

## Pseudoscalar Perturbations and Polarization of the Cosmic Microwave Background

Maxim Pospelov,<sup>1,2</sup> Adam Ritz,<sup>1</sup> and Constantinos Skordis<sup>2</sup>

<sup>1</sup>*Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, V8P 5C2, Canada*

<sup>2</sup>*Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada*

(Received 15 August 2008; revised manuscript received 11 December 2008; published 30 July 2009)

We show that models of new particle physics containing massless pseudoscalar fields superweakly coupled to photons can be very efficiently probed with CMB polarization anisotropies. The stochastic pseudoscalar fluctuations generated during inflation provide a mechanism for converting  $E$ -mode polarization to  $B$ -mode during photon propagation from the surface of last scattering. The efficiency of this conversion process is controlled by the dimensionless ratio  $H/(2\pi f_a)$ , where  $H$  is the Hubble scale during inflation, and  $f_a^{-1}$  is the strength of the pseudoscalar coupling to photons. The current observational limits on the  $B$  mode constrain this ratio to be less than 0.07, which in many models of inflation translates to a sensitivity to  $f_a$  exceeding  $10^{14}$  GeV, which surpasses other tests.

DOI: 10.1103/PhysRevLett.103.051302

PACS numbers: 98.70.Vc, 14.80.Mz, 98.80.Cq

*Introduction.*—Within the last decade, precision observations of fluctuations in the cosmic microwave background (CMB) have provided us with a powerful probe of cosmology, and been the driving force in pinning down the parameters in the concordance  $\Lambda$ CDM model. In recent times, gains in sensitivity have allowed us to complement observations of the temperature fluctuations with those in various components of polarization. Much of the focus has been aimed at the detection of the so-called  $B$  mode of polarization, as it is an important probe of primordial gravitational waves, and thus of the possible mechanism for inflation. However, precision probes of polarization in various modes also present us with a powerful set of tools to test for new degrees of freedom with which the CMB photons may interact. In this Letter, we show that existing upper bounds on the  $B$  mode of polarization already place stringent constraints on any new light pseudoscalar degrees of freedom through their stochastic fluctuations generated during inflation.

The interactions of a massless pseudoscalar field  $a$  with photons may be parametrized in terms of a dimensionful coupling constant  $f_a^{-1}$ ,

$$\mathcal{L}_{\gamma a} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a + \frac{a}{2f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (1)$$

where  $F_{\mu\nu}$  is the electromagnetic field strength tensor, and  $\tilde{F}_{\mu\nu}$  its dual. Note that the absence of a potential,  $V(a) = 0$ , is protected by the shift symmetry of this action,  $a \rightarrow a + \text{const}$ , and while it is also natural to expect that the pseudoscalar  $a$  would have derivative couplings to the spins of other particles, this does not affect the physics of the CMB. The presence of the photon coupling implies that a space-time variation of the field  $a$  will induce a rotation  $\Delta\psi$  of the polarization of an electromagnetic wave. Provided the photon wavelength is much smaller than the characteristic scale on which the field  $a$  varies [1], the rotation angle is frequency independent:

$$\Delta\psi = \frac{1}{f_a}\Delta a. \quad (2)$$

In the past, CMB polarization has been used to probe an induced rotation angle due to a spatially homogeneous pseudoscalar profile  $a = a(t)$  [1,2], with the latest data [3,4] giving comparable sensitivity to the strongest constraints from polarized extragalactic radio sources [5]. However, the time evolution of the background effectively amounts to a bulk violation of parity and/or Lorentz invariance. Here, we consider a scenario perfectly in keeping with standard cosmology, and focus on a stochastic background for the pseudoscalar as would arise from a period of inflation. We exploit the fact that all massless scalars have a universal amplitude for fluctuations generated during inflation,  $\delta a = H/(2\pi)$ , where  $H$  is the inflationary Hubble parameter at horizon crossing. Without loss of generality, we are allowed to choose  $a_{\text{now}} = 0$ , and then the additional rotation  $\psi$  (2) for a photon arriving from the direction  $\hat{n}$  is simply given by the randomly fluctuating value of  $A(\hat{n}, \tau) \equiv a(\hat{n}, \tau)/f_a$  at the surface of last scattering (LSS),  $\tau \approx \tau_{\text{LSS}}$ , where  $\tau$  is the conformal time. To quantify the effect, we introduce the dimensionless parameter  $c_a$ :

$$c_a = \left(\frac{H}{2\pi f_a}\right)^2, \quad |\Delta\psi| \sim \sqrt{c_a}. \quad (3)$$

It is then apparent that any  $O(1)$  probe of these fluctuations will provide impressive sensitivity to  $f_a$  on the order of the inflationary Hubble scale. For a scale-invariant source of inflationary perturbations in the field  $a$ , the evolution of the rotation angle  $\psi$  with the scale factor  $R(t)$  can be pictured as an effective random walk where each  $e$ -fold of expansion corresponds to a separate step, so that the cumulative effect can be further enhanced relative to (3):  $(\Delta\psi)^2 \sim c_a \ln(R_{\text{now}}/R)$ .

In what follows, we analyze the generation of the  $B$  mode of CMB polarization from the inflationary fluctua-

tions of a massless pseudoscalar. We use the  $B$ -mode spectrum to deduce a precise limit on  $c_a$ , and finally describe its implications for models of particle physics incorporating new pseudoscalar fields. We should note that the impact of inflationary axion perturbations has previously been discussed in the context of isocurvature perturbations on the CMB temperature spectrum [6], but this is not directly relevant for nonaxionic massless pseudoscalars. The technical aspects of our analysis do nonetheless have parallels with studies of the impact on the CMB spectra of stochastic magnetic fields [7], of scalar moduli [8], and of weak lensing, in that the convolution of an additional fluctuating source with the underlying density perturbations tends to mix the power spectra for temperature and  $E$ - and  $B$ -mode polarization.

*Generating B-mode polarization.*—At the LSS the underlying spectrum of scalar (and tensor) perturbations leaves its imprint on the CMB through anisotropies in temperature and linear polarization, the latter generated by Thomson scattering. Linear polarization is conveniently described in terms of the  $Q$  and  $U$  Stokes parameters, and while a single scalar perturbation mode with  $\vec{k} \parallel \hat{z}$  does not generate  $U$ , it generates  $Q$ . We generalize the approach of Zaldarriaga and Seljak [9] where, for a mode of momentum  $k$  directed along  $\hat{z}$ , the observed value of  $Q$  in a given direction  $\hat{n}$  can be written as an integral over conformal time  $\tau$  along the line of sight,

$$Q(k, \hat{n}) = \frac{3}{4}(1 - \mu^2) \int_0^{\tau_0} d\tau e^{ix\mu} g(\tau) \Pi(k, \tau). \quad (4)$$

Here  $x = k(\tau_0 - \tau)$  and  $\mu = \hat{k} \cdot \hat{n}$ . The source and visibility functions,  $\Pi(k, \tau)$  and  $g(\tau)$  [9], reflect the details of the generation of polarization via Thomson scattering at the LSS. In the presence of a small stochastic rotation (2), a  $U$  component would also be generated,  $U \simeq 2\Delta\psi Q = 2AQ$ . This rotation also depletes  $Q$ , but this is a less significant effect. It follows that after combining a Fourier mode of the underlying scalar perturbation, with  $\vec{k} \parallel \hat{z}$ , and a Fourier mode of the pseudoscalar perturbation with arbitrary direction  $\vec{q}$ , the result for  $U$  can be expressed as

$$U(k, q, \hat{n}) = \frac{3}{2}(1 - \mu^2) \times \int_0^{\tau_0} d\tau e^{ix\mu + iy\nu} g(\tau) \Pi(k, \tau) \Delta_A(\tau, q), \quad (5)$$

where  $y = q(\tau_0 - \tau)$  and  $\nu = \hat{q} \cdot \hat{n}$ .  $\Delta_A(q, \tau)$  is the transfer function, which has a generic form for any massless (pseudo)scalar field:  $\Delta_A(q, \tau)$  is normalized to one on large scales, and oscillates with a decaying envelope determined by the matter content on subhorizon scales. At the next step, we calculate basis-independent expansion coefficients by performing the projection onto spin 2-weighted spherical harmonics,  $a_{B\ell m} = -\int d\Omega (Y_{2,\ell m}^* + Y_{-2,\ell m}^*) U(\hat{n})/2$ , as described in [9],

$$a_{B\ell m} = \frac{3}{2} \left[ \frac{(l-2)!}{(l+2)!} \right]^{1/2} \int d\Omega_n d^3k d^3q Y_{0,\ell m}^*(\hat{n}) \times \int_0^{\tau_0} d\tau [m^2 - (1 + \partial_x^2)^2 x^2] e^{ix\mu + iy\nu} F(\tau, \vec{k}, \vec{q}). \quad (6)$$

Since the  $a_{B\ell m}$ 's are basis independent, we are now allowed to generalize the scalar-pseudoscalar source function to modes of arbitrary direction  $\vec{k}$  and  $\vec{q}$ , which are in turn determined by the primordial scalar  $\xi(\vec{k})$  and pseudoscalar  $\xi_A(\vec{q})$  perturbations:

$$F(\tau, \vec{k}, \vec{q}) = g(\tau) \Pi(k, \tau) \Delta_A(q, \tau) \xi(\vec{k}) \xi_A(\vec{q}). \quad (7)$$

The standard assumption that these perturbations are Gaussian random variables implies

$$\begin{aligned} \langle \xi^*(\vec{k}_1) \xi(\vec{k}_2) \rangle &= P_\phi(k_1) \delta^{(3)}(\vec{k}_1 - \vec{k}_2), \\ \langle \xi_A^*(\vec{q}_1) \xi_A(\vec{q}_2) \rangle &= P_A(q_1) \delta^{(3)}(\vec{q}_1 - \vec{q}_2), \end{aligned} \quad (8)$$

where  $P_\phi$  is the (now rather well-measured) primordial power spectrum for scalar perturbations. The scalar primordial power spectrum  $P_\phi$  sources the CMB anisotropy and indeed structure formation, while the pseudoscalar power spectrum  $P_A$  is, in our approach, directly linked to  $c_a$  in (3) by the standard inflationary prediction,

$$P_A(q) = \frac{c_a}{4\pi q^3} q^{n_a - 1}. \quad (9)$$

Here  $n_a$  is the spectral index of the pseudoscalar primordial power spectrum, with  $n_a = 1$  being scale invariant. The factor of  $4\pi$  reflects the normalization convention for Fourier transforms in [9] that we follow in this Letter.

To proceed, we expand the exponential  $e^{ix\mu + iy\nu}$  in (6) into a basis of products of spherical harmonics and, using their orthogonality and completeness, perform all the angular integrals. Ultimately we obtain

$$\begin{aligned} C_{B\ell} &= \frac{1}{2\ell + 1} \sum_m \langle a_{B\ell m}^* a_{B\ell m} \rangle \\ &= \frac{4(4\pi)^3 (l-2)!}{2\ell + 1 (l+2)!} \sum_{m, l_1, l_2} (2l_1 + 1)(2l_2 + 1) \binom{l}{0} \binom{l_1}{0} \binom{l_2}{0}^2 \int k^2 P_\Phi q^2 P_A dk dq |\Delta_{l_1 l_2 m}(k, q)|^2, \end{aligned} \quad (10)$$

with the generalized transfer function,

$$\begin{aligned} \Delta_{l_1 l_2 m}(k, q) &= \frac{3}{4} \int_0^{\tau_0} d\tau g(\tau) j_{l_1}(x) j_{l_2}(y) \left[ \frac{(l_1 + 2)!}{(l_1 - 2)!} \frac{1}{x^2} - m^2 \right] \\ &\quad \times \Delta_A(\tau, q) \Pi(\tau, k). \end{aligned} \quad (11)$$

The sum over  $m$  can be straightforwardly computed, while the summation range for  $l_1$  and  $l_2$  is restricted to  $|l_1 - l_2| \leq l \leq l_1 + l_2$ , which is enforced by the Wigner 3j symbol. It is important to note that the cross correlations  $TB$  and  $EB$  vanish on account of the overall conservation of parity.

At this point we assume a spectrum of scalar fluctuations with scalar spectral index  $n = 0.963$  chosen according to the best fit model for the WMAP5 CMB data, and take the pseudoscalar spectral index  $n_a$  to have the same value. We used our own code based on CMBFAST [10] to numerically calculate the functions  $\Pi(k, \tau)$  and  $\Delta_A(\tau, q)$  for the WMAP5 best fit model, and then to compute  $C_{Bl}$  using Eqs. (9)–(11). Figure 1 displays the results for  $C_{Bl}$  for a fiducial choice of  $c_a = 4.2 \times 10^{-3}$ . For comparison, the plot also shows  $C_{El}$  as well as the  $B$  modes generated by primordial tensor perturbations with  $r = 0.14$  and by lensing of the  $E$  mode. The qualitative form of  $C_{Bl}$  induced by pseudoscalar perturbations can be understood by looking at the dominant regions in the  $l_1$  and  $l_2$  sums. For large  $l$ , we find that  $l_1 \sim l$ , while the sum over  $l_2$  is effectively truncated at a lower value of order  $l_2^{\max} \sim (\tau_0 - \tau_{\text{LSS}})/\tau_{\text{LSS}} \sim 50$  (higher values of  $l_2$  contribute no more than about 2% to the  $C_{Bl}$ 's on small scales). Thus for large  $l \sim 1000$  the induced  $B$  mode closely tracks the underlying  $E$  mode. For lower values of  $l \sim O(1)$ , both  $l_1$  and  $l_2$  saturate at higher scales so the  $B$  mode is somewhat larger. Finally, the overall scale of the oscillations in  $C_{Bl}$  is slightly suppressed, as is to be expected from convoluting the underlying  $E$ -mode source with a Gaussian random variable.

It is important to keep in mind that our approximation, which assumes a small rotation angle, may break down once  $C_{Bl}$  becomes comparable to  $C_{El}$ . Consequently, for setting constraints on  $C_{Bl}$  we choose  $l$  in the interval 100–1000, where the most recent experimental results of the QUaD collaboration [11] probe the  $B$  modes well below the detected  $E$ -mode level,  $C_{Bl} \sim O(0.1) \times C_{El}$ . The QUaD limits, shown in Fig. 1, impose a stringent constraint on  $c_a$ :

$$c_a < 4.2 \times 10^{-3} \Rightarrow f_a > 2.4 \times 10^{14} \text{ GeV} \times H_{14}, \quad (12)$$

where we have introduced  $H_{14} \equiv H/10^{14} \text{ GeV}$ , a normalization inspired by the fact that  $H_{14} \sim O(1)$  is close to the maximal inflationary value of  $H$  allowed by observations. The inflationary Hubble scale can be traded for the tensor-to-scalar ratio  $r$ , commonly used to parametrize the strength of all massless perturbations including gravitational waves,  $r = 0.14H_{14}^2$ . Note that from Eq. (3), the conclusion that  $c_a \ll 1$  justifies *a posteriori* our perturbative treatment of the rotation of polarization. The constraint (12) is the main result of this Letter, and we will discuss its implications below. In Table I, we show the sensitivity to

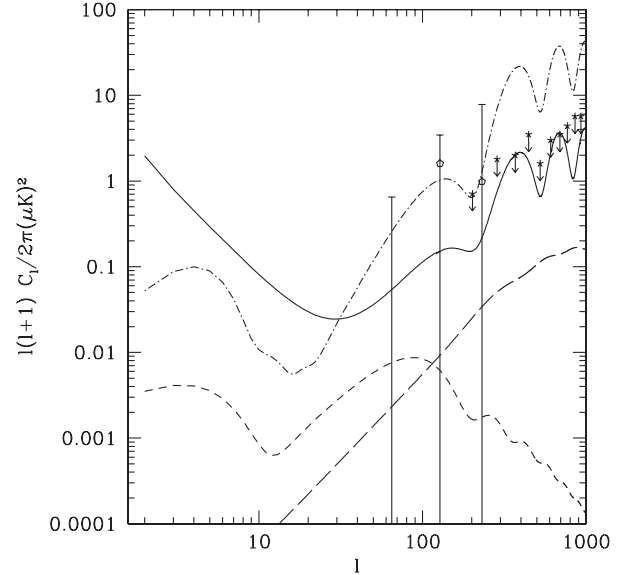


FIG. 1. We display the  $C_l$ 's for the  $E$ -mode (dash-dotted line) and induced  $B$ -mode polarization (solid line). The fiducial value  $c_a = 4.2 \times 10^{-3}$  was chosen to match the current upper bound on the  $B$  mode using data from the QUaD collaboration. Ninety-five percent upper limits from QUaD (stars) and  $2\sigma$  upper limits from WMAP5 (polygons) are shown for comparison. We also exhibit the  $C_{Bl}$ 's induced by lensing of the  $E$  mode (long-dashed line), and the contribution from primordial gravitational waves with  $r = 0.14$  (short-dashed line), corresponding to the choice  $H_{14} = 1$ .

$f_a$  of future experimental probes of  $B$ -mode polarization in relation to the sensitivity to  $r$ .

*Implications for particle physics.*—Given the conventional picture of inflationary cosmology, the constraint obtained above applies to massless pseudoscalars (or almost massless with a mass below the Hubble scale at decoupling), and it is important to consider how such new low-energy degrees of freedom may naturally arise. Recall that massless pseudoscalar fields coupled to the operator  $G_{\mu\nu}\tilde{G}^{\mu\nu}$  in QCD, namely, axions, resolve the strong  $CP$  problem in a natural way [12] while gaining an anomaly-induced mass  $m_a \sim m_\pi f_\pi/f_a$ . While this mass is large compared to the scales relevant here, this mechanism for inducing a mass is unique, and thus should two or more pseudoscalars couple to the QCD anomaly, only one linear combination would become massive [13]. Schematically, below the QCD scale, such models lead to an effective Lagrangian of the form

$$\left(\frac{a_1}{2g_1} + \frac{a_2}{2g_2}\right)G_{\mu\nu}\tilde{G}^{\mu\nu} + \left(\frac{a_1}{2f_1} + \frac{a_2}{2f_2}\right)F_{\mu\nu}\tilde{F}^{\mu\nu} \rightarrow \mathcal{L}_{\text{QCD}a} + \frac{a}{2f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (13)$$

where besides the Lagrangian  $\mathcal{L}_{\text{QCD}a}$  for the QCD axion, one has a massless pseudoscalar  $a$  as part of  $\mathcal{L}_{\gamma a}$  in (1) with  $f_a^{-1} = (g_2/f_2 - g_1/f_1)/\sqrt{g_1^2 + g_2^2}$ . In the absence of any special reasons for cancellation, the generic case is

$f_a^{-1} \neq 0$ . Although we refrain from assessing the likelihood of new high-energy physics leading to two or more light pseudoscalar fields, we can refer to various scenarios in string theory where multiple pseudoscalar moduli are a generic prediction (see, e.g., [14]). Therefore, the



TABLE I. Approximate sensitivity for future measurements of  $B$ -mode polarization, using the range  $l \sim 20\text{--}80$ .

| $r$   | $c_a \times 10^6$ | $f_a/H_{14}$ GeV     |
|-------|-------------------|----------------------|
| 0.1   | 135               | $1.4 \times 10^{15}$ |
| 0.01  | 13.5              | $4.3 \times 10^{15}$ |
| 0.001 | 1.35              | $1.4 \times 10^{16}$ |

Lagrangian (1) can easily originate from a more fundamental theory with multiple degrees of freedom. Note that it would take a significant increase in sensitivity before the limit in Eq. (12) could start probing  $f_a$  at the scales most relevant for string theory,  $f_a \sim 10^{16}$  GeV  $\times H_{14}$ .

Existing constraints on the coupling  $f_a$  arise via several known mechanisms. Most notably, the CAST experiment directly limits the emission of pseudoscalars from the solar interior and has obtained the limit  $f_a > 2 \times 10^{10}$  GeV [15], which is quite competitive with stellar constraints that typically require  $f_a \gtrsim 10^{11}$  GeV [16]. Pseudoscalar-photon conversion in the intergalactic magnetic field provides an additional means of probing  $f_a$  [17], with the typical sensitivity  $f_a \sim 10^{11}\text{--}10^{12}$  GeV  $\times (B_{\text{IG}}/\text{nG})$  strongly dependent on the strength of the intergalactic magnetic field  $B_{\text{IG}}$ , and on the number density of free electrons. Thus we observe that the constraint derived in this Letter (12) is complementary to other probes of the model, and for  $H_{14} \sim O(1)$  is superior to any other constraint by at least 2 orders of magnitude.

*Concluding remarks.*—Despite all the evidence for physics beyond the standard model in the neutrino sector and in cosmology, there is no compelling need for new degrees of freedom at or below the electroweak scale. Indeed, neutrino masses and dark matter can arise from heavy degrees of freedom, while dark energy is currently consistent with being a cosmological constant. Nonetheless, many models of new high-energy physics do lead to new light degrees of freedom whose mass may be protected by symmetry, as is the case for the pseudoscalars considered here, and we have shown that CMB physics can potentially be a very important probe of such models. The stochastic fluctuations of a light pseudoscalar field generated during inflation can induce the conversion of  $E$ -mode to  $B$ -mode polarization in the CMB, and with the current upper bounds this already leads to stringent constraints (12). We anticipate that forthcoming experiments will significantly improve the sensitivity to  $H/(2\pi f_a)$  provided the lensing background can be handled appropriately, and thus these experiments may provide the primary sensitivity to new physics of this type.

The search for primordial gravitational waves using  $B$  modes has intrinsic limitations in that the effect scales as  $(H/M_{\text{Pl}})^2$ , and so for  $H$  below  $10^{13}$  GeV the detection of tensor modes in the CMB becomes problematic. In this respect, the search for pseudoscalar fluctuations provides

an independent motivation for studying  $B$  modes, less predicated on the scale of inflation being large. Indeed, with the existing constraints on  $f_a$  [15],  $B$  modes may be generated at an observable level within a wide class of inflationary models with  $10^{10}$  GeV  $\leq H \leq 10^{14}$  GeV. However, since both pseudoscalars and gravitational waves (when combined with the background contribution from lensing) can have similar power spectra up to normalization, it will be important to investigate whether the two can be distinguished observationally.

We thank N. Afshordi, N. Kaloper, V. Mukhanov, and S. Speziale for helpful discussions. The work of M. P. and A. R. was supported in part by NSERC of Canada, and research at the Perimeter Institute is also supported in part by NSERC and by the Province of Ontario through MEDT.

- 
- [1] D. Harari and P. Sikivie, Phys. Lett. B **289**, 67 (1992).
  - [2] A. Lue, L. M. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
  - [3] E. Komatsu *et al.* (WMAP Collaboration), arXiv:0803.0547.
  - [4] B. Feng, M. Li, J. Q. Xia, X. Chen, and X. Zhang, Phys. Rev. Lett. **96**, 221302 (2006); G. C. Liu, S. Lee, and K. W. Ng, Phys. Rev. Lett. **97**, 161303 (2006).
  - [5] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990); S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
  - [6] M. Turner and F. Wilczek, Phys. Rev. Lett. **66**, 5 (1991).
  - [7] A. Kosowsky, T. Kahniashvili, G. Lavrelashvili, and B. Ratra, Phys. Rev. D **71**, 043006 (2005); T. Kahniashvili, Y. Maravin, and A. Kosowsky, arXiv:0806.1876; M. Giovannini and K. E. Kunze, Phys. Rev. D **78**, 023010 (2008).
  - [8] K. Sigurdson, A. Kurylov, and M. Kamionkowski, Phys. Rev. D **68**, 103509 (2003).
  - [9] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997).
  - [10] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).
  - [11] C. Pryke *et al.* (QUaD Collaboration), arXiv:0805.1944.
  - [12] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
  - [13] A. Anselm and N. Uraltsev, Phys. Lett. B **114**, 39 (1982).
  - [14] P. Svrcek and E. Witten, J. High Energy Phys. **06** (2006) 051.
  - [15] S. Andriamonje *et al.* (CAST Collaboration), J. Cosmol. Astropart. Phys. **04** (2007) 010.
  - [16] G. G. Raffelt, Annu. Rev. Nucl. Part. Sci. **49**, 163 (1999).
  - [17] C. Csaki, N. Kaloper, and J. Terning, Phys. Rev. Lett. **88**, 161302 (2002); C. Deffayet, D. Harari, J. P. Uzan, and M. Zaldarriaga, Phys. Rev. D **66**, 043517 (2002); L. Ostman and E. Mortzell, J. Cosmol. Astropart. Phys. **02** (2005) 005; A. Mirizzi, G. G. Raffelt, and P. D. Serpico, Phys. Rev. D **72**, 023501 (2005); M. Y. Piotrovich, Yu. N. Gnedin, and T. M. Natsvlishvili, arXiv:0805.3649.