Hall Effect of Spin Waves in Frustrated Magnets

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We examine a possible spin Hall effect for localized spin systems with no charge degrees of freedom. In this scenario, a longitudinal magnetic field gradient induces a transverse spin current carried by spin wave excitations with an anomalous velocity which is caused by a topological Berry-phase effect associated with spin chirality, in analogy with anomalous Hall effects in itinerant electron systems. Our argument is based on semiclassical equations of motion applicable to general spin systems. Also, a microscopic model of frustrated magnets which exhibits the anomalous spin Hall effect is presented.

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Introduction.—Spin transport phenomena in condensed matter systems have been attracting much interest because of potential applications to spintronics [1,2], and also because of their fundamental relation with the notion of topologically induced spin currents [3–5]. For semiconductors and metals with spin-orbit interactions, the spin Hall effect (SHE) in the absence of magnetic fields was predicted theoretically [1], and later experimentally realized [2]. This effect allows for topological interpretation in terms of the Berry phase; the spin-orbit interaction gives rise to the Berry curvature associated with the Bloch wave function, which results in the nonzero anomalous velocity producing a transverse force even in the absence of the Lorentz force [3]. This leads to the anomalous Hall effect (AHE) for the case without time-reversal symmetry [3–7], and to SHE for the case with time-reversal symmetry [1]. On the other hand, it was shown by several authors that the anomalous velocity is also generated by spin textures accompanying spin chirality, and the AHE occurs in itinerant electron systems coupled with localized spins, the orientation of which possesses spin chirality order [8]. In this Letter, we propose a possible analogue of this phenomenon for localized spin systems with no charge degrees of freedom. We demonstrate that, in analogy with metallic systems, the topological Berry-phase effect associated with spin chirality raises the dissipationless spin Hall current induced by a longitudinal magnetic field gradient: i.e., Hall effect of spin waves.

Spin current transport associated with spin chirality was also studied before for one-dimensional magnets; the Berry phase due to spin textures induces dissipationless spin currents [9,10], which parallel persistent charge currents in metal rings threaded by a magnetic flux [11]. However, the existence of a transverse force due to the topological Berry phase has not been noticed so far except in a pioneering work done by Haldane and Arovas [12], in which the Hall effect for spin currents in quantum spin systems was investigated for the case of two-dimensional (2D) chiral spin liquids with spin excitation gap, with emphasis on the quantum Hall effect characterized by the nonzero Chern number. However, unfortunately, up to now, there has been no experimental evidence for the realization of 2D spin liquid states with spin gap, though some very recent experiments suggest a possible existence of them in frustrated magnets [13]. Here, we demonstrate that such exotic transport phenomena are possible even in more conventional magnets; spin Hall currents carried by conventional spin wave excitations exist in magnetically ordered states [14]. We will, first, develop a general argument for the SHE in spin systems employing a semiclassical approach to spin dynamics, which is similar to a topological description for the Hall effect in electron systems developed by Sundaram and Niu [3]. Then, we will present an example of a microscopic model in which the Hall effect of spin waves occur.

Semiclassical spin dynamics with Berry-phase effects.—Here, we consider a semiclassical theory for spin dynamics which is in analogy with Sundaram and Niu's topological theory for the Hall effect of Bloch electrons [3]. In the Sundaram-Niu approach, the dynamics of canonically conjugate variables $\boldsymbol{x}$ and $\boldsymbol{k}$ are described by semiclassical equations of motion which include topological Berry-phase effects; i.e., $\dot{\boldsymbol{x}} = \nabla_k \varepsilon_k + \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}$, where $\boldsymbol{\Omega}$ is the Berry curvature. The second term is the origin of the intrinsic anomalous Hall effect. The purpose here is to establish an analogous topological description for spin transport in localized spin systems. We consider spin systems defined on a two-dimensional lattice with the Hamiltonian $\mathcal{H}$ which preserves the total spin projection $\sum_i S_i^z$. We assume that spins on nearest neighbor sites i and j interact with each other, and spin operators S_i^μ with $\mu = x, y, z$ appear in the Hamiltonian in the combinations $S_i^z S_j^z$, $S_i^+ S_j^-$, and $S_i^- S_j^+$. We also include the Zeeman term with a spatially varying magnetic field $\mathcal{H}_Z = -g\mu_B \sum S_i^z B_i^z$ with $\nabla B^z \neq 0$, which plays a role similar to an electric field in electron systems [12]. To identify the canonically conjugate variables associated with spin transport, we use the Haldane representation for spin operators [15], $S_i^+ = \sqrt{S + S_i^z} e^{i\hat{\phi}_i} \sqrt{S - S_i^z}$, where S_i^z and the phase operator $\hat{\phi}_i$ are canonically conjugate variables satisfying the commutation relation $[\hat{\phi}_i, S_j^z] = i\delta_{ij}$. In terms of S^z and $\hat{\phi}$, we define a new set of canonically conjugate variables,

$\hat{\chi}_i^\mu = \hat{\phi}_i - \hat{\phi}_{i+\hat{\mu}}$ and $\hat{\mathcal{T}}_i^\mu = -\sum_{\ell_\mu=1}^{i} S_\ell^z$, where $\hat{\mu}$ is a basic lattice vector along the μ direction with $\mu = x$ or y, and the sum over the lattice site ℓ in the definition of $\hat{\mathcal{T}}_i^\mu$ is taken only for the μ direction. These operators satisfy the commutation relation $[\hat{\mathcal{T}}_i^\mu, \hat{\chi}_j^\mu] = i\delta_{ij}$. The operators $\hat{\chi}^\mu$ and $\hat{\mathcal{T}}^\nu$ with $\mu \neq \nu$ do not commute with each other. However, this noncommutativity is not important for the following argument. The operators $\hat{\mathcal{T}}^\mu$ and $\hat{\chi}_i^\mu$ are, respectively, analogous to an electric dipole operator $e\boldsymbol{x}$ and momentum $\boldsymbol{k}$ for electron systems. In terms of the spin wave language, $\hat{\phi}$ is the U(1) phase for the spin wave boson, and thus $\hat{\chi}$, i.e., the gradient of $\hat{\phi}$, is just the U(1) gauge field, i.e., momentum. The time derivative of $\hat{\mathcal{T}}_i^\mu$ gives a spin current $J_i^\mu \equiv d\hat{\mathcal{T}}_i^\mu/dt$ which satisfies the continuity equation $\dot{S}_i^z + \sum_{\hat{\mu}}[J_{i+\hat{\mu}}^\mu - J_i^\mu] = 0$.

We consider the case that the low-energy states are dominated by gapless magnon excitations from the ground state, and the semiclassical approximation for magnons is applicable, which implies that the expectation values $\chi_i^\mu = \langle u|\hat{\chi}_i^\mu|u\rangle$ and $\mathcal{T}_i^\mu = \langle u|\hat{\mathcal{T}}_i^\mu|u\rangle$ with $|u\rangle$ the eigenstates of $\mathcal{H}_{\rm tot} = \mathcal{H} + \mathcal{H}_Z$ can be regarded as continuous variables. We also assume that different sets of $\{\chi_i^\mu, \mathcal{T}_i^\mu\}$ correspond to different eigenstates, which is valid within the conventional spin wave approximation. Then, we can introduce a state vector which is a linear combination of the eigenstates, and a solution for the time-dependent Schrödinger equation for $\mathcal{H}_{\rm tot}$: $|\Psi(\mathcal{T}^\mu)\rangle = \int d\chi a_\chi |u(\chi_i^\mu, \mathcal{T}_i^\mu)\rangle$, where $d\chi = \prod_i d\chi_i^\mu$ and the normalization condition $\int d\chi |a_\chi|^2 = 1$ is imposed. The state vector $|\Psi\rangle$ plays a role similar to the wave packet function for electron systems [3]. We consider the dynamics of the "wave packet", i.e., the equations of motion for $\langle\Psi|\hat{\chi}^\mu|\Psi\rangle$ and $\langle\Psi|\hat{\mathcal{T}}^\mu|\Psi\rangle$, under a spatially slowly varying external field which yields adiabatic changes of the states and the Berry-phase effect. We also introduce a new state $|\tilde{u}\rangle = \hat{U}|u\rangle$ with $\hat{U} = \exp(-i\sum_i \chi_i^\mu \hat{\mathcal{T}}_i^\mu) = \exp(i\sum_i \phi_i S_i^z)$. The operation of $\hat{U}$ amounts to the gauge transformation in the spin space which rotates the spin axis at the ith site around the z axis by an angle ϕ_i. Then, we obtain $\langle\Psi|\hat{\mathcal{T}}_i^\mu|\Psi\rangle = \int d\chi a_\chi^* i \frac{\partial a_\chi}{\partial \chi_i^\mu} + \int d\chi d\chi' a_{\chi'}^* a_\chi \langle\tilde{u}'| i \frac{\partial}{\partial\chi_i^\mu}|\tilde{u}\rangle$, with $|\tilde{u}'\rangle = |\tilde{u}(\chi_i^{\mu'}, \mathcal{T}_i^\mu)\rangle$. Here we have assumed the orthogonality relation $\langle u'|u\rangle = \delta(\chi_i^{\mu'} - \chi_i^\mu)$. The above equation implies that the variable $\hat{\mathcal{T}}^\mu$ acts on the function a_χ as an operator defined by $\{\hat{\mathcal{T}}_i^\mu\}_{\chi\chi'} = i\frac{\partial}{\partial\chi_i^\mu}\delta(\chi - \chi') + i\langle\tilde{u}'|\frac{\partial}{\partial\chi_i^\mu}|\tilde{u}\rangle$. This is regarded as the χ representation for the operator $\hat{\mathcal{T}}^\mu$, and analogous to the crystal momentum representation of the spatial coordinate $\boldsymbol{x}$ [16]. The second term $i\langle\tilde{u}'|\frac{\partial}{\partial\chi_i^\mu}|\tilde{u}\rangle$ is the Berry connection. In the case with multiple magnon modes, $\hat{\mathcal{T}}^\mu$ has the intramode components $\{\hat{\mathcal{T}}^\mu\}_{n\chi n\chi'}$ as well as the intermode components $\{\hat{\mathcal{T}}^\mu\}_{n\chi n'\chi'}$ where n is the index for the nth magnon mode. The intramode components satisfy the commutation relation $[\hat{\mathcal{T}}^\mu, \hat{\mathcal{T}}^\nu] = i\varepsilon_{\lambda\mu\nu}(\boldsymbol{\Omega}_{\chi\chi})_\lambda$, where $\boldsymbol{\Omega}_{\chi\chi}$ is the Berry curvature given by $(\boldsymbol{\Omega}_{\chi\chi})_\mu = i\frac{\epsilon_{\mu\nu\lambda}}{2}(\langle\frac{\partial\tilde{u}}{\partial\chi^\nu}|\frac{\partial\tilde{u}}{\partial\chi^\lambda}\rangle - \langle\frac{\partial\tilde{u}}{\partial\chi^\lambda}|\frac{\partial\tilde{u}}{\partial\chi^\nu}\rangle)$. This commutation relation is a key for topological spin dynamics, and is in analogy with the commutation relation of $\boldsymbol{x}$, $[x^\mu, x^\nu] = i\varepsilon_{\mu\nu\lambda}\Omega_\lambda$, which is the origin of the intrinsic AHE for Bloch electrons [16]. The spin dynamics and the spin current are derived from the Heisenberg equation of motion for $\hat{\mathcal{T}}^\mu$. The Zeeman term is rewritten as $\mathcal{H}_Z = -g\mu_B\sum_i S_i^z B_i^z = g\mu_B \sum_i \hat{\mathcal{T}}_i^\mu \nabla_\mu B^z$, which is analogous to the coupling between an electric dipole and an electric field. Then, from the commutation relation for $\hat{\mathcal{T}}^\mu$, the equations of motion for the expectation values of $\hat{\mathcal{T}}^\mu$ and $\hat{\chi}^\mu$ with respect to $\mathcal{H} + \mathcal{H}_Z$ are given by

$$\dot{\mathcal{T}}^\mu = \frac{\partial\mathcal{E}}{\partial\chi^\mu} + g\mu_B(\nabla B^z \times \boldsymbol{\Omega}_{\chi\chi})_\mu, \quad (1)$$

and $\dot{\chi}^\mu = g\mu_B\nabla_\mu B^z$, where $\mathcal{E} = \langle\psi|\mathcal{H}|\psi\rangle$. These equations are, indeed, spin analogues of the Sundaram-Niu equations for topological transport of Bloch electrons. The second term of (1) is the important Berry-phase effect. The equation for $\dot{\chi}^\mu$ describes the precession motion due to the external magnetic field parallel to the z axis, which was obtained previously by Haldane and Arovas [12]. It is apparent from (1) that the magnetic field gradient ∇B^z gives rise to the dissipationless spin Hall current flowing perpendicular to the direction of the field gradient provided that the Berry curvature $\boldsymbol{\Omega}_{\chi\chi}$ is nonzero. Note that this Hall effect is more analogous to the AHE for charge currents rather than the SHE in electron systems, because time-reversal symmetry is broken in our systems, and the spin Hall current is carried by the anomalous velocity.

We now discuss the condition for the nonzero Berry curvature $\boldsymbol{\Omega}_{\chi\chi}$. In the case of collinear order realized in magnets with spin rotational symmetry, the axis of spontaneous magnetization is parallel to the applied field along the z direction. Then, an infinitesimal rotation around the z axis which corresponds to the derivative $\partial|u\rangle/\partial\chi^\mu$ does not change the state, and thus, the Berry curvature vanishes, $\boldsymbol{\Omega}_{\chi\chi} = 0$. The existence of noncollinear order (or spin chirality), which is generally expected for frustrated magnets, is necessary for the nonzero $\boldsymbol{\Omega}_{\chi\chi}$. However, this is by no means a sufficient condition. As will be shown later, for the realization of the dissipationless SHE, it is required to introduce parity-breaking terms explicitly into the Hamiltonian.

Quantum spin model for anomalous spin Hall effect.— To demonstrate the existence of the Hall effect of spin waves in quantum spin systems, we present a microscopic model which exhibits the topological transport property predicted above. We consider a model of antiferromagnets on a bilayer triangular lattice with Bravais lattice vectors

$\boldsymbol{b}_1 = (1, 0)$, $\boldsymbol{b}_2 = (-1/2, \sqrt{3}/2)$. The Hamiltonian is $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{12}$ with $\mathcal{H}_n = J_n \sum_{\langle i,j \rangle} [S^x_{n,i} S^x_{n,j} + S^y_{n,i} S^y_{n,j} + \Delta_n S^z_{n,i} S^z_{n,j}] - h_z \sum_i S^z_{n,i}$, and $\mathcal{H}_{12} = K \sum_{i,\boldsymbol{a}_m} [S^x_{1,i} S^x_{2,i+\boldsymbol{a}_m} + S^y_{1,i} S^y_{2,i+\boldsymbol{a}_m}]$, where $\mathcal{H}_n$ $(n = 1, 2)$ is the Hamiltonian of a triangular antiferromagnet on each layer with an in-plane anisotropy $0 \le \Delta_n < 1$. The sum $\sum_{\langle i,j \rangle}$ is over pairs of the nearest neighbor sites i, j. $\mathcal{H}_{12}$ is an interaction between two layers, where the lattice vectors $\boldsymbol{a}_m$ $(m = 1, 2, 3)$ are defined by $\boldsymbol{a}_1 = \boldsymbol{b}_1$, $\boldsymbol{a}_2 = \boldsymbol{b}_2$, and $\boldsymbol{a}_3 = -\boldsymbol{b}_1 - \boldsymbol{b}_2$. The important feature of this model is that the coupling term $\mathcal{H}_{12}$ explicitly breaks inversion symmetry. As will be shown below, this parity-breaking term raises the anomalous velocity of spin waves associated with the topological Berry phase, which leads to the Hall effect of spin waves. In the following, we assume $K \ll J_n$ and treat $\mathcal{H}_{12}$ as a perturbation for simplicity. For $K = 0$, the ground state of $\mathcal{H}$ with $h_z = 0$ is the 120° order with all spins aligned parallel to the xy plane. For small but finite h_z, all spins are tilted by an angle φ_n toward the z direction. The mean field analysis gives $\sin\varphi_n = h_z/[3 J_n S_n (2\Delta_n + 1)]$ with S_n the size of spins. We consider spin transport in this φ-tilted ordered state, which possesses a nonzero scalar chirality order $\chi_S \equiv \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) = \frac{3\sqrt{3}}{2} S_n^3 \cos^2\varphi_n \sin\varphi_n$. To deal with low-energy spin excitations, we employ a standard spin wave theory within the Gaussian approximation [17]. We also postulate that infinite numbers of the bilayer systems are weakly

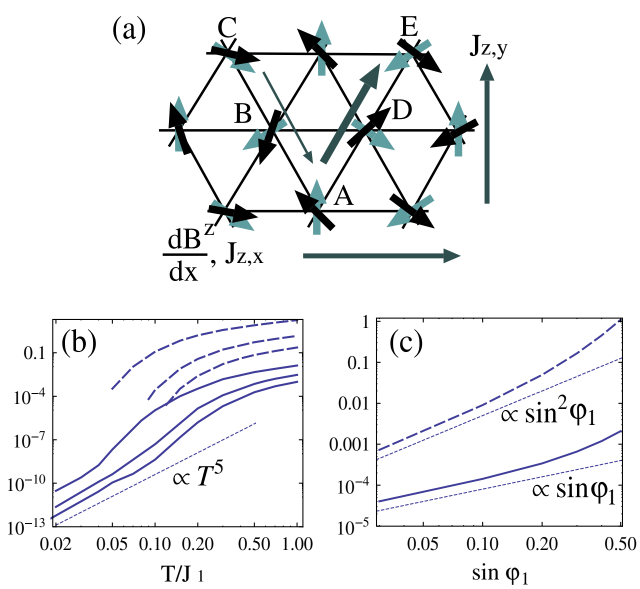

FIG. 1 (color online). (a) Schematic view of the origin of the Hall effect of spin waves. Short arrows represent spin structures. Long arrows represent spin current flows. See the text. (b) Examples of plots of $\sigma^{\mathrm{SHE}}_{xy}/K^2$ (solid line) and $\sigma^D_{xy}/\tau J_1^3$ (broken line) versus T/J_1 for several values of $\sin\varphi_1$. $\Delta_1 = \Delta_2 = 0.8$, $J_1 = 1.0$, $J_2 = 1.5$. $\sin\varphi_1 = 0.3, 0.5, 0.8$ (from bottom to top). (c) Plots of $\sigma^{\mathrm{SHE}}_{xy}/K^2$ (solid line) and $\sigma^D_{xy}/\tau J_1^3$ (broken line) versus $\sin\varphi_1$ for $\Delta_1 = \Delta_2 = 0.8$, $J_1 = 1.0$, $J_2 = 1.5$, and $T/J_1 = 0.8$.

coupled via a interbilayer interaction to stabilize the magnetic order at finite temperatures. In this system, only the in-plane spin wave mode is gapless. This in-plane mode carries a spin current with the magnetization parallel to the z axis. The spin current $J_z(i, j)$ on the bond (ij), which satisfies the continuity equation, is obtained from $\dot{S}^z_i = i[\mathcal{H}, S^z_i] = -\sum_m [J_z(i, i + \boldsymbol{a}_m) - J_z(i - \boldsymbol{a}_m, i)]$ [12]. The spin current consists of three parts, i.e., $J_z(i, j) = J^{11}_z(i, j) + J^{22}_z(i, j) + J^{12}_z(i, j)$. Here, the first two terms are intraplane spin currents: $J^{nn}_z(i, j) = J_n(\boldsymbol{S}_{n,i} \times \boldsymbol{S}_{n,j})_z$. The third term $J^{12}_z(i, j) = K(S^x_{1,i} S^y_{2,j} - S^y_{1,i} S^x_{2,j})$, which stems from the interlayer coupling $\mathcal{H}_{12}$, gives rise to the anomalous spin Hall current. The x and y components of the spin current are, respectively, $J_{z,x}(i) = J_z(i, i + \boldsymbol{a}_1) - \frac{1}{2}[J_z(i + \boldsymbol{a}_2, i) + J_z(i + \boldsymbol{a}_3, i)]$ and $J_{z,y}(i) = \frac{\sqrt{3}}{2} \times [J_z(i, i + \boldsymbol{a}_2) - J_z(i + \boldsymbol{a}_3, i)]$.

When there is a magnetic field gradient, e.g., $\partial B^z/\partial x \neq 0$, the spin Hall current $J_y = \sigma^{\mathrm{SHE}}_{yx} \partial B^z/\partial x$ appears. The origin of the spin Hall effect in this model is schematically understood as follows. As shown in Fig. 1(a), in the φ-tilted 120° ordered state (the spin structure is represented by gray arrows), the y component of the spin current carried by the in-plane spin wave mode flowing along the path $A \to B \to C$ and that flowing along the path $A \to D \to E$ cancel each other, and there is no net spin current along the y direction. When the field gradient parallel to the x axis, $\partial_x B^z$, is applied, the spin structure is changed to that represented by the black arrows. Then, the spins along these two paths feel a different torque, because of the difference of the field strength among the sites B, C, D, and E. As a result, the spin currents along the path $A \to B \to C$ and that along the path $A \to D \to E$ are unbalanced, and the spin Hall current along the y axis, $J_{z,y}$, appears. Using the Kubo formula, we calculate, up to the lowest order in K, the Hall conductivity for the spin current $J^{12}_{z,\mu} = \sum_i J^{12}_{z,\mu}(i)$ $(\mu = x, y)$, i.e. $\sigma^{\mathrm{SHE}}_{xy} = \lim_{\omega \to 0} \frac{i}{\omega} \times \int_0^\infty dt \langle [J^{12}_{z,x}(t), J^{12}_{z,y}(0)] \rangle e^{i\omega t}$, with $\langle \cdots \rangle$ the average with respect to $\mathcal{H}_1 + \mathcal{H}_2$. We found that $\sigma^{\mathrm{SHE}}_{xy}$ is nonzero when two layers are not equivalent; i.e., $J_1 S_1 \neq J_2 S_2$ or $\Delta_1 \neq \Delta_2$. This condition is analogous to the condition of spin imbalance for the anomalous Hall effect in electron systems [7]. This implies that the Hall effect of spin waves is analogous to the AHE for charge currents in electron systems rather than the SHE in electron systems with time-reversal symmetry. Actually, in our systems, time-reversal symmetry is broken by both magnetic order and magnetic fields. Also it is noted that the role played by spin degrees of freedom for the AHE in electron systems is played not by spins but by two-layer degrees of freedom in our spin system. In Fig. 1(b), we show an example of temperature dependence of $\sigma^{\mathrm{SHE}}_{xy}$ calculated numerically. At low T, $\sigma^{\mathrm{SHE}}_{xy} \propto T^5$. The Hall conductivity $\sigma^{\mathrm{SHE}}_{xy}$ does not depend on the relaxation time of magnons τ. The T dependence of $\sigma^{\mathrm{SHE}}_{xy}$ is merely due to thermally excited carriers, i.e., the Bose distribution function. This dissipationless Hall effect is raised by the Berry curvature $\Omega_{\chi\chi}$ associated with the scalar chirality, as predicted from the semiclassical analysis, Eq. (1).

Note that in our model, in addition to the dissipationless Hall effect, the dissipative Hall effect, which depends on the relaxation time τ, is also possible even when $K = 0$. The dissipative Hall effect is analogous to the extrinsic AHE of electron systems. However, the analogy is not complete. In contrast to the extrinsic AHE, the origin of the transverse force on spin waves is not asymmetric scattering but the existence of the spin scalar chirality. The dissipative Hall effect is more deeply related to spin currents induced by magnetic fields in one-dimensional magnets with noncoplanar spin texture [9]. In two- or three-dimensional systems, for a particular spin texture, it is possible that a field gradient induces transverse spin current as illustrated in Fig. 1(a). This effect is dissipative in the sense that the Hall conductivity generally depends on the relaxation time of magnons. The dissipative spin Hall conductivity σ_{xy}^D is calculated from $\sigma_{xy}^D = \lim_{\omega\to 0}\frac{i}{\omega}\times \int_0^\infty dt \sum_{n=1,2}\langle[J_{z,x}^{nn}(t), J_{z,y}^{nn}(0)]\rangle e^{i\omega t}$. The total Hall conductivity is, then, given by $\sigma_{xy}^{\mathrm{SHE}} + \sigma_{xy}^D$. We assume that τ is governed by impurity scattering, and independent of temperatures. Then, the numerically obtained σ_{xy}^D is orders of magnitude larger than $\sigma_{xy}^{\mathrm{SHE}}$ [Fig. 1(b)]. This makes it difficult to detect the dissipationless effect. Nevertheless, the dissipationless and dissipative contributions to the Hall effects are distinguishable from the different dependence on $\sin\varphi$, i.e., χ_S. We can show analytically that up to the lowest order in χ_S, $\sigma_{xy}^{\mathrm{SHE}} \propto \chi_S$ and $\sigma_{xy}^D \propto \chi_S^2$ hold. At high temperatures $T \sim J_1$, these relations are retained up to $\sin\varphi < 0.3$ [Fig. 1(c)]. Thus, the different dependence on χ_S clearly distinguishes the dissipationless Hall effect from the dissipative one.

Concluding remarks.—Although the model presented above is a toy model which does not describe actual magnetic systems known so far, its realization in real materials is feasible. Furthermore, it establishes the concept of the Hall effect of spin wave excitations caused by both the dissipationless and dissipative mechanisms. Our results open the possibility of exploring the SHE in localized spin systems with no charge degrees of freedom. The current study can be extended to some other transport properties such as thermal Hall currents. This issue will be addressed in the near future. Finally, it is noted that the nonzero Berry curvature in our spin system strongly suggests the existence of topological order, as in the case of the AHE [3,6]. It is curious to examine this scenario, when the spin wave excitations acquire a gap due to a spin anisotropy. In summary, we have demonstrated that, in a certain class of frustrated magnets, the topological Berry-phase effect associated with spin chirality gives rise to the Hall effect of spin waves, in analogy with the AHE in electron systems.

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