Time Modulation of the *K*-Shell Electron Capture Decay Rates of H-like Heavy Ions at GSI Experiments

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(Received 9 April 2009; published 6 August 2009)

According to experimental data at GSI, the rates of the number of daughter ions, produced by the nuclear *K* shell electron capture decays of the H-like heavy ions with one electron in the *K* shell, such as $^{140}Pr^{58+}$, $^{142}Pm^{60+}$, and $^{122}I^{52+}$, are modulated in time with periods T_{EC} of the order of a few seconds, obeying an A scaling $T_{EC} = A/20$ s, where A is the mass number of the mother nuclei, and with amplitudes $a_d^{EC} \sim 0.21$. We show that these data can be explained in terms of the interference of two massive neutrino mass eigenstates. The appearance of the interference term is due to overlap of massive neutrino mass eigenstate energies and of the wave functions of the daughter ions in two-body decay channels, caused by the energy and momentum uncertainties introduced by time differential detection of the daughter ions in GSI experiments.

DOI: 10.1103/PhysRevLett.103.062502 PACS numbers: 23.40.Bw, 13.15.+g, 14.60.Pq, 26.65.+t

Introduction.—Measurements of the K shell electron capture (EC) and positron (β^+) decay rates of the H-like heavy ions $^{142}Pm^{60+}$, $^{140}Pr^{58+}$, and $^{122}I^{52+}$ with one electron in their K shells have been recently carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt [1–4]. The measurements of the rates $dN_d^{EC}(t)/dt$ of the number $N_d^{EC}(t)$ of daughter ions $^{142}Nd^{60+}$, $^{140}Ce^{58+}$, and $^{122}Te^{52+}$ showed a time modulation of exponential decays with periods $T_{EC} = 7.10(22)$, $7.06(8)$, and $6.11(3)$ s and modulation amplitudes $a_d^{EC} = 0.23(4), 0.18(3)$, and $0.22(2)$ for $^{142}Pm^{60+}$, $^{140}Pr^{58+}$, and $^{122}I^{52+}$ [5], respectively.

Since the rates of the number of daughter ions are defined by

$$\frac{dN_d^{EC}(t)}{dt} = \lambda_d^{EC}(t) N_m(t), \qquad (1)$$

where $\lambda_d^{EC}(t)$ is related to the EC-decay rate and $N_m(t)$ is the number of parent ions, the time modulation of $dN_d^{EC}(t)/dt$ can be described in terms of a periodic time dependence of the EC-decay rate $\lambda_d^{EC}(t)$ [1–4]:

$$\lambda_d^{EC}(t) = \lambda_{EC}[1 + a_d^{EC} \cos(\omega_{EC} t + \phi_{EC})], \qquad (2)$$

where λ_{EC} is the EC-decay constant and a_d^{EC}, $T_{EC} = 2\pi/\omega_{EC}$, and ϕ_{EC} are the amplitude, period, and phase of the time-dependent term, respectively [1]. Furthermore, it was shown that the β^+-decay rate of $^{142}Pm^{60+}$, measured simultaneously with its modulated EC-decay rate, is not modulated with an amplitude upper limit $a_{\beta^+} < 0.03$ [2–4] (see also [5]).

The important property of the periods of the time modulation of the EC-decay rates is their proportionality to the mass number A of the nucleus of the parent H-like heavy ion. Indeed, the periods T_{EC} can be described well by the phenomenological formula $T_{EC} = A/20$ s. The proportionality of the periods of the EC-decay rates to the mass number A of the mother nuclei and the absence of the time modulation of the β^+-decay branch [2–4] can be explained, assuming the interference of neutrino mass eigenstates caused by a coherent superposition of monochromatic neutrino mass eigenstates with electron lepton charge [6,7].

Indeed, nowadays the existence of massive neutrinos, neutrino-flavor mixing, and neutrino oscillations is well established experimentally and elaborated theoretically [8]. The observation of the interference of massive neutrino mass eigenstates in the EC-decay rates of the H-like heavy ions sheds new light on the important properties of these states.

Amplitudes of EC decays of H-like heavy ions.—The Hamilton operator $H_W(t)$ of the weak interactions, responsible for the EC and β^+ decays of the H-like heavy ions, can be taken in the standard form [9], accounting for the neutrino-flavor mixing [7]. This gives $H_W(t) = \sum_j U_{ej} H_W^{(j)}(t)$, where U_{ej} are matrix elements of the neutrino mixing matrix U [8] and $H_W^{(j)}(t)$ is defined by

$$H_W^{(j)}(t) = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3x [\bar{\psi}_n(x)\gamma^\mu(1 - g_A\gamma^5)\psi_p(x)] \times [\bar{\psi}_{\nu_j}(x)\gamma_\mu(1-\gamma^5)\psi_{e^-}(x)], \qquad (3)$$

with standard notation [9]. The amplitude $A(m \to d + \nu_e)(t)$ of the EC decay $m \to d + \nu_e$ of a parent H-like ion m into a daughter ion d and an electron neutrino ν_e is defined as a sum of the amplitudes $A(m \to d + \nu_j)(t)$ of the emission of the massive neutrino mass eigenstates $m \to d + \nu_j$ as follows:

$$A(m \to d + \nu_e)(t) = \sum_j U_{ej} A(m \to d + \nu_j)(t), \qquad (4)$$

where the coefficients U_{ej} take into account that the electron couples to the electron neutrino only $|\nu_e\rangle = \sum_j U^*_{ej}|\nu_j\rangle$ [8] and t corresponds to the time of the observation of the decay of the parent ion into the daughter and electron neutrino state $d + \nu_e$.

According to time-dependent perturbation theory [10], the partial amplitudes $A(m \to d + \nu_j)(t)$, defined in the rest frame of the parent ion, are given by

$$A(m \to d + \nu_j)(t) = -i \int_{-\infty}^{t} d\tau e^{\varepsilon\tau} \langle d(\vec{q}_j)\nu_j(\vec{k}_j) \times |H^{(j)}_W(\tau)|m(\vec{0})\rangle, \quad (5)$$

where $\vec{k}_j$ is the 3-momentum of the neutrino mass eigenstate ν_j with mass m_j and $\vec{q}_j$ is the 3-momentum of the daughter ion produced entangled with the neutrino mass eigenstate ν_j. For the regularization of the integral over time, we use the $\varepsilon \to 0$ regularization procedure. Because of the factor $e^{\varepsilon\tau}$, the weak interaction switches on adiabatically up to the moment of the production of the parent ion, followed by its injection into the ESR, in which its decay is observed at the time $t_L = \gamma t$, where t_L is a laboratory time and $\gamma = 1.43$ is a Lorentz factor [1], relating times in the laboratory frame and the center of mass frame. In this connection we recall that at GSI the H-like parent ions are produced by a fragmentation reaction in a ^{9}Be target using a 500 MeV per nucleon bunched heavy ion beam with a bunch length of about 300 ns, separated in a fragment separator and injected into the ESR after about 500 ns transit time at 400 MeV per nucleon energy [1]. The upper limit $t = t_L/\gamma$ of the integral over time has the meaning of the time of the observation of the decay of the parent H-like ion into the final state $d + \nu_e$, which is also the time of the correlated appearance of the daughter ion d, which is actually observed, and the electron neutrino ν_e as a coherent superposition of neutrino mass eigenstates.

Following Ref. [9], we obtain the amplitude of the EC decay, which is a Gamow-Teller $1^+ \to 0^+$ transition, as a function of time t:

$$A(m \to d + \nu_e)(t) = -\delta_{M_F,-1/2}\sqrt{3}\sqrt{2M_m}\mathcal{M}_{\rm GT}\langle\psi^{(Z)}_{1s}\rangle \sum_j U_{ej}\sqrt{2E_d(\vec{k}_j)E_j(\vec{k}_j)}\frac{e^{i(\Delta E_j - i\varepsilon)t}}{\Delta E_j - i\varepsilon}\Phi_d(\vec{k}_j + \vec{q}_j), \quad (6)$$

where $\Delta E_j = E_d(\vec{q}_j) + E_j(\vec{k}_j) - M_m$ is the energy difference of the final and initial states, M_m is the parent ion mass, $E_d(\vec{q}_j)$ and $E_j(\vec{k}_j)$ are the energies of the daughter ion and massive neutrino ν_j, respectively, $\mathcal{M}_{\rm GT}$ is the nuclear matrix element of the Gamow-Teller transition $m \to d$, and $\langle\psi^{(Z)}_{1s}\rangle$ is the wave function of the bound electron in the H-like heavy ion m, averaged over the nuclear density [9].

We take the wave function of the detected daughter ion in the form of a wave packet. In this case the wave function $\Phi_d(\vec{k}_j + \vec{q}_j)$ describes a spatial smearing, caused by the differential time detection of the daughter ions [1]. The time differential detection of the daughter ions from the EC decays with a time resolution $\tau_d \simeq 320$ ms introduces an energy uncertainty $\delta E_d \sim 2\pi\hbar/\tau_d = 1.29 \times 10^{-14}$ eV, thus providing also a smearing of the 3-momenta of the daughter ions $|\delta\vec{q}_d| \sim 2\pi\hbar/v_d\tau_d = 1.82 \times 10^{-14}$ eV, where v_d is the velocity of the daughter ions, which is in the order of the velocity of the parent ions $v_d \simeq v_m = 0.71c$ in the laboratory frame [1]. This implies that the wave function $\Phi_d(\vec{k}_j + \vec{q}_j)$ is peaked in the vicinity of the zero value of the argument. In case the wave function of the daughter ion is approximately a plane wave, the wave function $\Phi_d(\vec{k}_j + \vec{q}_j)$ is proportional to the δ function.

Because of energy and momentum conservation in every EC-decay channel $m \to d + \nu_j$, the energy of massive neutrino ν_j is equal to

$$E_j(\vec{k}_j) = \frac{M^2_m - M^2_d + m^2_j}{2M_m}, \quad (7)$$

which leads to the energy difference of neutrino mass eigenstates

$$\omega_{ij} = E_i(\vec{k}_i) - E_j(\vec{k}_j) = \frac{\Delta m^2_{ij}}{2M_m}, \quad (8)$$

where $\Delta m^2_{ij} = m^2_i - m^2_j$. Note that ω_{ij} determines also the recoil energy difference of the observed daughter ions.

Since the experimental value of the matrix element $U_{13} = \sin\theta_{13}e^{i\delta}$, where δ is a CP-violating phase [8], is very close to zero, we set $\theta_{13} = 0$ and below deal with two neutrino mass eigenstates only with mixing matrix elements $U_{e1} = \cos\theta_{12}$ and $U_{e2} = \sin\theta_{12}$ [8].

EC-decay rates of H-like heavy ions.—In the following we derive the expression for the EC-decay rate of the H-like heavy ion with a bare daughter ion and a coherent superposition of electron neutrino mass eigenstates as the final state. The EC-decay rate is related to the expression

$$\lim_{\varepsilon\to 0}\frac{d}{dt}\frac{1}{2}\sum_{M_F}|A(m \to d + \nu_e)(t)|^2 = 3M_m|\mathcal{M}_{\rm GT}|^2|\langle\psi^{(Z)}_{1s}\rangle|^2\Big\{\sum_{j=1,2}|U_{ej}|^2 2E_d(\vec{k}_j)E_j(\vec{k}_j)2\pi\delta(\Delta E_j)|\Phi_d(\vec{k}_j + \vec{q}_j)|^2 + \sum_{i>j}U^*_{ei}U_{ej}\sqrt{2E_d(\vec{k}_i)E_i(\vec{k}_i)}\sqrt{2E_d(\vec{k}_j)E_j(\vec{k}_j)}\Phi^*_d(\vec{k}_i + \vec{q}_i)\Phi_d(\vec{k}_j + \vec{q}_j)[2\pi\delta(\Delta E_i) + 2\pi\delta(\Delta E_j)]\cos(\omega_{ij}t)\Big\}, \quad (9)$$

where ω_{ij} is given by Eq. (8). The first term in Eq. (10) is the sum of the two diagonal terms of the transition probability into the states $d + \nu_1$ and $d + \nu_2$, describing the incoherent contribution of neutrino mass eigenstates, while the second term defines the interference of states ν_i and ν_j with $i \neq j$ causing the periodic time dependence with the frequency ω_{ij}. The interference term, produced by the coherent contribution of the neutrino mass eigenstates, can be observed only due to the energy and momentum uncertainties, introduced by the time differential detection of the daughter ions as pointed out above. For the subsequent calculation of the EC-decay rate, we can take the massless limit everywhere except the modulated term $U^*_{ei}U_{ej}\cos(\omega_{ij}t)$. Because of the momentum uncertainty induced by the time differential detection of the daughter ions, we can set $\vec{k}_i \simeq \vec{k}_j \simeq \vec{k}$ and $\vec{q}_i \simeq \vec{q}_j \simeq \vec{q}$ with the consequence that $|\Phi_d(\vec{k}+\vec{q})|^2 = V(2\pi)^3\delta^{(3)}(\vec{k}+\vec{q})$, where V is a normalization volume and the δ function describes the conservation of momentum.

In such an approximation, using the definition of the EC-decay rate

$$\lambda_{\rm EC}(t) = \frac{1}{2M_m V}\int\frac{d^3q}{(2\pi)^3 2E_d}\frac{d^3k}{(2\pi)^3 2E_{\nu_e}} \times \lim_{\varepsilon\to 0}\frac{d}{dt}\frac{1}{2}\sum_{M_F}|A(m\to d+\nu_e)(t)|^2, \quad (10)$$

we obtain the following expression for the time modulated EC-decay rate:

$$\lambda_{\rm EC}(t) = \lambda_{\rm EC}[1 + a_{\rm EC}\cos(\omega_{21}t)], \quad (11)$$

where $\omega_{21} = \Delta m^2_{21}/2M_m$, $a_{\rm EC} = \sin 2\theta_{12}$, and $\lambda_{\rm EC}$ has been calculated in [9]. In the laboratory frame the EC-decay rate is time modulated with a frequency $\omega_{\rm EC} = \omega_{21}/\gamma$. Thus, the period $T_{\rm EC}$ of the time modulation is

$$T_{\rm EC} = \frac{2\pi}{\omega_{\rm EC}} = \frac{2\pi\gamma M_m}{\Delta m^2_{21}}, \quad (12)$$

where we have taken into account that the period $T_{\rm EC}$ is measured in the laboratory frame [1]. Since in the massless limit $\theta_{12}\to 0$, for $m_j \to 0$ we arrive at the time-independent decay rate $\lambda_{\rm EC}$.

From the experimental data on the periods of the time modulation of the EC-decay rates, the masses $M_m \simeq 931.494A$ of parent ions, and Eq. (12), we get the following values for quadratic mass difference Δm^2_{21} of massive neutrinos:

$$(\Delta m^2_{21})_{\rm GSI} = \begin{cases} 2.20(7)\times 10^{-4}\ {\rm eV}^2 & {}^{142}{\rm Pm}^{60+}, \\ 2.18(3)\times 10^{-4}\ {\rm eV}^2 & {}^{140}{\rm Pr}^{58+}, \\ 2.19(1)\times 10^{-4}\ {\rm eV}^2 & {}^{122}{\rm I}^{52+}. \end{cases} \quad (13)$$

The values are equal within their error margins and yield a combined squared neutrino mass difference $(\Delta m^2_{21})_{\rm GSI} = 2.19\times 10^{-4}\ {\rm eV}^2$. This confirms the proportionality of the period of time modulation to the mass number A of the mother nucleus $T_{\rm EC} = \kappa A$, where $\kappa = 4\pi\gamma\hbar M_m/A(\Delta m^2_{21})_{\rm GSI} = 0.050(4)$ s.

Amplitude of interference term.—The procedure for the calculation of the interference term proposed above leads to the amplitude $a_{\rm EC} = \sin 2\theta_{12}$ equal to $a_{\rm EC} \simeq 0.93$, if one takes into account the experimental value for the mixing angle of two neutrino mass eigenstates $\theta_{12} \simeq 34°$ deduced from solar neutrino experimental data [8].

We argue that the experimental amplitude of the time modulation $a^{\rm EC}_d \sim 0.21$ is not the amplitude of the EC-decay rate $a_{\rm EC}$ of Eq. (11) but presents the amplitude of the time modulation of the rate of the number of daughter ions. Indeed, in case the coherence of the contributions of neutrino mass eigenstates is retained in all EC decays of the parent ions injected into the ESR, one should measure the amplitude of the time modulation equal to $a_{\rm EC}$. However, stochastic processes, affecting a fraction of the H-like parent ions, can destroy the coherence of the contributions of neutrino mass eigenstates, and the EC-decay rate is defined by the first term in Eq. (9) only. As a result the EC-decay rate of this fraction of parent ions as well as the rate of the number of daughter ions do not show a time modulation. This diminishes effectively the amplitude of the time modulation of the rate of the number of daughter ions in Eq. (1), measured by the experiment [1–4].

For example, the H-like parent ions in the hyperfine ground state ${}^A X^{(Z-1)+}_{F=1/2}$ can be stochastically produced by the decay of the hyperfine excited states ${}^A X^{(Z-1)+}_{F=3/2}$ with the emission of a photon with an energy of $\omega \sim 1$ eV. These preceding electromagnetic decays can destroy the coherent contribution of neutrino mass eigenstates to the EC-decay rates of a fraction of parent ions in the process ${}^A X^{(Z-1)+}_{F=3/2} \to m + \gamma \to d + \nu_e + \gamma$ and lead to the amplitude $a^{\rm EC}_d$ of the time modulation of the rate of the number of daughter ion smaller than $a_{\rm EC}$. We propose to test this assertion by studying the EC decays of He-like heavy ions and the bound-state β^- decays of bare ions with no hyperfine structure.

Conclusion.—We have shown that the experimental data on the time modulation of the rates of the number of daughter ions in the EC decays of the H-like heavy ions, observed at GSI, can be explained by the interference of massive neutrino mass eigenstates.

The necessary condition for the appearance of the interference term in the EC-decay rates is the overlap of the energy levels of neutrino mass eigenstates. This is provided by the time differential detection of the daughter ions with a time resolution $\tau_d \simeq 320$ ms leading to an energy uncertainty $\delta E_d \sim 2\pi\hbar/\tau_d = 1.29\times 10^{-14}$ eV and a momentum uncertainty $|\delta\vec{q}_d| \sim 2\pi\hbar/v_d\tau_d = 1.82\times 10^{-14}$ eV of the daughter ions.

The application of this mechanism to the analysis of time modulation of the β^+-decay rates of H-like heavy ions [7] showed the absence of the time modulation due to the broad continuous neutrino spectrum in accordance with the experimental observation that the β^+-decay branch of the H-like $^{142}\mathrm{Pm}^{60+}$ ion decay shows no modulation with an upper limit of the amplitude $a_{\beta^+} < 0.03$ [2–4].

The value $(\Delta m^2_{21})_{\mathrm{GSI}} \simeq 2.19 \times 10^{-4}\ \mathrm{eV}^2$ is 2.9 times larger than that reported by the KamLAND Collaboration $(\Delta m^2_{21})_{\mathrm{KL}} = 7.59(21) \times 10^{-5}\ \mathrm{eV}^2$ [11]. A possible solution of this problem in terms of neutrino mass corrections, induced by the interaction of massive neutrinos with strong Coulomb fields of the daughter ions through virtual $\ell^- W^+$ pair creation, is proposed in [12].

Finally, we emphasize that the ''GSI oscillations,'' i.e., the time modulation or the periodic time dependence of the rates of the number of daughter ions of the EC decays of the H-like parent ions, have no relations to the ''neutrino oscillations,'' e.g., $\nu_e \leftrightarrow \nu_e$, $\nu_e \leftrightarrow \nu_\mu$, and $\nu_e \leftrightarrow \nu_\tau$. The period of ''neutrino oscillations'' is expected to be proportional to the neutrino energy or the Q value of the EC decay. This contradicts the experimental data showing no dependence of the modulation period on the Q value but a proportionality to the mass number A of the parent ion [1–4] in agreement with our approach.

Following the publication of the experimental data on the time modulation of the EC-decay rates of the H-like ions $^{140}\mathrm{Pr}^{58+}$ and $^{142}\mathrm{Pm}^{60}$ with periods $T_{\mathrm{EC}} \simeq 7$ s [1], Giunti [13] and Kienert *et al.* [14] proposed that this phenomenon is caused by quantum beats of two closely spaced mass eigenstates of the H-like parent ions with a mass splitting of the order of 10^{-15} eV of unknown origin. We notice only that such a mechanism, describing time modulation of the EC-decay rates of the H-like $^{140}\mathrm{Pr}^{58+}$ and $^{142}\mathrm{Pm}^{60}$ ions, does not reproduce the A scaling of the modulation periods, confirmed in the subsequent experiments on the EC-decay rates of the H-like $^{122}\mathrm{I}^{52+}$ ions [2–4]. The mass splitting can be attributed either to the nucleus mass or to the energy level of the bound electron. They would provide either a time modulation of positron decay rates with a period $T_{\beta^+} = T_{EC} \simeq 7$ s or time-independent decay rates for the EC and positron decay, which are both unobserved.

This research was partly supported by the DFG cluster of excellence ''Origin and Structure of the Universe'' of the Technische Universität München and by the Austrian ''Fonds zur Förderung der Wissenschaftlichen Forschung'' (FWF) under Contract No. P19487-N16.

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