Do Gluons Carry Half of the Nucleon Momentum?

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We examine the conventional picture that gluons carry about half of the nucleon momentum in the asymptotic limit. We show that this large fraction is due to an unsuitable definition of the gluon momentum in an interacting theory. If defined in a gauge-invariant and consistent way, the asymptotic gluon momentum fraction is computed to be only about one-fifth. This result suggests that the asymptotic limit of the nucleon spin structure should also be reexamined. A possible experimental test of our finding is discussed in terms of novel parton distribution functions.

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A classic prediction of perturbative quantum chromodynamics (QCD) is that about half of the nucleon momentum is carried by gluons in the asymptotic limit [1]. This renowned fraction has become a deeply rooted picture in the minds of hadron physicists, partially because it was supported [2], indirectly, by measurement of the quark momentum fraction in deeply inelastic scattering (DIS) of leptons off nucleons (which we will comment on later). Considered together with the nucleon spin problem discovered later, however, the taken-for-granted knowledge about the gluon contribution to the nucleon momentum becomes questionable: If the gluon momentum was known perfectly, why was severe difficulty (namely, with gauge invariance) encountered with even a sound theoretical definition of the gluon orbital angular momentum, which should be a direct extension of the gluon momentum?

To better appreciate and clarify the issue, let us recall that the conventional gluon momentum fraction is based on the following decomposition of the total momentum operator in QCD:

$$\vec{P}_{\rm total} = \int d^3x \psi^\dagger \frac{1}{i}\vec{D}\psi + \int d^3x \vec{E}\times\vec{B} \equiv \vec{\mathcal{P}}_q + \vec{\mathcal{P}}_g. \quad (1)$$

Here $\vec{D} = \vec{\nabla} - ig\vec{A}$ is the covariant derivative. (We suppress all color indices and generators.) The quark and gluon momentum fractions are defined through the nucleon-state expectation values of the renormalized operators $\vec{\mathcal{P}}_q^R$ and $\vec{\mathcal{P}}_g^R$. QCD prescribes the scale evolution of $\vec{\mathcal{P}}_q^R$ and $\vec{\mathcal{P}}_g^R$ according to the following well-known mixing matrix at leading order [1]:

$$\gamma^{\mathcal{P}} = -\frac{\alpha_s}{4\pi}\begin{pmatrix} -\frac{8}{9}n_g & \frac{4}{3}n_f \\ \frac{8}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}, \quad (2)$$

which leads to the well-known asymptotic limit, $\vec{\mathcal{P}}_g^R = \frac{2n_g}{2n_g+3n_f}\vec{P}_{\rm total}$, where $n_g = 8$ is the number of gluon fields and n_f is the number of active quark flavors. The most typical case of $n_f = 5$ gives $\vec{\mathcal{P}}_g^R \simeq \frac{1}{2}\vec{P}_{\rm total}$, which says that gluons carry about half the momentum of a nucleon (or virtually any hadron) in the asymptotic limit.

The doubtful point that we noted above is that the gluon momentum density $\vec{\mathcal{P}}_g(\vec{x}) = \vec{E}\times\vec{B}$ (the Poynting vector) in Eq. (1) cannot be used to construct a gluon orbital angular momentum. In fact, in a pure Yang-Mills theory (namely, without quarks), $\int d^3x\vec{x}\times\vec{\mathcal{P}}_g(\vec{x})$ gives the conserved total angular momentum of the gluon field, including both spin and orbital contributions:

$$\int d^3x\vec{x}\times(\vec{E}\times\vec{B}) = \int d^3x\vec{E}\times\vec{A} + \int d^3x\vec{x}\times E^i\vec{\nabla}A^i. \quad (3)$$

If we insist that a theoretically sound momentum density $\vec{P}(\vec{x})$ should match the orbital angular momentum in the standard form $\int d^3x\vec{x}\times\vec{P}(\vec{x})$, then Eq. (3) indicates that the gluon momentum density should be identified as $\vec{P}_g(\vec{x}) = E^i\vec{\nabla}A^i$. It has often been argued that there is no essential difference between $\vec{P}_g(\vec{x})$ and $\vec{\mathcal{P}}_g(\vec{x})$ since they give the same integrated gluon momentum. This equivalence, however, is limited to the pure Yang-Mills case:

$$\vec{E}\times\vec{B} = E^i\vec{\nabla}A^i - \nabla^i(E^i\vec{A}) + (\vec{\mathcal{D}}\cdot\vec{E})\vec{A}. \quad (4)$$

The second term is a total derivative and can be discarded in integration. By the QCD equation of motion, the third term (where $\vec{\mathcal{D}} = \vec{\nabla} - ig[\vec{A},\,]$ is the covariant derivative for the adjoint representation) is zero in the absence of quarks, but not zero otherwise. Hence we see that the quark-gluon interaction renders $\int d^3x\vec{\mathcal{P}}_g(\vec{x})$ and $\int d^3x\vec{P}_g(\vec{x})$ no longer equal. Then, naturally, we may find a totally different picture of the nucleon momentum partition if instead of $\int d^3x\vec{\mathcal{P}}_g(\vec{x})$ we define $\int d^3x\vec{P}_g(\vec{x})$ as the gluon momentum. This apparently interesting and important issue, however, has (to our knowledge) never been pushed to actual calculations. The obstacle is evident but not fully appreciated: that of gauge invariance, which we elaborate in some detail in the following.

The density $\vec{P}_g(\vec{x})$ is obviously gauge dependent. The gauge invariance of its integration, $\int d^3x \vec{P}_g(\vec{x})$, is guaranteed in a pure Yang-Mills theory by the equality to the gauge-invariant expression $\int d^3x \vec{\mathcal{P}}_g(\vec{x})$. When quarks are present, however, Eq. (4) says that $\int d^3x \vec{P}_g(\vec{x})$ loses its equality to $\int d^3x \vec{\mathcal{P}}_g(\vec{x})$, and thus loses simultaneously its property of gauge invariance.

Being a different density, $\vec{P}_g(\vec{x})$ does not give the same angular momentum $\int d^3x \vec{x} \times \vec{P}_g(\vec{x})$ as $\int d^3x \vec{x} \times \vec{\mathcal{P}}_g(\vec{x})$ even in a pure Yang-Mills theory, and is always gauge dependent. In fact, for several decades it was stated in common textbooks that the separation of spin and orbital angular momentum for a gauge field is conceptually not possible, even for the Abelian case [3]. If applied to our momentum discussion, this seems to dictate that the gluon momentum in the quark-gluon interacting case is intrinsically not meaningful, since without a proper orbital angular momentum definition we would have no means to match a momentum expression. In a pure Yang-Mills theory, it is possible to ignore the gauge dependence of $\int d^3x \vec{x} \times (E^i \vec{\nabla} A^i)$, and define the momentum density as $E^i \vec{\nabla} A^i$, which, though gauge dependent itself, does give a gauge-invariant integrated momentum. However, as we explained in the preceding paragraph, such a semijustification is lost in the quark-gluon interacting case.

It is quite tempting to agree with the common textbooks that in gauge theories the basic physical notions like the gluon or photon spin and orbital angular momentum are prohibited by gauge invariance, because construction of these quantities necessarily involves the gauge field $\vec{A}$. The gauge-invariance problem has long prohibited a consistent investigation of the nucleon spin structure [4] and, fairly speaking, was simply overlooked in the examination of the nucleon momentum.

Fortunately, a novel solution was achieved recently in Ref. [4], which supplies a unified and gauge-invariant treatment of the gluon momentum and angular momentum. The key to our solution is to recognize that in the gauge coupling $\bar{\psi}\gamma_\mu A^\mu \psi$, the gauge field A^μ plays a dual role: it provides a physical coupling to the Dirac field ψ, as well as a gauge freedom to compensate for the phase freedom of ψ. Our idea for solving the gauge-invariance problem is to decompose this dual role by seeking a unique separation $A^\mu = A^\mu_{\text{pure}} + A^\mu_{\text{phys}}$, with A^μ_{pure} a pure-gauge term transforming in the same manner as does the full A^μ, and always giving null field strength (i.e., $F^{\mu\nu}_{\text{pure}} \equiv \partial^\mu A^\nu_{\text{pure}} - \partial^\nu A^\mu_{\text{pure}} + ig[A^\mu_{\text{pure}}, A^\nu_{\text{pure}}] = 0$), and A^μ_{phys} a physical term transforming in the same manner as does $F^{\mu\nu}$. Namely, A^μ_{phys} is gauge invariant/covariant in the Abelian/non-Abelian case, while A^μ_{pure} has the same gauge freedom as A^μ and can be used instead of A^μ to construct a covariant derivative $D^\mu_{\text{pure}} \equiv \partial^\mu + igA^\mu_{\text{pure}}$ acting on the fundamental representation and $\mathcal{D}^\mu_{\text{pure}} \equiv \partial^\mu - ig[A^\mu_{\text{pure}},]$ acting on the adjoint representation. The field separation is to express A^μ_{phys} and A^μ_{pure} in terms of A^μ, thus not interfering with the gauge condition or canonical quantization, which manipulates the full A^μ. In this way, the spin and orbital angular momentum of the quark and gluon fields can all be constructed gauge invariantly at the density level:

$$\vec{J}_{\text{total}} = \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}} + \int d^3x \psi^\dagger \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^\dagger \frac{1}{i} \vec{D}_{\text{pure}} \psi. \quad (5)$$

Here we have improved over the expression in Ref. [4] by using the pure-gauge-covariant derivative for $\vec{A}_{\text{phys}}$ as well, which safely guarantees the gauge invariance of $E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}}$. This is somewhat a natural choice, since in QCD A^μ_{phys} is gauge covariant instead of invariant; thus as for ψ or $F^{\mu\nu}$, a covariant derivative should be applied. By doing so we are left with more freedom in choosing a defining equation for $\vec{A}_{\text{phys}}$ (e.g., $\vec{\mathcal{D}}_{\text{pure}} \cdot \vec{A}_{\text{phys}} = 0$), while in Ref. [4] the equation had to include $[\vec{A}_{\text{phys}}, \vec{E}] = 0$ so as to guarantee the gauge invariance of $E^i \vec{\nabla} A^i_{\text{phys}}$ with an ordinary derivative. We have checked that all consistencies in Ref. [4] are maintained in the present formulation. Moreover, superior to $[\vec{A}_{\text{phys}}, \vec{E}] = 0$, in the perturbative region $\vec{\mathcal{D}}_{\text{pure}} \cdot \vec{A}_{\text{phys}} = 0$ together with $F^{\mu\nu}_{\text{pure}} = 0$ can give explicit series expression, in powers of g, for A^μ_{phys} (which has trivial boundary behavior) and hence also $A^\mu_{\text{pure}} = A^\mu - A^\mu_{\text{phys}}$. As was noted in Ref. [4], a proper definition of A^μ_{phys} for the non-Abelian theory is quite subtle, but as we explain below, this subtlety does not affect the present calculation.

From Eq. (5), we can read out the corresponding gauge-invariant quark and gluon momentum operators:

$$\vec{P}_{\text{total}} = \int d^3x \psi^\dagger \frac{1}{i} \vec{D}_{\text{pure}} \psi + \int d^3x E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}} \equiv \vec{P}_q + \vec{P}_g. \quad (6)$$

It can be proven as in Ref. [4] that $\vec{P}_q + \vec{P}_g$ equals $\vec{\mathcal{P}}_q + \vec{\mathcal{P}}_g$ in Eq. (1). Nevertheless, the individual terms are distinct in the presence of quarks: $\vec{\mathcal{P}}_g$ differs from $\vec{P}_g$ by a gauge-invariant quark-gluon interaction term:

$$\int d^3x \vec{E} \times \vec{B} = \int d^3x E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}} + \int d^3x \psi^\dagger g \vec{A}_{\text{phys}} \psi. \quad (7)$$

Equipped with the theoretically sound gluon momentum definition which precisely matches an equally sound definition of the gluon orbital angular momentum, we can now take up the previously untouched calculation to reveal a possibly different picture of the nucleon momentum partition. The calculation is straightforward and parallels [1] that based on the operators in Eq. (1). Let us first display the mixing matrix for $\vec{P}_q$ and $\vec{P}_g$:

$$\gamma^P = -\frac{\alpha_s}{4\pi}\begin{pmatrix} -\frac{2}{9}n_g & \frac{4}{3}n_f \\ \frac{2}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}, \tag{8}$$

which gives a different asymptotic limit for the renormalized gluon momentum, $\vec{P}_g^R = \frac{n_g}{n_g+6n_f}\vec{P}_{\mathrm{total}}$. For the typical case of $n_f = 5$, this gives $\vec{P}_g^R \simeq \frac{1}{5}\vec{P}_{\mathrm{total}}$, as compared to $\vec{\mathcal{P}}_g^R \simeq \frac{1}{2}\vec{P}_{\mathrm{total}}$.

Some technical details are worth mentioning. Because the total momentum is conserved, the 2×2 evolution matrix of the quark and gluon momenta (in whatever definition) has only two independent elements, as can be seen in Eqs. (2) and (8). The easiest way of obtaining these two matrix elements is by calculating the expectation value of the gluon momentum operator in a quark state and the expectation value of the quark momentum operator in a gluon state, which both start at order α_s. By doing so, we avoid the task of computing the quark and gluon wave function renormalization if we consider only the leading-order evolution matrix. This strategy is advantageous in obtaining Eq. (2), and becomes almost indispensable when obtaining Eq. (8). The reason is that computation of the gluon wave function renormalization necessarily involves the non-Abelian three-gluon vertex, while for 1-loop calculation of the gluon matrix element in a quark state, the gluon field behaves like eight independent Abelian fields. Unlike in the non-Abelian case, the separation of an Abelian A^μ is unambiguous [4]. With vanishing boundary values for perturbative calculations, $\vec{A}_{\mathrm{pure}}$ is just the longitudinal field $\vec{A}_{\parallel} = \vec{\nabla}\vec{\nabla}^{-2}(\vec{\nabla}\cdot\vec{A})$, and $\vec{A}_{\mathrm{phys}}$ is just the transverse field $\vec{A}_{\perp} = -\vec{\nabla}\times\vec{\nabla}^{-2}(\vec{\nabla}\times\vec{A}) = \vec{A} - \vec{\nabla}\vec{\nabla}^{-2}(\vec{\nabla}\cdot\vec{A})$. ($A^0_{\mathrm{phys}}$ and A^0_{pure} are not relevant here.) Therefore the 1-loop insertion of $\vec{P}_g$ in a quark state becomes simplest in Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$, which gives $\vec{A}_{\mathrm{pure}} = \vec{0}$ and $\vec{A}_{\mathrm{phys}} = \vec{A}$. The calculation is then a standard textbook exercise, and leads to the first column in Eq. (8). These two matrix elements are $\frac{1}{4}$ of their counterparts in Eq. (2). As to the 1-loop insertion of $\vec{P}_q$ in a gluon state, we note that for a physical gluon, $\int d^3x \psi^\dagger \vec{A}_{\mathrm{phys}}\psi$ vanishes identically at 1-loop order. Then Eq. (7) means that $\vec{P}_q$ and $\vec{\mathcal{P}}_q$ give the same 1-loop expectation value in a physical gluon state, so the right-hand columns of Eqs. (2) and (8) are the same.

Thus we see that if the gluon momentum is defined properly, its contribution to the nucleon momentum is much smaller than conventionally accepted. The difference can be traced to the fact that the conventionally defined gluon momentum $\vec{\mathcal{P}}_g$ actually includes a quark-gluon interaction term, as indicated by Eq. (7), while $\vec{P}_g$ is constructed solely in terms of the physical degrees of freedom of the gluon field and is thus justified as a proper gluon momentum.

The novel and distinct asymptotic limit of the nucleon momentum partition we have presented naturally suggests that the asymptotic limit of the nucleon spin partition be reexamined according to the decomposition in Eq. (5), where the gluon spin and orbital angular momentum are constructed (as for $\vec{P}_g$) solely from the physical gluon configuration. We expect that this investigation would also reveal a distinct picture of the nucleon spin as compared to that in Ref. [5], which was based on a light-cone-gauge formulation and thus included nonphysical degrees of freedom in the gluon angular momentum.

In the remainder of this Letter, we discuss the delicate issue of possible experimental measurement of the proper quark and gluon momenta as defined by Eq. (6). To this end, it is very illuminating to first recall how the interaction-involving momenta in Eq. (1) are measured in hard processes. In fact, one may ask a key question: Why are interaction-involving quark and gluon momenta measured, instead of the proper ones? The answer lies, again, in gauge invariance.

Collision experiments do not measure the matrix elements of quark and gluon momentum operators directly. These matrix elements are extracted from the measured cross sections either via the operator product expansion analysis of DIS or via their relation to the quark and gluon parton distribution functions (PDFs) which can be shown to factorize in DIS and other hard processes [6]. The reason for the conventional operator product expansion approach to employ $\vec{\mathcal{P}}_q(\vec{x})$ and $\vec{\mathcal{P}}_g(\vec{x})$ is evident: Expansion of the products of the gauge-invariant quark electric currents requires gauge-invariant operators. The naively proper momentum operators $\psi^\dagger \frac{1}{i}\vec{\nabla}\psi$ and $E^i\vec{\nabla}A^i$ are gauge dependent. Conventionally, the only known way of accomplishing gauge invariance is by coupling a gauge field to ψ, and by using $F^{\mu\nu}$ instead of A^μ in constructing physical quantities of the gauge field. Either way, the physical content of the original quantity is substantially modified, because, as we explained earlier, a gauge field introduces not only a gauge freedom but also a physical coupling. Moreover, the restriction to $F^{\mu\nu}$ severely limits one's capability in constructing certain quantities, e.g., the spin. Now that we have decomposed the dual role of the gauge field by separating its pure-gauge component from its physical component, we can take exactly the needed part and discard the unwanted part for any desired goal. Namely, solely to restore gauge invariance, we use A^μ_{pure} instead of the full A^μ, while A^μ_{phys} is used instead of $F^{\mu\nu}$ whenever the gauge-field canonical variable is unavoidable in constructing a physical quantity. This is exactly how we succeed in constructing the gauge-invariant and proper momentum, spin, and orbital angular momentum of quarks and gluons in Eqs. (5) and (6). In principle, the gauge-invariant $\vec{P}_q(\vec{x})$ and $\vec{P}_g(\vec{x})$ can be used instead of $\vec{\mathcal{P}}_q(\vec{x})$ and

$\vec{\mathcal{P}}_g(\vec{x})$ in operator product expansion analysis of DIS, so that proper quark and gluon momenta can be measured.

It is less straightforward to see why the conventional factorization approach also refers to the interaction-involving quark and gluon momenta. In this approach, what one manipulates are the quark and gluon PDFs, which factorize in cross sections for certain hard processes [6] and can be integrated to give the quark and gluon momenta by moment relations [7]. Naively, one may expect that a quark or gluon PDF should provide an unambiguous representation of the quark or gluon property, but in conventional practice this is not the case. The key point is how one actually constructs the quark and gluon PDFs. Let us first look at the quark PDF in a target A [7]:

$$\mathcal{P}_{q/A}(\xi) = \int_{-\infty}^{\infty} \frac{dx^-}{4\pi} e^{-i\xi P^+ x^-} \Big\langle \bar{\psi}(0, x^-, 0_\perp)\gamma^+ \mathcal{P} \times \exp\Big\{ig \int_0^{x^-} dy^- A^+(0, y^-, 0_\perp)\Big\}\psi(0)\Big\rangle_A, \quad (9)$$

where a gauge link (Wilson line) is inserted to achieve gauge invariance. It is exactly this gauge link that makes the PDF defined by Eq. (9) an interaction-involving one, since the gauge field brings not only a gauge freedom but also a physical coupling to the quark field. The interaction term is more evident in the moment relation: $\int_{-\infty}^{\infty} d\xi \xi \mathcal{P}_{q/A}(\xi) = \frac{1}{2(P^+)^2}\langle \bar{\psi}\gamma^+ iD^+\psi\rangle_A$. This is just the $+$ component of $\vec{\mathcal{P}}_q(\vec{x})$ in Eq. (1). Here the gauge field in D^+ originates exactly from the gauge link in Eq. (9). It is often taken for granted that gauge invariance must be achieved at the price of a gauge coupling, and hence the measurable momenta are always the interaction-involving ones in Eq. (1).

But we do not really have to accept this confabulation for gauge invariance. Solely to guarantee gauge invariance, the gauge link can be constructed with the pure-gauge component $A^\mu_{\rm pure}$ instead of the full A^μ, and thus we can define a proper and gauge-invariant quark PDF,

$$P_{q/A}(\xi) = \int_{-\infty}^{\infty} \frac{dx^-}{4\pi} e^{-i\xi P^+ x^-} \Big\langle \bar{\psi}(0, x^-, 0_\perp)\gamma^+ \mathcal{P} \times \exp\Big\{ig \int_0^{x^-} dy^- A^+_{\rm pure}(0, y^-, 0_\perp)\Big\}\psi(0)\Big\rangle_A, \quad (10)$$

which integrates to give the desired proper quark momentum in Eq. (6): $\int_{-\infty}^{\infty} d\xi \xi P_{q/A}(\xi) = \frac{1}{2(P^+)^2} \times \langle \bar{\psi}\gamma^+ iD^+_{\rm pure}\psi\rangle_A$.

Analogously, the conventional gluon PDF

$$\mathcal{P}_{g/A}(\xi) = \int_{-\infty}^{\infty} \frac{dx^-}{2\pi\xi P^+} e^{-i\xi P^+ x^-} \Big\langle F^{+\nu}(0, x^-, 0_\perp)\mathcal{P} \times \exp\Big\{ig \int_0^{x^-} dy^- A^+(0, y^-, 0_\perp)\Big\} F_\nu{}^+(0)\Big\rangle_A \quad (11)$$

can be revised according to our strategy as

$$P_{g/A}(\xi) = \int_{-\infty}^{\infty} \frac{dx^-}{2\pi} e^{-i\xi P^+ x^-} \Big\langle F^{+i}(0, x^-, 0_\perp)\mathcal{P} \times \exp\Big\{ig \int_0^{x^-} dy^- A^+_{\rm pure}(0, y^-, 0_\perp)\Big\} A^i_{\rm phys}(0)\Big\rangle_A, \quad (12)$$

where, besides the pure-gauge link, the physical component $A^i_{\rm phys}$ is used instead of $F_\nu{}^+$ as the gauge-invariant canonical variable. The second moments of $\mathcal{P}_{g/A}$ and $P_{g/A}$ relate to the interaction-involving and proper gluon momentum in Eqs. (1) and (6), respectively.

It can be expected that the proper quark and gluon PDFs, $P_{q/A}$ and $P_{g/A}$, should (though probably nontrivially) factorize in the same processes to measure $\mathcal{P}_{q/A}$ and $\mathcal{P}_{g/A}$, so that the proper quark and gluon momenta in Eq. (6) can again be measured.

In closing, we remark that our approach is especially superior in gauge-invariant construction of polarized and transverse-momentum dependent PDFs with a clear particle number interpretation, and off-forward PDFs which can be measured to infer the orbital angular momenta in Eq. (5); e.g., the polarized gluon PDF can be defined gauge invariantly as

$$P_{\Delta g/A}(\xi) = \int_{-\infty}^{\infty} \frac{dx^-}{2\pi} e^{-i\xi P^+ x^-} \Big\langle F^{+i}(0, x^-, 0_\perp)\mathcal{P} \times \exp\Big\{ig \int_0^{x^-} dy^- A^+_{\rm pure}(0, y^-, 0_\perp)\Big\} \times \epsilon_{ij+} A^j_{\rm phys}(0)\Big\rangle_A, \quad (13)$$

with a first moment related to the gauge-invariant gluon spin in Eq. (5). Details will be reported elsewhere.

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