New Features of Shape Coexistence in ^{152}Sm

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Excited states in ^{152}Sm have been investigated with the ^{152}Sm$(n, n'\gamma)$ reaction. The lowest four negative-parity band structures have been characterized in detail with respect to their absolute decay properties. Specifically, a new $K^\pi = 0^-$ band has been assigned with its 1^- band head at 1681 keV. This newly observed band has a remarkable similarity in its $E1$ transition rates for decay to the first excited $K^\pi = 0^+$ band at 684 keV to the lowest $K^\pi = 0^-$ band and its decay to the ground-state band. Based on these decay properties, as well as energy considerations, this new band is assigned as a $K^\pi = 0^-$ octupole excitation based on the $K^\pi = 0_2^+$ state. An emerging pattern of repeating excitations built on the 0_2^+ level similar to those built on the ground state may indicate that ^{152}Sm is a complex example of shape coexistence rather than a critical point nucleus.

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The level structure of ^{152}Sm continues to provide a challenge to our understanding of nuclear structure. Much of the fascination is a result of the very rapid change in the shape of the nuclear ground state from the (apparently) spherical nucleus ^{150}Sm to the well-deformed nucleus ^{154}Sm. The rapid change in observables, such as the quadrupole moment and E_4^+/E_2^+ ratio, has lead to the suggestion [1–3] that a phase transition occurs in the shape degree of freedom across neutron number $N = 90$. A recent interpretation [4,5] of the low-lying levels in ^{152}Sm, and indeed in several nearby nuclei with $N = 90$, invokes these ideas suggesting that they lie at the critical point of a phase transition. A crucial question, however, is whether $N = 90$ nuclei truly represent a phase transition or are they more complex examples of shape coexistence. The distinction between the two descriptions is very subtle and perhaps best summarized by Heyde and co-workers [6]. In a phase-transition picture, the states of one limit of the Hamiltonian will spread out and eventually become the eigenstates of the other limit. For shape coexistence, one has a complete set of states, which arise from distinct Hilbert spaces, built on each shape, examples of which are well known throughout the nuclear chart [7].

In ^{152}Sm, the debate on the phase-transition vs shape-coexistence pictures has rested largely on the properties of the 0_2^+ band. If the 0_2^+ level arises from a coexisting shape, it will have a full spectrum of states built on it that will be distinct from those built on the ground state. If the shapes involved are not too drastically different, one may expect to observe a repeating pattern of excitations. Differentiating between the phase-transition and shape-coexistence pictures requires high precision and extensive spectroscopic data on highly nonyrast states. While much attention has been paid to the positive-parity levels, properties of the negative-parity levels, for which there also should be a pattern of repeating excitations in the shape-coexistence picture, have not been as thoroughly investigated. The present work focuses on the low-lying negative-parity bands, including a newly established $K^\pi = 0^-$ band built on the 1681-keV 1^- state that, combined with results from other studies [8], are suggestive of a pattern of repeating excitations on the 0_2^+ level similar to the pattern built on the ground state.

The experiments were performed at the University of Kentucky Accelerator Facility, where nearly monoenergetic neutrons, produced via the ^{3}H$(p, n)^3$He reaction with time-bunched beams of protons, bombarded a 36 g sample enriched to 96% in ^{152}Sm. The γ rays produced in the neutron bombardment of the sample were detected with Compton-suppressed HPGe detectors with relative efficiencies ranging from 52% to 57%. Excitation functions, using neutrons with energies varying from 1.2 to 3.0 MeV, were performed with a detector placed at an angle of 125° with respect to the incident proton beam. Angular distributions were performed at neutron energies of 2.05 and 2.7 MeV where the detector was rotated to angles between 40° and 155°. For these latter measurements, the energy calibration was monitored by using well-known transitions in ^{152}Sm and γ rays from a radioactive source placed near the scattering sample and in view of the detector. A $\gamma\gamma$ coincidence measurement was performed at a neutron energy of 3.2 MeV and used 4 HPGe detectors placed $\approx$5 cm from the sample.

The state at 1681 keV was previously assigned as spin 1 [9,10] (observed in ^{152}Pm $J^\pi = 1^+$ and ^{152}Eu $J^\pi = 0^-$ decays as summarized in the nuclear data sheets [10]), and

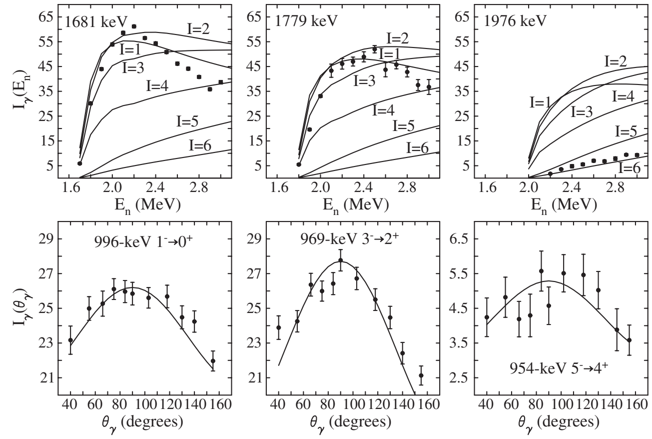

FIG. 1. Excitation functions (top panels) observed from the ^{152}Sm$(n, n'\gamma)$ reaction for the 1681-keV, 1779-keV, and 1976-keV levels. The lower panels display angular distributions for selected transitions. The curves are theoretical predictions for negative-parity states using the CINDY code [22] for the spin sequences given, not fits to the data.

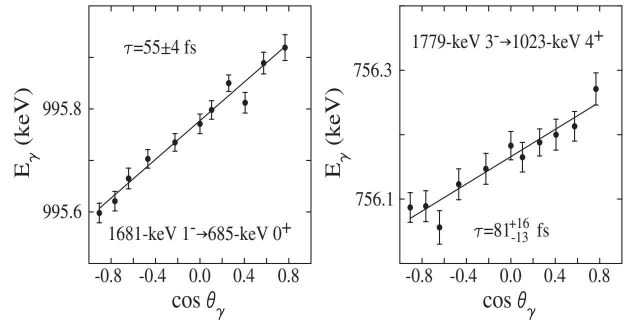

FIG. 2. Examples of Doppler shifts observed for selected transitions following the ^{152}Sm$(n, n'\gamma)$ reaction with 2.05 MeV neutrons.

was favored to have negative parity from its population in proton inelastic scattering [11]. Recent detailed spectroscopy [12] of ^{152}Eu decay, combined with data from the present work, as shown in Fig. 1, firmly establish an $I^\pi = 1^-$ assignment. The level at 1779 keV decays to lower-lying 2^+, 3^-, and 4^+ levels, which immediately restricts the spin parity to be 2^+, $3^\pm$, and 4^+. The lack of transitions to lower-lying 0^+ states suggests a spin 3 or 4 assignment, and combined with an analysis of the present data, strongly favors a $I^\pi = 3^-$ assignment. A new level has been established at 1976 keV, and while the excitation function exhibits better agreement with a spin 6 assignment, the angular distributions definitely rule this out and favor spin 5 for this weakly populated level. No evidence is found for additional spin 2^- or 4^- levels that could be members of this negative-parity band. The decay patterns of the 1^- and 3^- levels are consistent with those of the corresponding members of the first $K^\pi = 0^-$ band, and thus this band is assigned as the second $K^\pi = 0^-$ band.

A particular advantage of the $(n, n'\gamma)$ reaction is the possibility of determining level lifetimes through the Doppler shift attenuation method [13], as shown in Fig. 2 for selected transitions from the $K^\pi = 0_2^-$ band. The negative-parity levels, with their lifetimes established from the Doppler shift analysis, and the resulting $B(E1)$ values are listed in Table I. The current supersedes preliminary results presented in Ref. [14].

The $E1$ transition rates for decay of the $K^\pi = 0_1^-$ band to the ground-state band are shown in Fig. 3 and Table I, where lifetimes for the 1^- and 3^- band members were taken from Ref. [10]. It can be seen that the values range from 4 to 8 m W.u. (10^{-3} Weisskopf unit). Also shown are other $B(E1)$ values, on the order of 1 m W.u. and greater, observed in the decay of the assigned negative-parity bands. Large $B(E1)$ values are observed for (i) the decay of the odd-spin members of the $K^\pi = 1_1^-$ band to the ground-state band, (ii) the decay of the $K^\pi = 2_1^-$ band to the $K^\pi = 2_1^+$ band, and (iii) the decay of the $K^\pi = 0_2^-$ band to the second $K^\pi = 0^+$ band. The remaining $E1$ transitions, as listed in Table I, are significantly less than 1 m W.u., with many below 0.1 m W.u. or more than a full order of magnitude smaller than those displayed in Fig. 3.

At the beginning of the rare earth region, it is expected [15] that the octupole bands that arise from the coupling to a quadrupole deformed shape should be, in the order of increasing excitation energy, $K^\pi = 0^-$, 1^-, 2^-, and 3^-. With the exception of a candidate band for the $K^\pi = 3^-$ state, with a band head expected above 2 MeV [15] and thus difficult to identify, the observed bands are in line with this expectation. The 3^- members of the $K^\pi = 0_1^-$ and $K^\pi = 1^-$ bands were observed to have enhanced $B(E3)$ values of 14.7 single-particle units (s.p.u.) and 7.8 s.p.u. in the (d, d') reaction [16]. These values are consistent with the interpretation that they are octupole excitations built on the ground state. The large values observed for the $B(E1)$ values for the decay of members of the $K^\pi = 0_1^-$ and 1_1^- band to the ground-state band strongly supports the idea that enhanced $E1$ rates can be used as an indication of octupole correlations in these negative-parity bands. The magnitude of the $B(E1)$ values for decay from the $K^\pi = 1_1^-$ band are smaller than those of the $K^\pi = 0_1^- \rightarrow K^\pi = 0_1^+$ transitions, consistent with the observation that $\Delta K = 1$, $E1$ transitions are hindered compared to $\Delta K = 0$ transitions [17], although in the present case only by a factor of 5 to 10 which may be related to strong Coriolis coupling, as outlined in, e.g., Refs. [15,18], or the breakdown of K as a robust quantum number in a transitional nucleus such as ^{152}Sm.

The most remarkable features of the $E1$ rates are the large values observed for decay from the second $K^\pi = 0^-$ band to the second $K^\pi = 0^+$ band. These particular $\Delta K = 0$ transitions dominate by 2 orders of magnitude over the $\Delta K = 0$ transitions to either the ground-state band or the $K^\pi = 0_3^+$ band, and are nearly equal to the $E1$ rates determined for decays from the $K^\pi = 0_1^-$ band to the ground-state band. It is further noted that the energy difference between the $K^\pi = 0_1^-$ and $K^\pi = 0_2^-$ bands is 718 keV,

TABLE I. Level lifetimes (τ in fs), $B(E1)$ (in m W.u.) and $B(E2)$ (in W.u. assuming $E2$ multipolarity in the case of mixed $E2/M1$ transitions) values determined for $K^\pi = 0^-$, 1^-, and 2^- bands identified in ^{152}Sm below 2 MeV. The $M2$ admixtures in the $E1$ transitions were consistent with zero (where measured). The final column presents the corresponding transition rates calculated with the *spdf*IBM. Values with the "*" indicate that the calculations predict the reverse ordering in excitation energy of the levels. Values in bold are discussed in the text.

E_i/τ; K_i^π, I_i^π (keV)/(fs)	E_f; K_f^π, I_f^π (keV)	$B(E\lambda\downarrow)$	*spdf* IBM
963.4; $0_1^-, 1^-$	121.8; $0_1^+, 2^+$	$7.80^{+0.73}_{-0.64}$	10.9
	0.0; $0_1^+, 0^+$	$4.18^{+0.39}_{-0.34}$	4.6
1041.1; $0_1^-, 3^-$	366.5; $0_1^+, 4^+$	$8.4^{+1.9}_{-1.3}$	8.3
	121.8; $0_1^+, 2^+$	$8.0^{+1.8}_{-1.3}$	4.9
1221.7; $0_1^-, 5^-$	706.8; $0_1^+, 6^+$	$5.0^{+1.2}_{-1.1}$	6.9
$\tau = 105^{+23}_{-18}$	366.5; $0_1^+, 4^+$	$4.1^{+0.9}_{-0.8}$	4.3
1510.8; $1_1^-, 1^-$	1292.8; $0_3^+, 2^+$	0.175(26)	0.078*
$\tau = 132(9)$	1085.8; $2_1^+, 2^+$	**0.0066(28)**	**0.28**
	963.4; $0_1^-, 1^-$	**3.5(6)**	**14**
	810.5; $0_2^+, 2^+$	**0.089(10)**	**0.41**
	684.8; $0_2^+, 0^+$	$0.150^{+0.016}_{-0.015}$	0.29
	121.8; $0_1^+, 2^+$	$0.906^{+0.065}_{-0.064}$	1.1
	0.0; $0_1^+, 0^+$	0.0056(6)	0.0032
1529.8; $1_1^-, 2^-$	1292.8; $0_3^+, 2^+$	0.017(3)	0.031*
$\tau = 395^{+93}_{-64}$	1233.8; $2_1^+, 3^+$	0.59(12)	0.25
	1085.8; $2_1^+, 2^+$	**1.17(23)**	**0.11**
	1041.1; $0_1^-, 3^-$	26^{+4}_{-6}	10.3
	963.4; $0_1^-, 1^-$	$4.1^{+0.7}_{-1.0}$	4.6
	810.5; $0_2^+, 2^+$	**0.0089(17)**	**0.11**
	121.8; $0_1^+, 2^+$	$0.260^{+0.051}_{-0.050}$	0.24
1579.4; $1_1^-, 3^-$	1371.7; $2_1^+, 4^+$	$\mathbf{1.37^{+0.12}_{-0.11}}$	**0.0026***
$\tau = 104(8)$	1292.8; $0_3^+, 2^+$	**0.156(30)**	**0.018***
	1233.8; $2_1^+, 3^+$	0.369(43)	0.12
	1085.8; $2_1^+, 2^+$	$0.412^{+0.036}_{-0.035}$	0.19
	1041.1; $0_1^-, 3^-$	7.8(8)	2.6
	1023.0; $0_2^+, 4^+$	$0.165^{+0.015}_{-0.014}$	0.59
	963.4; $0_1^-, 1^-$	8(1)	12.7
	810.5; $0_2^+, 2^+$	0.416(34)	0.68
	366.5; $0_1^+, 4^+$	$1.30^{+0.11}_{-0.10}$	4.1
	121.8; $0_1^+, 2^+$	$0.265^{+0.022}_{-0.021}$	0.62
1682.0; $1_1^-, 4^-$	1371.7; $2_1^+, 4^+$	<0.11	0.088*
$\tau > 860$	1233.8; $2_1^+, 3^+$	**<1.3**	**0.049**
	366.5; $0_1^+, 4^+$	<0.11	0.25
1764.3; $1_1^-, 5^-$	1023.0; $0_2^+, 4^+$	$0.61^{+0.47}_{-0.35}$	0.74
$\tau = 111^{+134}_{-48}$	706.8; $0_1^+, 6^+$	$\mathbf{1.4^{+1.1}_{-0.8}}$	**6.9**
	366.5; $0_1^+, 4^+$	$0.41^{+0.32}_{-0.23}$	1.2
1649.9; $2_1^-, 2^-$	1292.8; $0_3^+, 2^+$	$0.213^{+0.038}_{-0.036}$	0.062
$\tau = 236^{+47}_{-35}$	1233.8; $2_1^+, 3^+$	$\mathbf{2.41^{+0.43}_{-0.41}}$	**0.020**
	1085.8; $2_1^+, 2^+$	$\mathbf{4.40^{+0.78}_{-0.74}}$	**0.58**
	1041.1; $0_1^-, 3^-$	$\mathbf{1.1^{+0.2}_{-0.3}}$	**0.032**
	963.4; $0_1^-, 1^-$	$\mathbf{10.2^{+0.2}_{-0.3}}$	**0.39**
	810.5; $0_2^+, 2^+$	0.048(8)	0.17
	121.8; $0_1^+, 2^+$	$\mathbf{0.117^{+0.021}_{-0.020}}$	**0.0072**
1730.2; $2_1^-, 3^-$	1371.7; $2_1^+, 4^+$	$1.99^{+0.28}_{-0.26}$	0.56
$\tau = 118^{+15}_{-13}$	1233.8; $2_1^+, 3^+$	$\mathbf{2.5^{+0.9}_{-0.8}}$	**0.12**
	1085.8; $2_1^+, 2^+$	$1.45^{+0.22}_{-0.21}$	0.66
	1023.0; $0_2^+, 4^+$	$\mathbf{0.228^{+0.032}_{-0.030}}$	**1.5**
	963.4; $0_1^-, 1^-$	**6.8(11)**	**0.61**
	810.5; $0_2^+, 2^+$	0.46(6)	0.81
	366.5; $0_1^+, 4^+$	**0.53(7)**	**0.11**
	121.8; $0_1^+, 2^+$	0.068(9)	0.023
1680.6; $0_2^-, 1^-$	1292.8; $0_3^+, 2^+$	0.066(26)	0.24*
$\tau = 55(4)$	1085.8; $2_1^+, 2^+$	**0.057(17)**	**0.68**
	1082.9; $0_3^+, 0^+$	**0.116(20)**	**0.79**
	1041.1; $0_1^-, 3^-$	8.4(17)	9.5
	963.4; $0_1^-, 1^-$	11(2)	3.5
	810.5; $0_2^+, 2^+$	$5.40^{+0.48}_{-0.43}$	6.2
	684.8; $0_2^+, 0^+$	$2.12^{+0.21}_{-0.19}$	2.5
	121.8; $0_1^+, 2^+$	$\mathbf{0.076^{+0.009}_{-0.008}}$	**1.5**
	0.0; $0_1^+, 0^+$	$\mathbf{0.0407^{+0.0058}_{-0.0056}}$	**0.76**
1779.1; $0_2^-, 3^-$	1041.1; $0_1^-, 3^-$	$\mathbf{46^{+8}_{-10}}$	**1.9**
$\tau = 81^{+16}_{-13}$	1023.0; $0_2^+, 4^+$	$3.94^{+0.76}_{-0.66}$	1.3
	810.5; $0_2^+, 2^+$	$2.39^{+0.46}_{-0.40}$	0.81
	366.5; $0_1^+, 4^+$	$\mathbf{0.0188^{+0.0037}_{-0.0033}}$	**0.53**
	121.8; $0_1^+, 2^+$	$\mathbf{0.0189^{+0.0041}_{-0.0037}}$	**0.11**

only slightly larger than the energy difference between the ground state and the 0_2^+ level of 684 keV. These facts lead to the conclusion that the $K^\pi = 0_2^-$ band is an octupole excitation built on the 0_2^+ state; this is the first firm evidence for such an excitation in a deformed nucleus.

In order to test the observations of the present work with the phase-transition picture, calculations with the *spdf* version of the interacting boson model (IBM), as described in Ref. [19], were performed. The transition rates resulting from this calculation are presented in Table I. The negative-parity states in the calculation are assigned to bands based on the dominant $E2$ branch. The agreement of the $E1$ rates for the $K^\pi = 0_1^-$ band is not surprising since the e_1 effective charge was adjusted to fit the known $B(E1)$ values for the 1_1^- and 3_1^- states in the Sm isotopes, and the $\chi_{\rm sp}$ and $\chi_{\rm df}$ parameters for the $E1$ transition operator were adjusted to reproduce the known $K^\pi = 0_1^-$ $B(E1)$ ratios in ^{152}Sm [19] (approximately reproduced by ratios of Clebsch-Gordan coefficients). Of the remaining $E1$ transitions, nearly half have a discrepancy of 4 or greater in their rate (indicated by bold entries in Table I). The $B(E1)$ values for the $K^\pi = 1^-$ band are reproduced reasonably well, with a few notable exceptions such as the strong decays observed (but not predicted) into the γ band. The description of the 2^- and 3^- members of the $K^\pi = 2^-$ band, however, is rather poor. The second $K^\pi = 0^-$ band

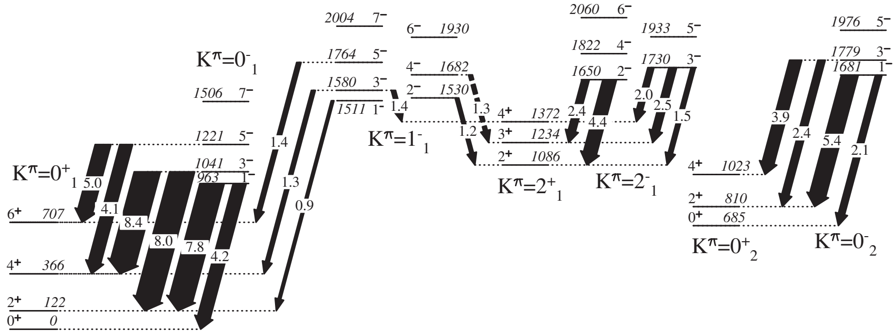

FIG. 3. Partial level scheme showing the negative-parity bands assigned in ^{152}Sm with transitions labeled with their absolute $B(E1)$ values (in units of m W.u.), with arrow widths proportional to the $B(E1)$, deduced using lifetimes from Ref. [10] and the present work and with γ-ray branching ratios from Ref. [12] and this study. Only those transitions with $B(E1)$ values on the order of 1 m W.u. and greater are shown.

in the calculations has the character of a state built on the 0_2^+ level, and while it is gratifying that the enhanced $E1$ transitions to the $K^\pi = 0_2^+$ band are reproduced, the calculations also indicate strong $E1$ transitions to the ground-state band, which are overpredicted by more than an order of magnitude on average.

Table I also lists the $E2$ transition rates, with the same e_2 effective boson charges as used for the positive-parity states [19]. There are a number of strongly enhanced $E2$ transitions observed connecting the negative-parity bands, and while in some cases these may involve mixed $E2/M1$ transitions for which the δ value is unknown, others are $\Delta J = 2$ transitions that must be purely $E2$ in nature. Of particular interest are the enhanced $E2$ transitions between the $K^\pi = 0_2^-$ band and the $K^\pi = 0_1^-$ band, with the $3^- \rightarrow 3^-$ transition perhaps as large as that of the $K^\pi = 0_2^+ \rightarrow 2^+_{\rm gsb}$ transition. As can be seen in Table I, some transitions are more than an order of magnitude stronger than predicted. The serious discrepancies observed for both the $E1$ and $E2$ transition rates reveal that the IBM calculations at the critical point do not reproduce in detail the nature of the negative-parity levels in ^{152}Sm.

In the traditional interpretation of the structure of ^{152}Sm, the 0_2^+ level might be regarded as an excellent candidate for a β vibration, as it has a large $B(E2; 0_2^+ \rightarrow 2_1^+) = 33.3 \pm 1.3$ W.u. [10] and a large $\rho^2(E0)$ value of $58 \pm 6 \times 10^{-3}$ [20], in line with expectations [21]. However, the large two-neutron-transfer cross section implies significant, if not dominant, pairing components [21]. In a recent study [8] of ^{152}Sm via Coulomb excitation, no candidates for higher-lying multiphonon β vibrations were found up to an energy of nearly $4 \times E(0_2^+)$, providing additional support that the 0_2^+ level cannot be interpreted as a β vibration. Together with the $K^\pi = 2^+$, $0_2^+ \otimes \gamma$ band discovered in Ref. [8], a picture is emerging of a series of repeating rotational bands built on the 0_2^+ state similar to those built on the ground state.

In summary, the $(n, n'\gamma)$ reaction has been used to study the negative-parity excitations below 2 MeV in ^{152}Sm. A new $K^\pi = 0^-$ band is established with a band head at 1681 keV that bears a striking similarity in its decay to the $K^\pi = 0_2^+$ band to the $K^\pi = 0_1^-$ band and its decay to the ground-state band. The $K^\pi = 0_2^-$ band is assigned as an octupole excitation built on the 0_2^+ state and is thus the first firm example of such an excitation in a deformed nucleus. Calculations with the $spdf$ IBM with values of parameters close to the critical point for the $U(5)$-$SU(3)$ phase transition display serious discrepancies with the data. The emerging pattern of repeating excitations built on the 0_2^+ level may indicate rather that ^{152}Sm is a complex example of shape coexistence.

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