Spin Torque and Waviness in Magnetic Multilayers: A Bridge between Valet-Fert Theory and Quantum Approaches

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We develop a simple theoretical framework for transport in magnetic multilayers, based on the Landauer-Buttiker scattering formalism and random matrix theory. A simple transformation allows one to go from the scattering point of view to theories expressed in terms of local currents and the electromagnetic potential. In particular, our theory can be mapped onto the well-established classical Valet-Fert theory for collinear systems. For noncollinear systems, in the absence of spin-flip scattering, our theory can be mapped onto the generalized circuit theory. We apply our theory to the angular dependence of spin accumulation and spin torque in noncollinear spin valves.

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The discovery of Giant Magnetoresistance (GMR) [1] and its subsequent counterpart, the current induced spin torque [2], was at the origin of a new field which aims at controlling the magnetization dynamics of small metallic devices through standard electronics. The theory of transport in those systems is now well developed and includes a number of approaches that range from classical Valet-Fert (VF) theory in the diffusive regime [3], the Boltzmann equation [4] to quantum approaches (original [5] and generalized [6]), circuit theory, random matrix theory (RMT) for the scattering matrix [7], and *ab initio* based models [8,9]. Many connections exist between these different approaches. One popular route [10] starts from the Keldysh Green's function formalism. In the quasiclassical approximation it yields the Boltzmann equation and the (VF) diffusive equation. A different strategy where local equilibrium is assumed only in certain points leads to circuit theory. An alternative route is the Landauer-Büttiker formalism which expresses the problem of transport inside a quantum conductor as a scattering problem. This approach is well suited for a coherent system and is equivalent to the Keldysh approach. However, the classical concepts of chemical potential or local equilibrium do not arise naturally in the scattering approach so that classical intuitions do not easily transfer into its language.

In this Letter, we take the scattering formalism as our starting point and develop a theory which fully captures VF and (generalized) circuit theory. Our theory (here after referred as C-RMT for Continuous Random Matrix Theory) can be tabulated by the same set of (experimentally accessible) parameters as VF [11]. On the other hand, it properly includes Sharvin resistance and allows for noncollinear and even a one-dimensional texture of magnetization (i.e., domain walls). We apply C-RMT to the discussion of the angular dependence of spin torque ("waviness" [12,13]) in asymmetric spin valves, see Fig. 1.

Two quantum conductors in series.—Before introducing C-RMT, we start with the pedagogical example of transport in a nonmagnetic metal [14,15]. C-RMT is a mere extension of the concepts developed below to properly take into account the electronic spins. In a coherent quantum system, the transport properties can be characterized by the scattering matrix S_A,

$$S_A = \begin{pmatrix} r'_A & t_A \\ t'_A & r_A \end{pmatrix}, \tag{1}$$

where r_A and t_A (t'_A and r'_A) describe the reflection or transmission amplitudes to the right (left) of the sample. When putting two conductors A and B in a series, one has to add the amplitudes of the direct transmission process $t_B t_A$, of the process with one reflection on each conductor $t_B(r_A r'_B)t_A$, and so on. This leads to a geometrical series which can be resumed into

$$t_{AB} = t_B[1 - r_A r'_B]^{-1} t_A \tag{2}$$

$$r_{AB} = r_B + t_B[1 - r_A r'_B]^{-1} r_A t'_B. \tag{3}$$

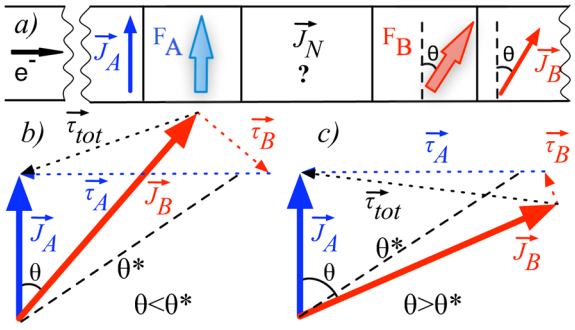

FIG. 1 (color online). (a) Schematic of a spin valve with two ferromagnetic layers F_A and F_B whose magnetization makes an angle θ. (b) and (c): geometric construction of the spin torque, see text. $\vec{J}_A$, $\vec{J}_N$, and $\vec{J}_B$ are the spin currents along the valve while $\vec{\tau}_A$ and $\vec{\tau}_B$ are the torque on the two magnetic layers.

The conductance of such a system is given by the Landauer formula $g_{AB} = (e^2/h)T_{AB}$ where $T_{AB} = |t_{AB}|^2$ is the probability for an electron to be transmitted. In the semiclassical limit for a large number $N_{\rm ch}$ of propagating channels, one gets,

$$T_{AB} = T_B(1 - R_A R'_B)^{-1} T_A \tag{4}$$

which has a simple interpretation [similar to Eq. (2)] in term of adding the *probabilities* of the various transmission/reflection processes [15] (Here, $R_A = |r_A|^2 = 1 - T_A$ is the reflection probability). Equation (4) can be recast into $\mathcal{R}_{AB} = \mathcal{R}_A + \mathcal{R}_B - \mathcal{R}_{\rm sh}$ where we have introduced the Sharvin resistance $\mathcal{R}_{\rm sh} = h/(e^2 N_{\rm ch})$. This addition law for resistances is very close to Ohm law except for the presence of the Sharvin (or contact) resistance. An important point here is that "Ohm law" is derived for fully coherent conductors. There is no need for a well-defined chemical potential (local equilibrium) in between the two conductors even though everything happens as if such a chemical potential existed.

RMT.—We now proceed with the extension of the above ideas to magnetic systems where the S matrix now includes a spin grading. Our starting point is an extension of RMT that was introduced in Ref. [7]. The theory has a structure almost identical to the scattering approach except that a conductor (or a part of it) is now described by 4×4 "hat" matrices $\hat{r}$, $\hat{r}'$, $\hat{t}$, and $\hat{t}'$ defined in term of the $(4N_{\rm ch} \times 4N_{\rm ch})$ S matrix,

$$\hat{r} = \frac{1}{N_{\rm ch}} {\rm Tr}_{N_{\rm ch}} \begin{pmatrix} r_{\uparrow\uparrow} r_{\uparrow\uparrow}^\dagger & r_{\uparrow\uparrow} r_{\uparrow\downarrow}^\dagger & r_{\uparrow\downarrow} r_{\uparrow\uparrow}^\dagger & r_{\uparrow\downarrow} r_{\uparrow\downarrow}^\dagger \\ r_{\uparrow\uparrow} r_{\downarrow\uparrow}^\dagger & r_{\uparrow\uparrow} r_{\downarrow\downarrow}^\dagger & r_{\uparrow\downarrow} r_{\downarrow\uparrow}^\dagger & r_{\uparrow\downarrow} r_{\downarrow\downarrow}^\dagger \\ r_{\downarrow\uparrow} r_{\uparrow\uparrow}^\dagger & r_{\downarrow\uparrow} r_{\uparrow\downarrow}^\dagger & r_{\downarrow\downarrow} r_{\uparrow\uparrow}^\dagger & r_{\downarrow\downarrow} r_{\uparrow\downarrow}^\dagger \\ r_{\downarrow\uparrow} r_{\downarrow\uparrow}^\dagger & r_{\downarrow\uparrow} r_{\downarrow\downarrow}^\dagger & r_{\downarrow\downarrow} r_{\downarrow\uparrow}^\dagger & r_{\downarrow\downarrow} r_{\downarrow\downarrow}^\dagger \end{pmatrix}, \tag{5}$$

with similar definitions for $\hat{r}'$, $\hat{t}$, and $\hat{t}'$. The main result of Ref. [7] is that the addition law for the "hat" matrices is exactly given by Eqs. (2) and (3) except that now the transmission and reflection matrices are to be replaced by their "hat" counterparts. The conductance is given by $g = (\hat{t}_{11} + \hat{t}_{14} + \hat{t}_{41} + \hat{t}_{44})/\mathcal{R}_{\rm sh} = (1/\mathcal{R}_{\rm sh}) \sum_{\sigma\sigma'} T_{\sigma\sigma'}$, where $T_{\sigma\sigma'} = (1/N_{\rm ch}) {\rm Tr} t_{\sigma\sigma'} t_{\sigma\sigma'}^\dagger$ is the probability for an electron with spin σ' to be transmitted with a spin σ. The "hat" matrices for the interfaces between two metals can be obtained from *ab initio* calculation [16,17], or tabulated from the experiments (see below).

C-RMT.—To obtain the "hat" matrices for the bulk parts of the metallic layers, we need to proceed further. Assuming we know the "hat" matrix $\hat{S}(L)$ for a system of size L, we can add an infinitely small layer of size δL and compute $\hat{S}(L + \delta L)$ from the knowledge of $\hat{S}(\delta L)$ and the addition law for "hat" matrices. The generic form for $\hat{S}(\delta L)$ is given by

$$\hat{t} = 1 - \Lambda^t \delta L, \qquad \hat{r} = \Lambda^r \delta L \tag{6}$$

where the two matrices Λ^t and Λ^r entirely characterize the bulk properties of the material. Expanding the addition law Eq. (2) in δL provides

$$\partial \hat{r}/\partial L = \Lambda^r - \Lambda^t \hat{r} - \hat{r}\Lambda^t + \hat{r}\Lambda^r \hat{r} \tag{7}$$

$$\partial \hat{t}/\partial L = -\Lambda^t \hat{t} + \hat{r}\Lambda^r t. \tag{8}$$

The various elements entering in the "hat" matrices (say $\hat{t}$) are vastly inequivalent. The main elements are the probabilities $T_{\sigma\sigma'}$. Second comes the element on the diagonal, the so called (complex) mixed transmission $T_{\rm mx} = (1/N_{\rm ch}) {\rm Tr} t_{\uparrow\uparrow} t_{\downarrow\downarrow}^\dagger$ which measures how a spin transverse to the magnetic layer can be transmitted through the system. These elements are usually small [17,18] in magnetic systems, but can play a role in noncollinear configurations nevertheless. Last, the other elements involve some coherence between spin-flip and non spin-flip processes and are likely to be even smaller. In the basis parallel to a layer magnetization, they can be disregarded. We parametrize a bulk layer by four parameters $\Gamma_\uparrow$, $\Gamma_\downarrow$ $\Gamma_{\rm sf}$, and $\Gamma_{\rm mx}$,

$$\Lambda^t = \begin{pmatrix} \Gamma_\uparrow + \Gamma_{\rm sf} & 0 & 0 & -\Gamma_{\rm sf} \\ 0 & \Gamma_{\rm mx} & 0 & 0 \\ 0 & 0 & \Gamma_{\rm mx}^* & 0 \\ -\Gamma_{\rm sf} & 0 & 0 & \Gamma_\downarrow + \Gamma_{\rm sf} \end{pmatrix} \tag{9}$$

$l_\sigma = 1/\Gamma_\sigma$ is the mean-free path for spin σ. In a ferromagnet, $\Gamma_{\rm mx} = 1/l_\perp + i/l_L$ where $l_\perp$ is the penetration length of transverse spin current inside the magnet while l_L is the Larmor precession length. Those lengths, which are roughly equal, are the smallest characteristic lengths with typical values smaller than 1 nm. In a normal metal, $\Gamma_\uparrow = \Gamma_\downarrow = \Gamma$ and $\Gamma_{\rm mx} = \Gamma + 2\Gamma_{\rm sf}$ so that the "hat" matrices remain invariant upon arbitrary rotation of the spin quantization axis. Λ^r is given by the same parametrization as Eq. (9) with $\Gamma_{\rm sf}$ being replaced by $-\Gamma_{\rm sf}$ (in order to fulfill current conservation) and neglecting $\Gamma_{\rm mx}$ for ferromagnets (as the mixing conductance is essentially of ballistic origin). This completes the formulation of the theory. Equations (7) and (8) can be integrated and a given multilayer is then constructed by using the addition law Eqs. (2) and (3) for the various bulk layers and the corresponding interfaces.

Link with Valet-Fert theory.—Let us introduce the 4-vector $\mathbf{P}_\pm(x) = (P_{\pm,\uparrow}, P_{\pm,{\rm mx}}, P^*_{\pm,{\rm mx}}, P_{\pm,\downarrow})$ where $P_{+,\uparrow}$ ($P_{-,\downarrow}$) is the probability to find a left (right) moving electron with spin up (down) at point x. The addition law Eqs. (2) and (3) (for "hat" matrices) is equivalent to state that $\mathbf{P}_\pm(x_1)$ and $\mathbf{P}_\pm(x_2)$ on two sides of a conductor A are related through its "hat" scattering matrix $\hat{S}_A$ as

$$\begin{bmatrix} \mathbf{P}_-(x_1) \\ \mathbf{P}_+(x_2) \end{bmatrix} = \hat{S}_A \begin{bmatrix} \mathbf{P}_+(x_1) \\ \mathbf{P}_-(x_2) \end{bmatrix}. \tag{10}$$

In the physical picture where one represents the scattering processes as random events with certain transmission or reflection probabilities, the above equation has a simple interpretation when one focuses on its first and fourth raw $P_{\pm,\uparrow}$ and $P_{\pm,\downarrow}$: it accounts for the conservation of probability in the scattering events; i.e., it is the master equation of the underlying Brownian motion undertaken by the incident electrons. Let us now introduce two new 4-vectors,

$\mathbf{j}(x)$ and $\boldsymbol{\mu}(x)$ defined as

$$\mathbf{j}(x) = [\mathbf{P}_+(x) - \mathbf{P}_-(x)]/(e\mathcal{R}_{\rm sh}) \tag{11}$$

$$\boldsymbol{\mu}(x) = [\mathbf{P}_+(x) + \mathbf{P}_-(x)]/2 \tag{12}$$

(with $e < 0$). Now, using the parametrization Eq. (6) and (9) for infinitely thin layer, and writing Eq. (10) in term of $\mathbf{j}(x)$, $\boldsymbol{\mu}(x)$, we arrive at

$$j_\sigma = -1/(e\Gamma_\sigma \mathcal{R}_{\rm sh})\partial_x \mu_\sigma \tag{13}$$

$$\partial_x j_\sigma = 4\Gamma_{\rm sf}/(e\mathcal{R}_{\rm sh})[\mu_{-\sigma} - \mu_\sigma] \tag{14}$$

which are precisely the VF equations [3]. Hence, for a collinear system, C-RMT simply reduces to VF theory. In its original form, however, VF theory does not account for the presence of Sharvin resistance. Here, the boundary conditions are given by the Landauer formula: the presence of a potential drop eV between the reservoirs located at, say, $x = 0$ and $x = L$, imposes $P_{+\sigma}(0) = eV$ and $P_{-\sigma}(L) = 0$ which translates into

$$\mu_\sigma(0) + (e\mathcal{R}_{\rm sh}/2)j_\sigma(0) = eV \tag{15}$$

$$\mu_\sigma(L) - (e\mathcal{R}_{\rm sh}/2)j_\sigma(L) = 0. \tag{16}$$

These mixed boundary conditions allows VF theory to properly include the Sharvin resistance of the system and correspond to adding half of the Sharvin resistance on both sides of the sample. For a collinear system without interface spin-flip scattering, we also recover the results of Ref. [19].

Link with generalized circuit theory.—Let us now consider Eq. (10) for a conductor whose transmission and reflection matrices are purely diagonal, i.e., without any spin-flip scattering. Let us further suppose that $R_{\rm mx}$ might be nonzero but $T_{\rm mx} = 0$. Then, Eq. (10) takes the form

$$j_\sigma(x_1) = j_\sigma(x_2) = \frac{1}{e\mathcal{R}_{\rm sh}}\frac{T_\sigma}{1-T_\sigma}[\mu_\sigma(x_1) - \mu_\sigma(x_2)] \tag{17}$$

$$j_{\rm mx}(x_{1,2}) = \pm\frac{2}{e\mathcal{R}_{\rm sh}}\frac{1-R^{1,2}_{\rm mx}}{1+R^{1,2}_{\rm mx}}\mu_{\rm mx}(x_{1,2}) \tag{18}$$

where T_σ is the transmission of an electron with spin σ and $R^{1,2}_{\rm mx}$ are the mixing reflections from left to left ($R^1_{\rm mx}$) and right to right ($R^2_{\rm mx}$). Equations (17) and (18) define the generalized circuit theory [6] so that in the absence of spin-flip scattering, C-RMT and generalized circuit theory are completely equivalent. In fact, the renormalization coefficients of generalized circuit theory [6] were chosen such that the calculation of the conductance with RMT and generalized circuit theory fully agree with each other. We find that the point of view of scattering taken in this Letter is fully equivalent to the alternative view in term of local current and chemical potential (VF, circuit theory), and one can change from one to the other simply using Eqs. (11) and (12).

Tabulation of C-RMT with VF set of parameters.—As C-RMT and VF are equivalent, we can use the huge corpus of experimental data that has been interpreted within VF to parametrize C-RMT. The VF resistivities $\rho_{\uparrow(\downarrow)} = 2\rho^*(1 \mp \beta)$ and spin-diffusion length $l_{\rm sf}$ are in a one-to-one correspondence with C-RMT: $1/l_{\rm sf} = 2\sqrt{\Gamma_{\rm sf}}\sqrt{\Gamma_\uparrow + \Gamma_\downarrow}$, $\beta = (\Gamma_\downarrow - \Gamma_\uparrow)/(\Gamma_\uparrow + \Gamma_\downarrow)$, and $\rho^*/\mathcal{R}_{\rm sh} = (\Gamma_\uparrow + \Gamma_\downarrow)/4$. Similarly, within VF, interfaces are described by a set of three parameters, $r^b_{\uparrow(\downarrow)} = 2r^{b*}(1 \mp \gamma)$ and δ which correspond to an effective layer of thickness d which is taken to be infinitely thin while keeping the parameters $\delta = d/l_{\rm sf}$ and $r^b_\sigma = \rho_\sigma d$ constant. Figure 2(a) shows the spin accumulation profile of a Co/Py spin valve calculated with C-RMT. For the parallel and antiparallel configuration, we also show an (independent) VF calculation using the CNRS-Thales software developed by one of us (H. J.). Both match perfectly as expected.

Application to spin torque: Wavy or not wavy?—We consider the valve depicted in Fig. 1(a) and note $\vec{J}_A$, $\vec{J}_N$, and $\vec{J}_B$ the spin currents just before, in between, and after the two magnetic layers F_A and F_B. For noncollinear magnetization, spin current is not conserved, and the spin torque on F_A and F_B is defined as

$$\vec{\tau}_A = \vec{J}_A - \vec{J}_N, \qquad \vec{\tau}_B = \vec{J}_N - \vec{J}_B. \tag{19}$$

Let us start with a simple geometric construction that allows us to get a physical picture for the torque in a rather general way. To do so, we need two hypotheses: (i) the mixing transmission are small (it is the case for metallic magnetic layers) so that $\vec{J}_A$ and $\vec{J}_B$ are parallel to the magnetization of F_A and F_B respectively; (ii) the system is thin enough for spin-flip scattering to be ignored in the

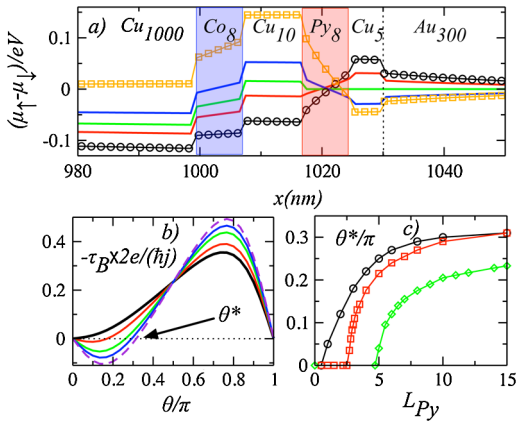

FIG. 2 (color online). (a) Spin accumulation in the middle of a spin valve $A = {\rm Cu}_{1000}{\rm Co}_8{\rm Cu}_{10}{\rm Py}_8{\rm Cu}_5{\rm Au}_{300}$ (thickness in nm) for different angle $\theta = 0$ (circles), $\pi/4$, $\pi/2$, $3\pi/4$, and π (squares). Symbols stand for VF calculations while lines correspond to C-RMT. (b) torque $\tau_B(\theta)$ (per total current j) on the Py-layer of A for various Py-thickness $L_{\rm Py}$ from 0.5 nm (thick line) to 15 nm (dashed line). (c) Stability angle θ^* as a function of $L_{\rm Py}$ for A (circles), $B = {\rm Cu}_{1000}{\rm Co}_8{\rm Cu}_{10}{\rm Py}_{L_{\rm Py}}{\rm Cu}_{1000}$ with $\delta = 0$ (squares) and B with $\delta_{\rm CoCu} = \delta_{\rm CoPy} = 0.25$ (diamonds).

active region so that $\vec{\tau}_A$ ($\vec{\tau}_B$) is perpendicular to $\vec{J}_A$ ($\vec{J}_B$). The construction goes as follows, see Figs. 1(b) and 1(c): first, we plot $\vec{J}_A$ and $\vec{J}_B$ which make the same angle θ as the magnetization of the respective magnetic layers; then, we note that $\tau_{\rm tot} \equiv \vec{\tau}_A + \vec{\tau}_B = \vec{J}_A - \vec{J}_B$ does not depend on the unknown $\vec{J}_N$ and points from the tip of $\vec{J}_B$ to the tip of $\vec{J}_A$. The construction of the torque is then straightforward: the two vectors $\vec{\tau}_A$ and $\vec{\tau}_B$ are chosen such that they are perpendicular to their respective layer and their sum goes from the tip of $\vec{J}_B$ to the tip of $\vec{J}_A$. This simple construction gives, in particular, the sign of the torque as a function of the angle θ. We find that when, say, $J_B > J_A$, the torque on the layer with the highest polarization (F_B) can become wavy [12,13], i.e., instead of favoring the parallel or antiparallel configurations, the torque stabilizes (or destabilizes depending of the direction of the current) a configuration with a finite angle θ^*. The critical angle θ^* where the torque vanishes verifies $|J_A/J_B| = \cos\theta^*$. On the other hand, at small angle, one has the following development $|J_A/J_B|(\theta) = 1 - \eta\theta^2/2 + \ldots$ (Current conservation imposes $J_A = J_B$ at $\theta = 0$ and the ratio is an even function of θ) so that "waviness" is found when $\eta > 1$. In a symmetric structure, $\eta = 0$ so that a finite asymmetry is needed to enforce waviness.

Without spin-flip scattering, we find $\eta = [\gamma_B r_B^* - \gamma_A r_A^* + (\gamma_B - \gamma_A) r_A^* r_B^* / \mathcal{R}_{\rm sh}]/(\gamma_B r_B^* + \gamma_A r_A^*)$ where the effective parameters γ_A and r_A^* include both the interface and bulk properties of layer A. More generally, the crossover between normal and wavy can be discussed by looking at the small angle expression derived by Fert *et al.* [Eq. (5) in [20], we omit the ballistic corrections]. It can be obtained by relating the spin accumulation in the spacer for $\theta \ll 1$ to spin current and spin accumulation at $\theta = 0$, and then applying Eq. (18). It reads

$$\left.\frac{d\tau_B}{d\theta}\right|_{\theta=0} = -\frac{\hbar}{e}\left[\left.\frac{j_\uparrow - j_\downarrow}{4}\right|_{\theta=0} + \left.\frac{\mu_\uparrow - \mu_\downarrow}{2e\mathcal{R}_{\rm sh}}\right|_{\theta=0}\right]. \quad (20)$$

When, for instance, one crosses the asymmetry border from $J_A > J_B > 0$ to $J_B > J_A > 0$, the spin accumulation, proportional to the gradient of the spin current, changes from negative to positive, and the second term in Eq. (20) becomes negative and begins to compensate the first one. The crossover from normal to wavy occurs when, by a further increase of asymmetry, the spin accumulation term wins and reverse the sign of $d\tau_B/d\theta|_{\theta=0}$ (see Gmitra and Barnaś [21] for an extensive discussion of the normal to wavy crossover).

Typical examples of our numerical results are presented in Fig. 2 for Co/Cu/Py samples in which the asymmetry comes from the short $l_{\rm sf}$ and large polarization and resistivity of Py. Starting from a small value of $L_{\rm Py}$, an increase of the asymmetry and finally a crossover to wavy ($\theta^* \neq 0$) can be obtained by increasing $L_{\rm Py}$ as shown in Figs. 2(b) and 2(c). By comparing the curves in samples with and without Au on the right of the valve, one sees that the short $l_{\rm sf}$ of Au in a layer close to Py tends to increase the asymmetry and the waviness. On the other hand, interface spin-flip is found to favor a normal spin torque. In the experimental results of Boulle *et al.* [13], a wavy behavior was found for Cu/Co/Cu/Py/Cu/Au structures with equal thicknesses (8 nm) for Co and Py.

Conclusion.—Our approach, C-RMT, not only provides a conceptual connection between quantum and classical approaches, but also allows for new practical developments. For instance the numerical solution shown in Fig. 2, based on the "hat" version of Eqs. (2) and (3) is immediate to implement and extremely fast. The generalization of C-RMT to three dimensions will allow real time coupling between transport and micromagnetic simulations. Other applications will include multiscale modeling (to treat purely quantum regions like MgO tunneling barriers) and superconductivity [22].

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