in general, there is direct CP violation in some decays, our relations apply for those modes where Eq. (8) holds.

From Experiment to Theory.—Given experimental constraints on x, y, $|q/p|$, and ϕ, we can use Eq. (7) to constrain x_{12} and ϕ_{12} and subsequently the new physics model parameters. In particular, we derived the following equations for each of x_{12} and ϕ_{12}, first in terms of x, y, and ϕ:

$$\begin{aligned} x^2_{12} &= \frac{x^4\cos^2\phi + y^4\sin^2\phi}{x^2\cos^2\phi - y^2\sin^2\phi}, \\ \sin^2\phi_{12} &= \frac{(x^2 + y^2)^2\cos^2\phi\sin^2\phi}{x^4\cos^2\phi + y^4\sin^2\phi}, \end{aligned} \tag{9}$$

and, second, in terms of x, y, and $|q/p|$:

$$\begin{aligned} x^2_{12} &= x^2\frac{(1 + |q/p|^2)^2}{4|q/p|^2} + y^2\frac{(1 - |q/p|^2)^2}{4|q/p|^2}, \\ \sin^2\phi_{12} &= \frac{(x^2 + y^2)^2(1 - |q/p|^4)^2}{16x^2y^2|q/p|^4 + (x^2 + y^2)^2(1 - |q/p|^4)^2}. \end{aligned} \tag{10}$$

A Model Independent Relation.—The fact that we are able to express the four experimental parameters in terms of three theoretical ones means that the experimental parameters fulfill a model independent relation. It depends solely on our assumption that direct CP violation can be neglected.

The relation can be extracted from Eqs. (9) and (10):

$$\frac{(1 - |q/p|^4)^2}{\sin^2\phi} = \frac{16(y/x)^2|q/p|^4 + [1 + (y/x)^2]^2(1 - |q/p|^4)^2}{1 + (y/x)^4\tan^2\phi}. \tag{11}$$

The relation becomes very simple in two limits. Fortunately, each of the four neutral meson systems is subject to at least one of these two approximations. First, consider a system where

$$y_{12} \ll x_{12}. \tag{12}$$

This approximation applies to the B and B_s systems. It gives, to leading order in y_{12}/x_{12}:

$$\begin{gathered} y/x = \cos\phi_{12}y_{12}/x_{12}, \qquad |q/p| - 1 = (y_{12}/x_{12})\sin\phi_{12}, \\ \tan\phi = -\tan\phi_{12}. \end{gathered} \tag{13}$$

The derivation of the sign for the CP violating observables starts from the definition of q/p (see, for example, [4]).

Second, consider a system where CP violation is small,

$$|\sin\phi_{12}| \ll 1. \tag{14}$$

This situation applies to the K system. Very recent measurements imply that it also applies (with limits of order 0.2) to the D system [5]. We obtain, to leading order in $|\sin\phi_{12}|$,

$$y/x = \mathrm{sgn}(\cos\phi_{12}) y_{12}/x_{12}, \quad |q/p| - 1 = \frac{(y/x)\tan\phi_{12}}{1+(y/x)^2},$$
$$\tan\phi = \frac{-\tan\phi_{12}}{1+(y/x)^2}. \qquad (15)$$

The two sets of equations, (13) and (15), lead to the same simple relation:

$$\frac{y}{x} = \frac{1-|q/p|}{\tan\phi}. \qquad (16)$$

Equation (16) is the main theoretical result of this work. If it is found to be violated, then new physics will have to provide not only indirect CP violation, but also a direct one. That would exclude many classes of candidate theories.

In what follows, we analyze the applicability and implications of this relation in each of the four neutral meson systems.

$K^0 - \bar{K}^0$ mixing.—The two ingredients that go into the relation (16)—small CP violation and the absence of direct CP violation—hold in the $K \to \pi\pi$ decays. Thus, this relation should hold in the neutral K system. Neglecting direct CP violation, and defining

$$A_0 = \langle(\pi\pi)_{I=0}|\mathcal{H}|K^0\rangle, \qquad \lambda_0 = (q/p)(\bar{A}_0/A_0), \quad (17)$$

the CP violating ϵ parameter corresponds to [6]

$$\epsilon = \frac{1-\lambda_0}{1+\lambda_0}. \qquad (18)$$

Then we have

$$\mathrm{Re}\,(\epsilon) \approx \tfrac{1}{2}(1-|q/p|), \qquad \mathrm{Im}(\epsilon) \approx -\tfrac{1}{2}\tan\phi. \quad (19)$$

The relation (16) translates into the prediction

$$\arg(\epsilon) \approx \arctan(-x/y) = 43.5°, \qquad (20)$$

where, for the numerical value, we used [7] $\Delta m_K = 0.5290 \times 10^{10}\ \mathrm{s}^{-1}$ and $\Delta\Gamma_K = -1.1163 \times 10^{10}\ \mathrm{s}^{-1}$. Indeed, the experimental value is [7]

$$\arg(\epsilon) = 43.51 \pm 0.05°. \qquad (21)$$

Thus, the relation (16) is tested in the neutral kaon system and works very well.

$B^0 - \bar{B}^0$ mixing.—In the neutral B system, the width difference is constrained to be small (and consistent with zero within the present accuracy), $\Delta\Gamma/\Gamma = 0.01 \pm 0.04$, while the mass splitting is measured to be much larger, $\Delta m/\Gamma = 0.78 \pm 0.01$ [7]. Thus $y_{12}/x_{12} \ll 1$ and Eqs. (13) apply. One has to note, however, that the equation for ϕ holds only for modes where Eq. (8) applies. Since [8–12]

$$\arg(\Gamma_{12}) \approx \arg[(V_{tb}V_{td}^*)^2], \qquad (22)$$

the phase ϕ relates to modes whose phase is dominated by $\arg(V_{tb}V_{td}^*)$. [The weak phase of $B \to \psi K_S$ is dominated by $\arg(V_{cb}V_{cd}^*)$ and, therefore, $S_{\psi K_S}$ cannot be used to test (16).] The problem is that the approximation (22) gives $1-|q/p| = 0$ and $\phi = 0$, so that $y\tan\phi = x(1-|q/p|)$ is fulfilled in a rather trivial way.

If one wants to go beyond (22), the large relative phase between $V_{tb}V_{td}^*$ and $V_{cb}V_{cd}^*$ has to be taken into account. It enters Γ_{12} and $\bar{A}_f/A_f$ in different ways, and thus direct CP violation plays a role and (16) is violated. Nevertheless, the relation (16) could in principle provide interesting predictions if M_{12} had significant contributions from new physics carrying a new phase. Experimental data constrain, however, such contributions to be smaller than $\mathcal{O}(0.2)$ [13,14], which is the same order as the direct CP violating effects in Γ_{12} [8–12].

$B_s - \bar{B}_s$ mixing.—Within the standard model (SM), the discussion of the B_s system follows a line of reasoning that is very similar to our discussion of the B_d system. However, in contrast to the B_d system, a situation where the indirect CP violation is entirely dominated by new physics in M_{12} is still possible for $B_s - \bar{B}_s$ mixing. Actually, recent measurements in D0 and CDF provide hints at a level higher than 2σ that this is indeed the case [5]. If so, then the relation (16) provides a very interesting probe of the new physics. Neglecting $\beta_s = \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$, the relation reads

$$A_{\mathrm{SL}}^s = -\mathrm{sgn}(\cos\phi)(2y/x)S_{\psi\phi}/(1-S_{\psi\phi}^2)^{1/2}$$
$$= -2|y/x|S_{\psi\phi}/(1-S_{\psi\phi}^2)^{1/2} \qquad (23)$$

where A_{SL}^s is the CP asymmetry in semileptonic (SL) decays, and $S_{\psi\phi}$ is the CP violating parameter in the decays into $(\psi\phi)_{CP=+}$. The second equality assumes that neither Γ_{12} nor $b \to c\bar{c}s$ decays are significantly affected by new physics, which implies that $\mathrm{sgn}(y\cos\phi) = \mathrm{sgn}(y\cos\phi)^{\mathrm{SM}} = +1$. The experimental data read [7] $\Delta\Gamma/\Gamma = -0.07 \pm 0.06$, $\Delta m/\Gamma = 26.1 \pm 0.5$, which give

$$y/x = -0.0014 \pm 0.0012. \qquad (24)$$

If the central value is approximately correct, then $S_{\psi\phi} = \mathcal{O}(0.3)$ would imply $A_{\mathrm{SL}}^s = \mathcal{O}(-10^{-3})$. We can expect a significant improvement in the measurements of y and of $S_{\psi\phi}$. (Hopefully, the hints for a signal in $S_{\psi\phi}$ will not disappear as the experimental accuracy improves.) Then, we will obtain a much sharper prediction for A_{SL}^s. A failure of this test would imply that the new physics introduces both direct and indirect CP violation.

A relation very similar to (23) was previously presented in Refs. [15,16]. Their relation can be written as $A_{\mathrm{SL}}^s/S_{\psi\phi} = \mathrm{Re}(\Gamma_{12}^{\mathrm{SM}}/M_{12}^{\mathrm{SM}})|M_{12}^{\mathrm{SM}}/M_{12}|$. What we add here to their results are the following two points: (1) The right-hand side of this relation, which is calculated from theory, can be replaced by the experimentally measurable factor $-2y/(x\cos\phi)$. Thus, this becomes a theory-independent (in both the electroweak model and QCD uncertainty aspects) relation. (2) We make it clear that a failure of this relation must imply new direct CP violation.

$D^0 - \bar{D}^0$ mixing.—Within the standard model, CP violation in $D^0 - \bar{D}^0$ is negligibly small (see, for example,

[17]). Thus, any signal of CP violation requires new physics. It is quite likely that such new physics will contribute negligibly to tree level decay amplitudes, though new direct CP violation is not impossible [18]. Measurements of the time-dependent decay rates will allow us to extract ϕ and $1-|q/p|$ and put (16) to the test.

Experimentally, there has been a very significant progress in determining the mixing parameters in the neutral D system [5]:

$$x=(1.00\pm 0.25)\times 10^{-2}, \qquad y=(0.77\pm 0.18)\times 10^{-2},$$
$$1-|q/p|=+0.06\pm 0.14, \qquad \phi=-0.05\pm 0.09. \tag{25}$$

The CP violating parameters are constrained to be small, and consistent with zero. In case, however, that CP violation is observed in the future, the fact that

$$y/x\approx 0.8\pm 0.3 \tag{26}$$

suggests that the CP violation in mixing is comparable in size to the CP violation in the interference of decays with and without mixing. Whether or not the relation (16) is fulfilled will teach us about the new physics and will disfavor or support models of the type discussed in Ref. [18], where direct CP violation can be generated.

Conclusions.—CP asymmetries in neutral meson decays where direct CP violation is negligible obey a relation. The relation involves four experimentally measurable parameters and is thus independent of the electroweak model and clean of QCD uncertainties. It applies to neutral K and D decays in the form (16). If new physics provides a large phase to $B_s-\bar{B}_s$ mixing, then the same relation applies also to B_s decays.

The phenomenological implications of this relation are the following: (i) The relation is already successfully tested in K decays. (ii) If a large CP violating effect is measured in $B_s\to\psi\phi$, then there is a clear prediction for the CP asymmetry in semileptonic decays $A^s_{\rm SL}$ that is strongly enhanced compared to the SM. (iii) If, for neutral D decays, CP violation in either mixing or the interference of decays with and without mixing is observed, there is a clear prediction for CP violation of the other type, of comparable size. (iv) If the relation fails in D decays, it will be an unambiguous evidence that the new physics generates also CP violation in the decay amplitudes.

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