

Radial Spreading of Drift-Wave–Zonal-Flow Turbulence via Soliton Formation

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The self-consistent spatiotemporal evolution of a drift-wave (DW) radial envelope and a zonal-flow (ZF) amplitude is investigated in a slab model. The stationary solution of the coupled partial differential equations in a simple limit yields the formation of DW-ZF soliton structures, which propagate radially with speed depending on the envelope peak amplitude. Additional interesting physics, e.g., the generation, destruction, collision, and reflection of solitons, as well as turbulence bursting can also be observed due to the effects of linear growth or damping, dissipation, equilibrium nonuniformities and soliton dynamics. The propagation of soliton causes significant radial spreading of DW turbulence and therefore can affect transport scaling with the system size by broadening of the turbulent region. The correspondence of the present analysis with the description of DW-ZF interactions in toroidal geometry is also discussed.

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Explaining the size scaling of confinement properties in magnetized plasmas is one of the crucial and challenging problems of fusion energy research. It has been pointed out that turbulence spreading is responsible for local turbulence intensity dependence on global equilibrium properties [1], i.e., the system size, and, thus, for the size scaling of turbulent transport coefficients. Therefore, the nonlocal character of turbulent intensity plays a crucial role in the breakdown of gyro-Bohm scaling of turbulent transport and transition to Bohm scaling, as observed in several numerical simulations [2–4].

The radial propagation of drift-wave (DW) turbulence in tokamak plasmas was first investigated by Garbet [5], in the absence of zonal flow (ZF). Turbulence spreading was investigated also in Refs. [6,7]. Later on, using a single model equation for the local turbulence intensity, Hahn *et al.* [8] considered the “minimal problem” for turbulence spreading, which is about spatiotemporal diffusive propagation of a patch of turbulence as a fluctuation front from an unstable to a stable or a weaker drive region. A mean field theory of turbulent transport has been developed and extensively studied. By performing a Fokker-Planck analysis on the evolution of turbulence energy density, or applying quasilinear theory to the wave kinetic equation, one can derive a simple equation for the mean turbulence energy density. This approach leads to a reaction diffusion equation similar to the well-known Fisher equation [9,10]. Gürçan *et al.* [11] obtained an exact solution for this model, which describes a ballistic front propagation with speed given by the geometric mean of diffusion coefficient and linear growth. In this work it was pointed out that ballistic spreading is possible even without toroidal coupling effects. A more systematic approach [12] was proposed to explain turbulence spreading in terms of nonlinear mode couplings using a two-field Hasegawa-Wakatani

model (kinetic and internal energy) recovering the previous one-field model [11] in the proper limit, where the fluxes due to nonlinear interaction are written in the Fick’s law form. Analyses of turbulence spreading based on solutions of a bi-variate Burgers equation [13] for the evolution of the DW plasmon density were reported in Ref. [14]. Garbet *et al.* [15] also developed a two-field critical gradient model that couples a heat equation to an evolution equation for the turbulence intensity. It is shown that this model exhibits the dual character of turbulent dynamics, diffusive or ballistic, depending on parameters such as the heat flux and the wave number.

In spite of great effort, the fundamental dynamics of turbulence spreading is still not well understood. Although turbulence is truly a microscopic phenomenon, spreading or propagation of turbulence is usually related to meso-scale dynamics, e.g., intermittency, formation of avalanches, transport barriers and other coherent structures, which cannot be described by linear excitation and nonlinear wave-wave couplings via triad interaction processes only. ZFs are frequently assumed to be less or not important at all in the spreading process [8–10], based on the argument that large scale radially extended eddies are most effective at spreading turbulence, while ZFs inhibit spreading by destroying these structures [16,17]. However, slower DW turbulence spreading, observed in global gyrokinetic simulations when ZFs are included, has been attributed to the suppression of DW intensity by the ZFs [18,19] and not to their dynamic role.

In the present work, we study the nonlinear DW-ZF interplay in a simple slab geometry in order to elucidate the underlying physics mechanisms responsible for turbulence spreading. A general two-field DW-ZF model is derived for the spatiotemporal evolution of the DW radial envelope and ZF amplitude, which reduces to previous

descriptions [2,20,21] when ZF induced modulations on a given DW pump are considered with its sidebands (4-wave). Since the total energy cascades into shorter radial wavelengths via the coherent nonlinear DW-ZF modulation interaction, the local DW envelope nonlinearly steepens and the DW linear dispersion becomes stronger. Time scales for nonlinear interaction and linear dispersion eventually become comparable, showing analogies to the Langmuir soliton problem. Coherent structures are, thus, expected to form, such as DW-ZF solitons which will propagate radially. Turbulence spreading may then occur via DW-ZF soliton propagation with $x \sim t$, which is faster than any diffusive process.

The coherent 4-wave DW-ZF modulational interaction model for toroidal plasmas [21] has been derived based on the first-principles nonlinear gyrokinetic equation [22]. In this work, the same theoretical approach is applied in a simplified slab geometry [23], where x, y, z coordinates correspond to, respectively, the toroidal coordinates r, θ , and ζ . In particular, the radial wave number k_x may be understood as the wave number of the DW radial envelope, $k_r \equiv n\theta_k dq/dr$ [2,20,21]. In this respect, the results obtained in the present simplified slab model [24] can be expected to hold, at least qualitatively, in toroidal plasmas.

We start from the slab analysis of the electrostatic DW-ZF interaction model proposed in [23]. Similar to the Hasegawa-Mima model, using two-fluid description and quasineutrality condition, one can straightforwardly derive the DW evolution equation in the form [23]:

$$(1 - \rho_s^2 \nabla^2) \partial_t \phi_d - (c_s^2 / \Omega_i) \nabla \phi_d \times \hat{z} \cdot \nabla \ln n_0 - (c_s^2 / \Omega_i) \nabla \phi_z \times \hat{z} \cdot \nabla \phi_d + (c_s^2 \rho_s^2 / \Omega_i) \nabla \cdot [\nabla \phi_z \times \hat{z} \cdot \nabla \nabla \phi_d + \nabla \phi_d \times \hat{z} \cdot \nabla \nabla \phi_z] = 0; \quad (1)$$

where $c_s = \sqrt{T_e/m_i}$, $\rho_s = c_s / \Omega_i$ is the ion Larmor radius at the sound speed and the scalar potential is normalized to T_e/e ; the ZF potential $\phi_z = \langle \phi \rangle$, where $\langle \cdot \cdot \rangle$ represents flux surface averaging [(y, z) plane]. The last two terms on the right-hand side correspond to higher order Reynolds stress corrections due to nonlinear polarization drift, $O(k^4 \rho_s^4)$, which can be ignored when $|k \rho_s| \ll 1$. Meanwhile, ZF has $k_\theta = k_\parallel = 0$; thus, electrons do not behave adiabatically in the ZF potential. We can describe ZFs by the condition of no net radial flux:

$$\partial_t \nabla^2 \phi_z - (c_s^2 / \Omega_i) \langle \nabla \cdot [\nabla \phi_d \times \hat{z} \cdot \nabla \nabla \phi_d] \rangle = 0. \quad (2)$$

As in Refs. [2,20,23], we consider a coherent drift wave with single toroidal number n , or constant k_y in slab geometry. Thus, the 2-field coupled set of DW-ZF evolution equations are readily cast in the form

$$(1 + k_y^2 - \partial_x^2) \partial_t \phi_d + i \frac{\omega_*(x)}{\omega_*(0)} \phi_d = -iC \phi_d \partial_x \phi_z, \quad (3)$$

$$\partial_t \phi_z = iC \langle \phi_d \partial_x \phi_d^* - \text{c.c.} \rangle; \quad (4)$$

where $\omega_* = -k_y c_s / L_n$ is the diamagnetic drift frequency

and $L_n = (d \ln x / dx)^{-1}$ measures the nonuniformity scale; $C = k_y \Omega_i / \omega_*(0)$ is a constant, while space and time have been normalized to ρ_s and $\omega_*^{-1}(0)$, respectively. Note the structural similarity of Eqs. (3) and (4) with Eq. (4) of [2] in toroidal geometry where the most general form of the leading order radial envelope wave-packet propagation is given by Eq. (5) of [2]. Numerical simulations of the above coupled system show that DW-ZF can form solitary structures, which coherently propagate with characteristic speed (Fig. 1). These coherent structures are envelope solitons with wavelength of the carrier wave comparable to the envelope width, suggesting that turbulence spreading can be caused by soliton formation due to balance between DW dispersion and trapping by nonlinearly generated ZFs.

For the sake of simplicity, we initially ignore linear growth or damping and dissipation of both DW and ZF. For now, we also take ω_* constant. The $\omega_*(x)$ profile introduces extra effects of finite system size, which will be discussed elsewhere. Furthermore, we assume a coherent DW form $\phi_d(x, t) = A_d u_d(x, t) \exp(ik_x x - i\omega t)$, in which A_d is the maximum perturbation amplitude, usually $\approx 10^{-4} - 10^{-2}$, the normalized envelope function $u_d(x, t)$ is chosen to be real and long-scale $|\partial_x^2 u_d| \ll k_y^2 |u_d|$, the phase $\varphi = k_x x - \omega t$ describes fast oscillations in time but not necessarily in space, k_x is the radial wave number and ω is the DW frequency. Substituting the DW form given above into Eqs. (3) and (4), the coupled partial differential equations (PDEs) can be rewritten in the following nonlinear Schrödinger equation form

$$(1 + k_\perp^2)(\partial_t + v \partial_x) u_d + (i\omega - \partial_t) \partial_x^2 u_d - i\omega \lambda u_d = -iC \partial_x \phi_z u_d, \quad (5)$$

$$\partial_t \phi_z = 2C k_x A_d^2 u_d^2; \quad (6)$$

where $v = -2k_x \omega / (1 + k_\perp^2)$, $\lambda = (1 + k_\perp^2) - 1/\omega$, and $k_\perp^2 = k_x^2 + k_y^2$. For constructing a stationary solution, we

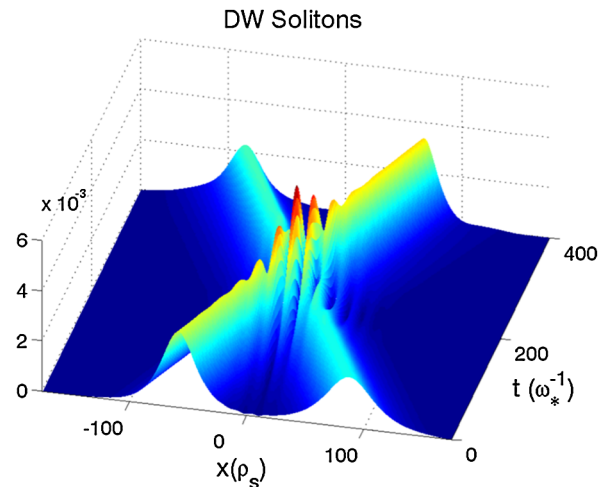


FIG. 1 (color online). Collision between two different DW-ZF solitons, where $A_{d1} = 0.002$, $k_x \rho_s = 0.3$ and $A_{d2} = 0.0003$, $k_x \rho_s = 0.3$.

introduce $\xi = (x - vt)\delta$ and take $u_d(x, t) = u_d(\xi)$ and $\phi_z(x, t) = \phi_z(\xi)$, such that $\partial_t = -v\delta\partial_\xi$ and $\partial_x = \delta\partial_\xi$. Here, the small parameter δ corresponds to the slowly varying envelope scale. Finally, substituting ϕ_z from Eq. (6) into Eq. (5), we derive one single ordinary differential equation (ODE) for the DW perturbation:

$$\delta^2 u_d'' - \lambda u_d + (C^2/\omega^2)(1 + k_\perp^2)A_d^2 u_d^3 = 0; \quad (7)$$

where terms $\propto \partial_t + v\partial_x$ cancel by construction and $\partial_x^2 \partial_t u_d$ is ignored assuming that the envelope transient time is much longer than the DW oscillation period, e.g. $v\delta \ll \omega$, which can be justified *a posteriori*. The above second order ODE clearly indicates the competition between linear dispersion and nonlinear self-trapping process. When the DW amplitude A_d increases, its envelope becomes nonlinearly steeper, i.e. δ increases; meanwhile, the DW dispersion also becomes stronger and tends to inhibit the focusing process. Formally, this corresponds to equating the three coefficients of u_d'' , u_d and u_d^3 , i.e.,

$$\delta^2 = 1 + k_\perp^2 - 1/\omega = C^2/(2\omega^2)(1 + k_\perp^2)A_d^2. \quad (8)$$

The DW wave-packet frequency ω is then readily obtained from the above quadratic equation, i.e., $\omega = [1 + \sqrt{1 + 2(1 + k_\perp^2)^2 C^2 A_d^2}]/[2(1 + k_\perp^2)]$. Note that the right-hand side contains the nonlinear frequency shift due to finite DW turbulence amplitude. Similarly, the parameter δ can also be determined as

$$\delta = \pm \sqrt{(1 + k_\perp^2)/2C(A_d/\omega)}. \quad (9)$$

This derivation is subject to our *a priori* assumption $\delta \ll \omega v^{-1}$, which guarantees that the DW oscillation ω occurs on the fastest time scale. Substituting δ , ω , v as functions of k_x and A_d , this assumption is equivalent to $k_x C A_d \ll \sqrt{(1 + k_\perp^2)/2\omega(k_x, A_d)}$. Finally, given Eq. (8), Eq. (7) becomes a dimensionless ODE governing the stationary envelope function u_d ,

$$u_d'' - u_d + 2u_d^3 = 0. \quad (10)$$

This ODE is analogous to that of an oscillator in the so called ‘‘Sagdeev potential’’ $\Phi(u_d) = (-u_d^2 + u_d^4)/2$, whose solution can be written as hyperbolic secant function $u_d(\xi) = \text{sech}(\xi)$, when appropriate boundary conditions are imposed, *viz.* $u_d \rightarrow 0$ at $|\xi| \rightarrow \infty$. Meanwhile, the ZF solution is obtained straightforwardly by integrating Eq. (6) once, such that $\phi_z(\xi) = \int 2k_x C/(v\delta) A_d^2 \text{Sech}^2(\xi) d\xi = \mp \sqrt{2(1 + k_\perp^2)A_d} \tanh(\xi)$ which satisfies the causality constraint, i.e., $\partial_\xi \phi_z \rightarrow 0$ when $|\xi| \rightarrow \infty$ for any initially localized DW turbulence. Expressions for DW and ZF in the laboratory frame are

$$\phi_d(x, t) = A_d \text{Sech} \left[\delta \left(x + \frac{2k_x \omega}{1 + k_\perp^2} t \right) \right] e^{ik_x x - i\omega t}, \quad (11)$$

$$\phi_z(x, t) = \sqrt{2(1 + k_\perp^2)A_d} \tanh \left[\delta \left(x + \frac{2k_x \omega}{1 + k_\perp^2} t \right) \right]. \quad (12)$$

From Eqs. (11) and (12), it generally follows that ZF potentials have radially moving structures of hyperbolic tangent shape; meanwhile, $E_z = -\partial \phi_z / \partial x$ manifests itself as scalar-potential wells in the background plasma and trap the corresponding DW packets. The radial scale of the soliton is $\rho_s^2/(L_n A_d)$ according to Eq. (9). Figure 1 shows the spatiotemporal evolution of two counter-propagating DW-ZF solitons, which are solutions of the original coupled PDEs, given $k_y = 0.3$, $\Omega_i/\omega = 100$. For consistency with our analytic approach, we have chosen initial k_x and A_d to satisfy the *a priori* assumption $\delta \ll \omega v^{-1}$. Furthermore, we assumed no ω_* equilibrium variation and no growth or damping and dissipation. Note that the two envelope solitons remain unchanged in both real and k space after the collision, although the dynamics during the collision can be quite complicated. This is one of the soliton’s essential features.

The radial propagation velocity of DW-ZF solitary structures, v , depends on both the radial wave number and the DW amplitude. It is different from the group velocity, $v_g = \partial \omega / \partial k_x$, which is determined by k_x through the linear dispersion relation only. Therefore, the solution of Eqs. (11) and (12) gives a two-parameter, k_x and A_d , family of solitons. Figure 2 shows the relation between v and k_x for small initial amplitude $A_d = 0.003$. Numerical and analytical results agree well when $k_x \leq 0.4$. The discrepancy when $k_x > 0.4$ originates from the breaking of the *a priori* assumption that $\partial_x^2 \partial_t \phi_d$ can be ignored in Eq. (5). Moreover, when A_d increases to about 10^{-2} , the analytical result is no longer valid either, since the ignored term $O(v\delta^3)$ modifies the solution at larger k_x or A_d , according to Eq. (9).

Our numerical simulation results for $A_d \geq 0.01$ show that the dominant asymptotic ($t \rightarrow \infty$) DW turbulence behavior is still of soliton type and the propagation velocity v increases with the DW amplitude A_d . We observe that the DW radial wave number no longer corresponds to its initial value but rather to δ , which is mainly determined by the

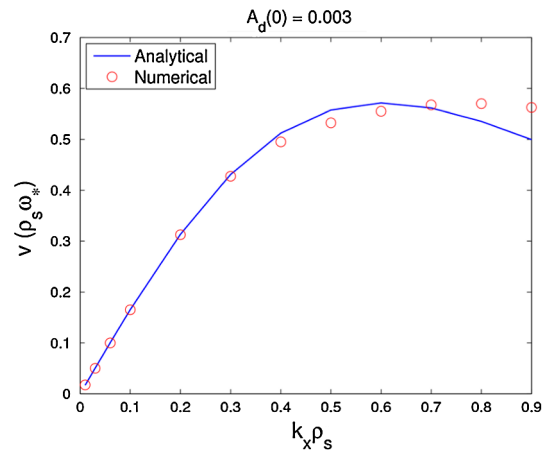


FIG. 2 (color online). The relation between v and k_x for $A_d = 0.003$.

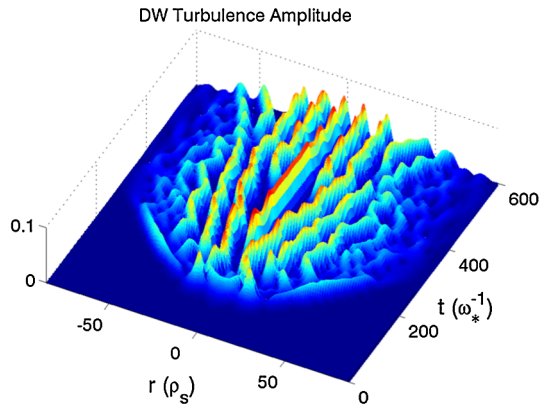


FIG. 3 (color online). DW turbulence spreading in the presence of growth or damping, dissipation and finite system size effects.

amplitude A_d alone. There seems to be a transition from a two-parameter to a one-parameter family of soliton solutions of the coupled system. If the amplitude becomes even larger, e.g. $A_d \geq 0.02$, the initially localized DW-ZF soliton breaks into many pieces in the form of solitons and wave trains; similar to Gardner's work [25] on the Korteweg–deVries equation, in which it is shown that a localized but otherwise arbitrary initial perturbation will generate a conventional wave train, quickly destroyed by dispersion, and a finite number of solitons, which characterize the asymptotic solution.

We studied the DW-ZF initial value problem in more general cases as well, i.e., in the presence of linear growth or damping, dissipations, and variation of equilibrium profiles. Figure 3 shows the evolution of DW turbulence out of initial random noise, with strong DW growth rate $\gamma_d(0) = 0.1$, uniform ZF damping rate $\gamma_z = 0.075$ and $L_p = 150\rho_s$, which represents the system size. Dissipations are also included. The drift frequency $\omega_*(x)$ has Gaussian shape centered at $x = 0$; DW turbulence is linearly unstable in the central region ($|x| < 80\rho_s$) but is damped in the outer region ($|x| > 80\rho_s$), i.e. $0.15 \exp(-x^2/(75\rho_s)^2) - 0.05$, while the ZF is uniformly damped. Figure 3 clearly shows formation and propagation of solitons, which however exhibit more complicated dynamic behaviors, for instance, growing amplitudes, slowing down of propagation speed, soliton breaking, turbulence bursting and more. Since coupled PDEs generally describe infinite-dimensional dynamical systems, DW turbulence dynamics appears mostly chaotic in the corresponding parameter space, (γ_d, γ_z) . Solitons may bounce back at their turning points, possibly enhancing nonlinear interactions inside the turbulent region and impacting the size scaling of turbulence transport. Figure 3 also demonstrates that the nonlinearly saturated turbulence has spread into a much broader region than that of its linear mode

structure sampling global equilibrium properties and, again, reflecting finite system size.

In summary, we have demonstrated the novel result that coherent structures such as radial envelope solitons can be constructed self-consistently in a two-field DW-ZF model and cause significant radial turbulence spreading in a slab plasma. Horton and Meiss [26] considered poloidal DW solitons in the absence of ZF and dispersion due to polarization drift. Despite the difference in the underlying physics, the inverse scattering method and statistical approach they adopted could also be applied here to resolve the radial DW-ZF dynamics. We have also shown the structural analogy of the underlying coupled PDEs for the nonlinear evolution of the DW radial envelope and ZF amplitude with the corresponding equations derived in toroidal geometry [2,20], demonstrating the generality of the present results and the possibility of readily extending them in future works. The size scaling of DW turbulence will also be discussed in detail in a separate work.

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