Scaling Laws of Turbulence and Heating of Fast Solar Wind: The Role of Density Fluctuations

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Incompressible and isotropic magnetohydrodynamic turbulence in plasmas can be described by an exact relation for the energy flux through the scales. This Yaglom-like scaling law has been recently observed in the solar wind above the solar poles observed by the Ulysses spacecraft, where the turbulence is in an Alfvénic state. An analogous phenomenological scaling law, suitably modified to take into account compressible fluctuations, is observed more frequently in the same data set. Large-scale density fluctuations, despite their low amplitude, thus play a crucial role in the basic scaling properties of turbulence. The turbulent cascade rate in the compressive case can, moreover, supply the energy dissipation needed to account for the local heating of the nonadiabatic solar wind.

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The interplanetary space is permeated by the solar wind [1], a magnetized, supersonic flow of charged particles originating in the high solar atmosphere and blowing away from the Sun. Low frequency fluctuations of solar wind variables are often described in the framework of fully developed hydromagnetic (MHD) turbulence [2,3]. The large range of scales involved, spanning from 1 AU ($\simeq 1.5 \times 10^8$ km) down to a few kilometers, make the solar wind the largest "laboratory" where MHD turbulence can be investigated using measurements collected *in situ* by instruments on board a spacecraft [3]. MHD turbulence is often investigated through the Elsässer variables $\mathbf{z}^{\pm} = \mathbf{v} \pm (4\pi\rho)^{-1/2}\mathbf{b}$, computed from the local plasma velocity $\mathbf{v}$ and magnetic field $\mathbf{b}$, ρ being the plasma mass density. In terms of such variables, MHD equations can be rewritten as $\partial_t \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} = -\nabla P/\rho + \text{diss}$, where P is the total hydromagnetic pressure and diss indicates dissipative terms involving the viscosity and the magnetic diffusivity. As in the Navier-Stokes equations for neutral fluids, the nonlinear terms $\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm}$ cause the turbulent energy transfer between different scales, at high Reynolds numbers where dissipative terms can be neglected. However, in the MHD case, they couple the two Elsässer variables, so that the Alfvénic MHD fluctuations $\mathbf{z}^{\pm}$, propagating along the background magnetic field, are advected by fluctuations $\mathbf{z}^{\mp}$ propagating in the opposite direction. The presence of strong correlations (or anticorrelations) between velocity and magnetic fluctuations, along with a nearly constant magnetic intensity and low amplitude density fluctuations, is usually referred to as the *Alfvénic state* of turbulence and implies that one of the two modes $\mathbf{z}^{\pm}$ should be negligible, making the nonlinear term of MHD equations vanish for pure Alfvénic fluctuations. In that case, the turbulent energy transfer should also disappear [4]. Alfvénic turbulence is observed almost ubiquitously in fast wind. This holds both in the ecliptic fast streams and in the high latitude wind blowing directly from the Sun coronal holes [3,5,6]. As pointed out in Ref. [4], the observation of Alfvénic state turbulence in the solar wind represents therefore a paradoxical "contradiction in terms."

MHD turbulence, however, satisfies an important *analytical* relation, which is the equivalent for magnetized fluids of the Kolmogorov or the Yaglom relations. Under suitable hypotheses, it has been shown [7,8] that the pseudoenergy fluxes $Y^{\pm}(\ell)$ through the scale ℓ of the increments of the Elsässer fields $\Delta\mathbf{z}^{\pm}(\ell) = \mathbf{z}^{\pm}(\mathbf{x}+\ell) - \mathbf{z}^{\pm}(\mathbf{x})$ follow a linear scaling relation

$$Y^{\pm}(\ell) \equiv \langle |\Delta\mathbf{z}^{\pm}|^2 \Delta z_{\parallel}^{\mp} \rangle = -\tfrac{4}{3}\epsilon^{\pm}\ell. \tag{1}$$

Here $\Delta z_{\parallel}^{\pm}$ represents the component of the increment $\Delta\mathbf{z}^{\pm}$ along the direction ℓ, and $\epsilon^{\pm}$ are the dissipation rates per unit mass of the pseudoenergies $\langle |\mathbf{z}^{\pm}|^2 \rangle/2$ ($\langle\cdot\rangle$ indicates space averages). The scaling law (1) has been recently observed experimentally in polar wind [8] and in the ecliptic plane [9,10]. The confirmation of the scaling law (1) is an important step towards the solution of the apparent paradox of the Alfvénic turbulent state, because it unambiguously shows that an MHD cascade is present, maybe with a weak transfer rate, despite the strong velocity-magnetic fields correlations.

Relation (1), which is of general validity within MHD turbulence, is not always realized in the solar wind observations [8]. Indeed, it is possible that local characteristics of the solar wind plasma do not always satisfy the assumptions required for (1) to be valid, namely, large-scale homogeneity, isotropy, and incompressibility. The role of anisotropy of solar wind turbulence, which is expected to be important, is not considered in this Letter. Density fluctuations in solar wind have a low amplitude, so that nearly incompressible MHD framework is usually considered [11,12]. However, compressible fluctuations are observed, typically convected structures characterized by anticorrelation between kinetic pressure and magnetic pressure [13]. Properties and interaction of the basic MHD modes in the compressive case have also been considered in the past [14,15]. In the present Letter, we show that density fluctuations, despite their very low amplitude, play a central role in the turbulent energy transfer and that a phenomenological scaling law obtained by taking into account density fluctuations is observed in a much larger proportion of fast solar wind. We also show that the turbulent dissipation can account for a large fraction of the local heating causing a slower than expected decrease of temperature with distance. A first attempt to include density fluctuations in the framework of fluid turbulence was due to Lighthill [16]. He pointed out that, in a compressible energy cascade, the mean energy transfer rate *per unit volume* $\epsilon_V \sim \rho v^3/\ell$ should be constant in a statistical sense (v being the characteristic velocity fluctuations at the scale ℓ), obtaining $v \sim (\ell/\rho)^{1/3}$. Fluctuations of a density-weighted velocity field $\mathbf{u} \equiv \rho^{1/3}\mathbf{v}$ should thus follow the usual Kolmogorov scaling $u^3 \sim \ell$. The same phenomenological conjecture can be introduced in MHD turbulence by considering the pseudoenergy dissipation rates per unit volume $\epsilon_V^{\pm} \equiv \rho\epsilon^{\pm}$ and introducing density-weighted Elsässer fields, defined as $\mathbf{w}^{\pm} \equiv \rho^{1/3}\mathbf{z}^{\pm}$. The equivalent of the Yaglom-type relation

$$W^{\pm}(\ell) \equiv \langle|\Delta\mathbf{w}^{\pm}|^2\Delta w_{\parallel}^{\mp}\rangle\langle\rho\rangle^{-1} = -\tfrac{4}{3}\epsilon^{\pm}\ell \qquad (2)$$

should then hold for the density-weighted increments $\Delta\mathbf{w}^{\pm}(\ell)$. Note that we have defined the flux $W^{\pm}(\ell)$ so that it reduces to $Y^{\pm}(\ell)$ in the case of constant density, allowing for comparisons between the compressible scaling (2) and the purely incompressible one (1). Despite its simple phenomenological derivation, the introduction of the density fluctuations in the Yaglom-type scaling (2) seems to describe correctly the turbulent cascade for compressible fluid (or magnetofluid) turbulence. The law for the velocity field has been observed in recent numerical simulations [17,18].

We will now study the cascade properties of compressive MHD turbulence from solar wind data collected by the spacecraft Ulysses. In order to avoid as far as possible variations due to solar activity, or other ecliptic disturbances such as slow wind sources, coronal mass ejection, and current sheets, we concentrate our analysis on pure Alfvénic state turbulence observed in high speed polar wind. We use here measurements from the Ulysses spacecraft in the first six months of 1996. This period was characterized by low solar activity, so that solar origin disturbances were almost absent. Moreover, the spacecraft orbit was at high and slowly decreasing heliolatitude, from about 55° to 30°, and presented small variations of the heliocentric distance r, from 3 AU to 4 AU. Since the mean wind speed $\langle\mathbf{v}\rangle$ in the spacecraft frame is much larger than the typical velocity fluctuations and is nearly aligned with the radial direction R, space scales ℓ can be viewed as time scales τ, related through the Taylor hypothesis by $\ell = -\langle v_R\rangle\tau$. We then used 8 minutes averaged time series of both Elsässer variables $\mathbf{z}^{\pm}(t)$ and density $\rho(t) = n_p + 4n_{\mathrm{He}}$ (obtained as the sum of proton density and 4 times He density), to compute the density-weighted time series $\mathbf{w}^{\pm}(t)$. From this time series we calculate the increments $\Delta\mathbf{w}^{\pm}(\tau) = \mathbf{w}^{\pm}(t+\tau) - \mathbf{w}^{\pm}(t)$ for different time lags τ and the third-order mixed structure functions $W^{\pm}(\tau) = \langle|\Delta\mathbf{w}^{\pm}(\tau)|^2\Delta w_R^{\mp}(\tau)\rangle_t$ by time averaging $\langle\cdot\rangle_t$ over windows of fixed duration t. The same procedure has also been used to calculate the quantities $Y^{\pm}(\tau)$ using the time series of the Elsässer fields $\mathbf{z}^{\pm}(t)$. In order to eliminate instationarities, heliolatitude and heliocentric distance changes, and to explore the wind properties locally, averages are computed over a moving window of about 11 days, consisting of 2048 data points. Accuracy of the third-order moments estimate [19] was tested with such a sample size [20]. We found that the third-order structure functions $W^{\pm}(\tau)$ computed from the Ulysses data show a linear scaling

$$W^{\pm}(\tau) \sim \tfrac{4}{3}\epsilon^{\pm}\langle v_R\rangle\tau \qquad (3)$$

during a considerable fraction of the period under study. In particular, we observed linear scaling of $W^{+}(\tau)$ in about half of the signal, while $W^{-}(\tau)$ displays scaling on about a quarter of the sample. As a comparison, the corresponding incompressive scaling law for $Y^{\pm}(\tau)$ was observed only in a third of the whole period, considerably smaller than the compressible case [8]. The portions of wind where the scaling is present are distributed in the whole period, and their extensions span from 6 hours up to 10 days. The linear scaling law generally extends on about 2 decades, from a

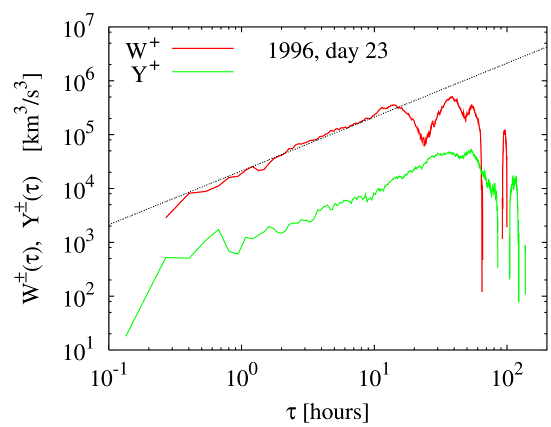

FIG. 1 (color online). One example of the mixed third-order compressible pseudoenergy flux $W^{+}(\tau)$ as computed from the Ulysses data during days 23–32 of 1996. The incompressible flux $Y^{+}(\tau)$ in the same time window and a linear fit are also indicated. In this case, both compressible and incompressible fluxes obey a Yaglom-like law.

few minutes to one day or more. For the compressible scaling, the two fluxes $W^{\pm}(\tau)$ coexist in a large number of cases. This does not hold for the incompressive scaling, where in general the scaling periods for the two fluxes $Y^{\pm}(\tau)$ are disjoint.

Figure 1 shows one example of both mixed third-order structure functions $W^{+}(\tau)$ and $Y^{+}(\tau)$ computed in the same 11 day windows where the scaling was observed. Figure 2 shows two more examples of scaling, observed for both $W^{+}(\tau)$ and $W^{-}(\tau)$, in two different time windows. The $W^{+}(\tau)$ scaling extends over 2 decades, while $W^{-}(\tau)$ behaves linearly on the whole range of scales considered here (3 decades). In the last example, the scaling is not present for the incompressible fluxes $Y^{\pm}(\tau)$. This example shows that the inclusion of compressible effect through the density-weighted fluctuations improves the scaling (2) and modifies the energy cascade. The scaling relation (2) also allows a direct estimate of the pseudoenergy transfer rates in the compressible case. A fit of the linear law (3) provides the local values of the amount of pseudoenergy transferred from large to small scales by the turbulent MHD cascade. This was already measured in the incompressive case [8,21], so that it is possible to compare the transfer rates in the two cascades. The mean values, computed over the 46 observed scaling cases at different radial distances from the Sun ($\pm$ their dispersion, in [$\mathrm{J\,kg^{-1}\,sec^{-1}}$]), for the compressible cascade are $\epsilon^{+} = 3668 \pm 1900$ (29 cases)

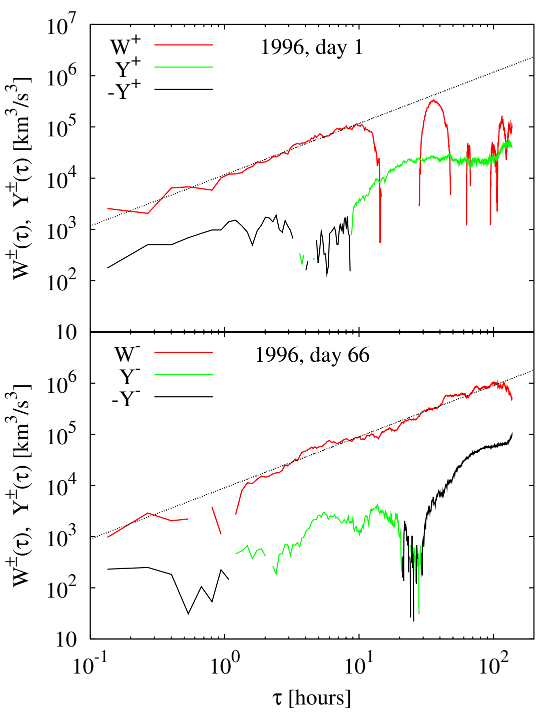

FIG. 2 (color online). Top panel: An example of the third-order compressible pseudoenergy flux $W^{+}(\tau)$ during days 1–10 of 1996. Bottom panel: $W^{-}(\tau)$ for days 66–75 of the same year. In both panels, the corresponding incompressible fluxes $Y^{\pm}(\tau)$ (no scaling present) and a linear fit are displayed.

and $\epsilon^{-} = 3536 \pm 2500$ (17 cases). Both values are considerably larger than the corresponding values for the incompressive case ($\epsilon_I^{+} = 182 \pm 73$, 24 cases, and $\epsilon_I^{-} = 156 \pm 50$, 11 cases [21]). This result shows again that the cascade in the solar wind is strongly enhanced by density fluctuations, despite their small amplitude. Note that the new variables are built by coupling the Elsässer fields with the density, before computing the scale-dependent increments. Moreover, the third-order moments are very sensitive to intense field fluctuations (intermittency), that could arise when density fluctuations are correlated with velocity and magnetic field. Similar results, but with a considerably smaller effect, were found in numerical simulations of compressive MHD [22]. We should point out that experimental values of energy transfer rate in the incompressive case had been also estimated with different techniques from different data sets [9,10]. Those values are not in agreement with the present (incompressive case) results. However, the different nature of wind (ecliptic vs polar, fast vs slow, at different radial distances from the Sun) makes such a comparison only indicative.

An interesting open question is the problem of the solar wind heating. The first models of solar wind assumed an adiabatic cooling due to spherical expansion of plasma blowing out of the Sun. This would result in a radial decrease of the proton temperature $T(r) \sim r^{-\xi}$, with $\xi = 4/3$. On the contrary, spacecraft measurements [23] have shown that the temperature decay is slower than the adiabatic prescription, with $\xi \in [0.7, 1]$. This implies that some local heating mechanism is present. One standing hypothesis is that the heating could be provided by energy dissipation occurring at the small scales of a turbulent cascade [9,24,25]. By using Eq. (1), the rates at which the incompressible turbulent pseudoenergy is transported down the scales, and eventually dissipated at a small scale, can be measured directly from data. This has recently been used to investigate whether or not a turbulent cascade can heat the solar wind. Results were, however, not conclusive. In fact, the measured incompressible dissipation rate of pseudoenergies can account for only up to 50% of the solar wind heating [10,21].

Figure 3 shows the radial profiles of the pseudoenergy transfer rates for both the compressive and incompressive cascades. In the same figure, we show the profiles of the heating rates needed to obtain the observed temperatures, as estimated from heating models [9,21,25] and from the measured temperatures (the two different values refer to the different estimates of the temperature obtained from Ulysses instruments). It is evident that, while the incompressive cascade cannot provide all of the energy needed to heat the wind, the density fluctuations coupled with magnetohydrodynamic turbulence can supply the amount of energy required. This evidence shows the importance of the density fluctuations in polar, fast solar wind turbulence, confirming that it should be considered as an example of

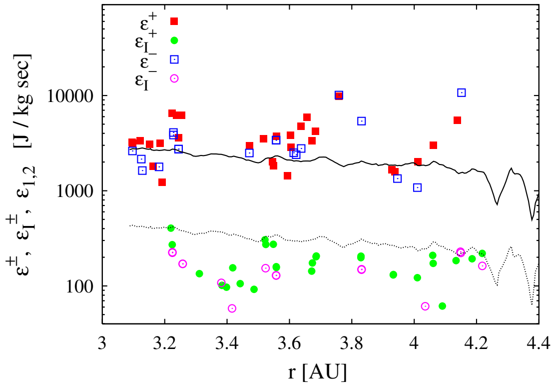

FIG. 3 (color online). Radial profile of the pseudoenergy transfer rates obtained from the turbulent cascade rate through the Yaglom relation, for both the compressive and the incompressive case. The solid lines represent the radial profiles of the heating rate required to obtain the observed temperature profile.

compressive fully developed MHD turbulence. Note that, since in a few samples we measured both ϵ^+ and ϵ^- in the same period, the values of the energy $\epsilon = (\epsilon^+ + \epsilon^-)/2$ and cross-helicity $\epsilon_H = (\epsilon^+ - \epsilon^-)/2$ transfer rates can be disentangled. From the values obtained, it is clear that the cross-helicity contribution, indicating the importance of the Alfvénic state of turbulence, can vary from a negligible fraction (less than 1%) to a considerable 25% of the energy contribution. Since its amplitude does not appear to be correlated with the observation of the cascade, Alfvénicity seems not to play a crucial role in the cascade at the observed scales. This would be in agreement with previous analysis of solar wind turbulence anisotropy, where the Alfvénic contribution to the field fluctuations is small [26,27].

In summary, we used the density-weighted Elsässer fields $\mathbf{w}^\pm$ to show for the first time that a phenomenological compressive Yaglom-like relation is verified to a large extent within the solar wind turbulence. This implies that low amplitude density fluctuations play a crucial role for scaling laws of solar wind turbulence [22]. This observation also confirms the recent results for the Kolmogorov 4/5 law from numerical simulations of compressible turbulence [17], while no experimental evidences from real fluids had been found so far. This could be attributed to the incompressible nature of flows in ordinary fluids accessible to laboratory experiments. Here, in fact, we present the first experimental observation of relation (2) in real systems. Using solar wind data, we have access to a sample of weakly compressible MHD turbulence in nature. The scaling law is found to be quite common and extends on a large range of scales, indicating not only that a nonlinear MHD cascade for pseudoenergies is active in the solar wind turbulence but also that compressible effects are an important ingredient of the cascade. We point out that the observed departures from the scaling law could be due to the presence of inhomogeneity and anisotropy in the solar wind [28]. The compressive corrections to the cascade also cause the transfer of a considerably larger amount of energy toward the small scales, where it can be dissipated to heat the plasma locally. The role of anisotropy in the solar wind turbulent cascade still remains an open question.

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