Strong Magnetic Coupling of an Ultracold Gas to a Superconducting Waveguide Cavity

J. Verdú,[1] H. Zoubi,[2] Ch. Koller,[1] J. Majer,[1] H. Ritsch,[2] and J. Schmiedmayer[1]

[1]Atominstitut der Österreichischen Universitäten, TU-Wien, 1020 Vienna, Austria

[2]Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 21a, 6020 Innsbruck, Austria

(Received 12 September 2008; published 24 July 2009)

Placing an ensemble of 10^6 ultracold atoms in the near field of a superconducting coplanar waveguide resonator with a quality factor $Q \sim 10^6$, one can achieve strong coupling between a single microwave photon in the coplanar waveguide resonator and a collective hyperfine qubit state in the ensemble with $g_{\rm eff}/2\pi \sim 40$ kHz larger than the cavity linewidth of $\kappa/2\pi \sim 7$ kHz. Integrated on an atomchip, such a system constitutes a hybrid quantum device, which also can be used to interconnect solid-state and atomic qubits, study and control atomic motion via the microwave field, observe microwave superradiance, build an integrated micromaser, or even cool the resonator field via the atoms.

DOI: 10.1103/PhysRevLett.103.043603 PACS numbers: 42.50.Pq, 03.65.−w, 03.67.−a, 37.30.+i

In the past decade, important breakthroughs in implementing quantum information processing were made in different physical implementations [1], each showing advantages and shortcomings. For quantum information to emerge as a valuable technology, it is mandatory to pool their strengths. Solid-state systems allow fast processing and dense integration; atom- or ion-based systems are slower but exhibit long qubit coherence times. Here we analyze a device to *quantum interconnect* superconducting solid-state qubits to an atomic ensemble. Ensembles of atoms constitute a quantum memory, the information from which can be read out using photons [2] that can then be transmitted over long distances [3].

The challenge in transferring the state of a solid-state qubit to atoms is bridging the tremendous gap in time scales that govern solid-state and atomic physics devices. This difference can be overcome by employing the long microwave photon lifetime in a superconducting coplanar waveguide resonator (CPWR) [4–6] which can be efficiently electrically coupled to superconducting qubits [7–11]. The small effective mode volume in the CPWR allows a strong coupling between a microwave photon and a collective state in an atomic ensemble.

Various ways have been proposed to couple solid-state quantum devices to atomic and molecular systems [12–18]. In this Letter, we concentrate on the magnetic coupling of a microwave photon in a CPWR to a collective hyperfine qubit in an ensemble of ultracold atoms. We show below that, even though the magnetic coupling strength is much weaker than the optical dipole coupling, one can achieve strong coupling with currently available technology of circuit cavity quantum electrodynamics and ultracold atomic ensembles on an atomchip.

As a particular qubit example, we consider an ensemble of ultracold ^{87}Rb atoms and the hyperfine transition between $|F = 2, m_F\rangle$ and $|F = 1, m_F\rangle$ states at a frequency of 6.83 GHz, which is ideally suited for a CPWR. In principle, both systems can be integrated in a hybrid device on a single superconducting atomchip [19]. Besides the transfer of a single photon to the atomic ensemble as a quantum memory and back, such a hybrid quantum system opens up many different other possibilities. For example, nondestructive microwave detection of the atomic density will allow the continuous monitoring of Bose-Einstein condensation (BEC) formation or, by changing operating parameters, one can achieve a superradiant microwave source (a micromaser). Optically pumped atoms are a heat bath close to $T = 0$ and will strongly suppress thermal photons in the coupled resonator mode. In addition, adiabatic microwave potentials will allow the coupling of the quantum properties of the resonator mode to the mechanical motion of the atoms.

The CPWR developed for circuit cavity QED consists of three conducting stripes: the central conductor plus two ground planes [Fig. 1(a)]. Its electromagnetic field is strongly confined near the gaps between the conductor and the ground planes. By using atomchip technology [20], a large number of ultracold atoms ($N \approx 10^6$) can be positioned only a few micrometers above a gap [21,22], where they experience the very strong localized magnetic field of the CPWR. The high concentration of field energy near the surface results in a dramatic reduction of the effective volume $V_{\rm eff} \sim \frac{\pi}{2}\lambda l^2$ of the resonator mode. For the ^{87}Rb microwave transition at 6.83 GHz (wavelength $\lambda \sim 3$ cm) and a typical decay length $l \sim 3$ μm of the field of the order of the gap size W, one expects an enhancement of the atom-photon coupling strength of $(\lambda/l) \sim 10\,000$. A full calculation of the local electromagnetic field following Refs. [23,24] and normalizing to a single photon results in the field shown in Figs. 1(b)–1(d). We obtain at a distance of 1 μm above the surface a field of >40 μG for a single photon, which confirms the estimates above.

For a detailed treatment of the coupling we write the single-mode electromagnetic field operators as

$$\vec{E}^{\,\gamma}(\vec{r}, t) = \frac{\vec{e}^{\,\gamma}_{\rm tr}(x, y)}{\sqrt{2}}(a_\gamma e^{i(k_\gamma z - \omega_\gamma t)} + a^\dagger_\gamma e^{-i(k_\gamma z - \omega_\gamma t)}),$$

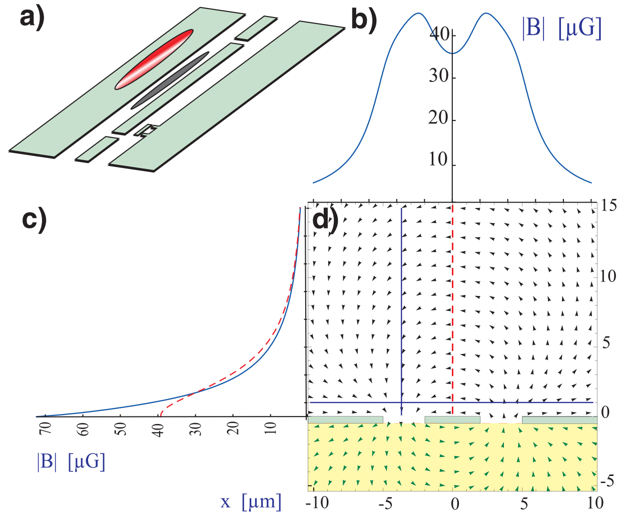

FIG. 1 (color online). (a) Schematic of the CPWR including a solid-state qubit and a cloud of ultracold atoms trapped above one of the gaps. (b) The magnetic field strength of a single photon as a function of lateral distance 1 μm from the chip surface and (c) as a function of distance to the chip surface at the gap (full line) and at the central conductor (dashed line). (d) Vector plot of the magnetic field in the resonator.

$$\vec{B}^{\gamma}(\vec{r},t) = i\frac{\vec{b}^{\gamma}_{\rm tr}(x,y)}{\sqrt{2}}(a_\gamma e^{i(k_\gamma z-\omega_\gamma t)} - a^\dagger_\gamma e^{-i(k_\gamma z-\omega_\gamma t)}),$$

where $a^\dagger_\gamma$ and a_γ represent the boson creation and destruction operators, respectively, for the microwave photons. $\omega_\gamma = 2\pi\nu$ is the angular frequency of the microwave, and $k_\gamma = 2\pi/\lambda$ is the propagation constant with wavelength $\lambda = c/\sqrt{\epsilon_{\rm eff}}$. The effective relative dielectric constant $\epsilon_{\rm eff}$ has a value between the substrate value and 1 (vacuum) and depends on the actual dimensions of the CPWR [25].

The corresponding mode functions $\vec{e}^{\gamma}_{\rm tr}(x,y)$ and $\vec{b}^{\gamma}_{\rm tr}(x,y)$ vary strongly in space, depending on the CPWR geometry, and have to be determined numerically. To satisfy the proper field commutators they have to be normalized to

$$\frac{1}{2}\int dV\epsilon(\vec{r})|\vec{e}^{\gamma}_{\rm tr}|^2 = \frac{1}{2\mu_0}\int dV|\vec{b}^{\gamma}_{\rm tr}|^2 = \frac{1}{2}\hbar\omega_\gamma. \qquad (1)$$

The mode functions represent the field amplitude per photon. Note that the permittivity has to be included in the integral. In the following, we assume the substrate to be nonmagnetic. The field Hamiltonian is then $\mathcal{H}_\gamma = \hbar\omega_\gamma(a^\dagger_\gamma a_\gamma + \frac{1}{2})$, where ω_γ is the cavity resonance frequency.

For a ground state Rb atom the dominant interactions with a microwave field are the magnetic dipole (M1) transitions between the atomic hyperfine states $|F=2,m_F\rangle \leftrightarrow |F=1,m'_F\rangle$. This leads to the interaction Hamiltonian:

$$\mathcal{H}_{\rm int} = \vec{\mu}\cdot\vec{B}^{\gamma} = \frac{\mu_B}{\hbar}\left(g_S\vec{S} - \frac{\mu_N}{\mu_B}g_I\vec{I}\right)\cdot\vec{B}^{\gamma}. \qquad (2)$$

By assuming an external bias field $\vec{B}_0$ as the quantization axis for the atomic magnetic moment, transitions driven by a transverse field $\vec{B}^{\gamma} \perp \vec{B}_0$ follow the selection rules $\Delta m_F = m_F - m'_F = \pm 1$. Longitudinal fields $\vec{B}^{\gamma} \| \vec{B}_0$ induce $\Delta m_F = 0$ transitions.

The transverse fields generated by the quasi-TEM mode of the CPWR cavity couples therefore to the two transitions $\Delta m_F = 1$ and $\Delta m_F = -1$. By adjusting the Zeeman splitting of the hyperfine states via a typical longitudinal Ioffe bias field of $B_0 \sim 1$ Gauss, we can ensure that the CPWR is resonant only with one of those two transitions. Thus the atom can be modeled effectively by a two level system, and we simply denote the two coupled atomic states by $|2\rangle = |F=2,m_F\rangle$ and $|1\rangle = |F=1,m'_F\rangle$ with energies E_2 and E_1.

For the internal dynamics of an ensemble of N atoms, we thus get an effective Hamiltonian in a standard Jaynes-Cummings form [26]:

$$H_{\rm atom} = \sum_i \hbar\omega_a \hat{\pi}^\dagger_i\hat{\pi}_i - \sum_i\{\hat{\mu}_i\cdot\vec{B}^{\gamma}(\vec{r}_i) + \hat{\mu}^*_i\cdot\vec{B}^{\gamma\dagger}(\vec{r}_i)\}.$$

Here ω_a is the atomic transition frequency, $\hat{\pi}^\dagger_i$ is the excitation operator of the ith atom, and $\hat{\mu}_i = \vec{\mu}^*_i\hat{\pi}_i + \vec{\mu}_i\hat{\pi}^\dagger_i$, with $\vec{\mu}_i$ the transition matrix element of the magnetic dipole moment as defined in Eq. (2).

A typical ensemble of ultracold Rb atoms confined in an elongated trap on the atomchip has a transverse extension of $d < 1$ μm and a length of up to a few millimeters. On these length scales the variation of the microwave field of the CPWR is small if the atomic ensemble is positioned parallel to the CPWR and longitudinally at the maximum of the microwave field. We therefore neglect the change in the magnetic microwave field over the size of the atomic ensemble and fix $\vec{b}^{\gamma}_{\rm tr}(r_i) \approx \vec{b}^{\gamma}_{\rm tr}(\bar{R})$ for all atoms. Here $\bar{R}$ is the mean transverse position. This allows us to define a simple collective atomic excitation operator of the form $\tilde{\pi} = (1/\sqrt{N})\sum_i\hat{\pi}_i$ and to rewrite the interaction Hamiltonian in the Tavis-Cummings form [27]:

$$H = \hbar\omega_\gamma a^\dagger_\gamma a_\gamma + \hbar\omega_a\tilde{\pi}^\dagger\tilde{\pi} + \hbar g_{\rm eff}\tilde{\pi}^\dagger a_\gamma + \hbar g_{\rm eff} a^\dagger_\gamma\tilde{\pi}, \qquad (3)$$

where $g_{\rm eff} = \sqrt{N}g$ and $\hbar g = (1/\sqrt{2})[\vec{b}^{\gamma}_{\rm tr}(\bar{R})\cdot\vec{\mu}]$. From this we can immediately read out the effective coupling strength $g_{\rm eff}$ for the first symmetric atomic excitation, which is enhanced by a factor of $\sqrt{N}$.

As a concrete example we obtain a matrix element of $0.86\mu_B$ for a $m_F = 2$ to $m'_F = 1$ transition in the ^{87}Rb ground state. Taking into account the calculated field of a CPWR (see Fig. 1), we obtain a single-photon–single-atom Rabi frequency of typically $g/2\pi \sim 40$ Hz at a height of a few micrometers. For an atomic ensemble of $N \sim 10^6$ ^{87}Rb atoms, the coherent collective coupling $g_{\rm eff}/2\pi = \sqrt{N}g/2\pi \sim 40$ kHz dominates the cavity decay $\kappa/2\pi = \nu/Q \sim 7$ kHz, and one would get several exchanges between a microwave photon in the cavity and a collective atomic excitation before the photon decays.

The Hamiltonian [Eq. (3)] can be diagonalized by eigenstates forming a weighted coherent superposition of collective atomic excitations and a photon, depending on system parameters. Controlling the relative weights of the superposition (via atomic or cavity tuning) adiabatically switches excitation between the microwave and the collective atomic qubit state, which is a delocalized symmetric superposition of all possible single-atom excitations. The strong coupling regime allows this transfer fast enough to avoid decoherence.

The best choice for the qubit states in the quantum memory are the tappable clock states $|F = 1, m_F = -1\rangle$ and $|F = 2, m_F = 1\rangle$ where very long coherence times of >1 s were demonstrated for hyperfine excitations on an atomchip even at close proximity to the surface [28]. To write such a qubit, one has to add a second radio frequency photon to complete the Raman transition [29]. By using a sizable radio frequency drive for the second part of the Raman transition, the overall strength is given by the weak single photon transition. Such a stored collective qubit can be transferred into an optical photon by forward coherent Raman scattering [2], which completes the transfer from solid-state qubits via an atomic quantum memory to photons as flying qubits.

Besides implementing a quantum interconnect between solid-state qubits, atoms, and even photons, our system offers many further interesting possibilities, which we will discuss in short examples below.

Whenever the effective Rabi splitting g_{eff} is larger than the cavity linewidth κ, it manifests itself in the spectrum of the transmitted and reflected fields [see Fig. 2(a)] and allows for nondestructive probing of the integrated atomic density in the mode.

Let us note that here, compared to standard cavity QED, the linewidth of the atomic excitation is not limited by spontaneous decay as, for all practical purposes, both hyperfine states can be regarded as stable. Hence the decay rate will effectively be given by nonradiative losses such as the lifetime of atoms in the trap, which can be seconds. In view of the large difference of atomic and cavity decay, the single-atom cooperativity parameter $C = g^2/(\kappa\gamma)$, which can reach $C \sim 1$, is only partly meaningful. While a single excited atom will emit a photon predominantly into the cavity mode, the large cavity linewidth prevents direct single-atom detection via resonator transmission.

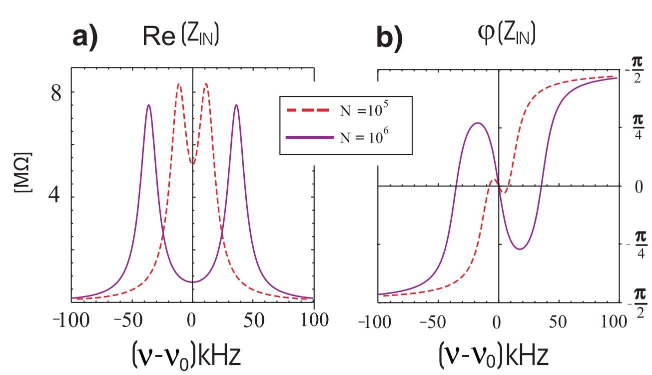

FIG. 2 (color online). Response of the atom-cavity system as illustrated by its complex impedance Z: (a) The real part of Z is related to the spectrum, and (b) the phase of Z directly illustrates the phase shift of the transmitted microwave radiation. The calculations are for $Q = 10^6$, $N = 10^6$, and $N = 10^5$ and plotted vs incident field detuning from the resonance frequency.

In the microwave regime, the transmitted field amplitude and phase are directly measurable. The corresponding phase shift of the transmitted field is plotted in Fig. 2(b), for 10^6 (solid line) and 10^5 (dashed line) atoms coupled to the microwave mode of the cavity. Note that measuring the phase shift will not only be a sensitive probe of the atom number but, as the phase changes sign when the atoms are transferred to the upper hyperfine state, it can also be used for preparation and readout of spin states.

One important aspect neglected so far is thermal photons. The number of photons in the mode of the CPWR is given by $\bar{n}_T = [\exp(\frac{\hbar\omega_\gamma}{k_B T}) - 1]^{-1}$. With $\frac{\omega_\gamma}{2\pi} = 6.83$ GHz corresponding to a temperature $T \sim 350$ mK, cooling to below 100 mK is required to have an empty cavity ($\bar{n}_T < 0.1$). However, a perfectly polarized BEC with all atoms in the lower hyperfine state has a very low effective internal temperature. A relative purity in the population of the hyperfine states of 10^{-5} corresponds to $-\Delta E_{\text{hf}}/k_B T = \ln(10^{-5}) = -11.5$, which relates to a temperature of $T \sim 30$ mK for ^{87}Rb (hyperfine splitting $\Delta E_{\text{hf}} = 6.83$ GHz). Coupling the two systems can lead to an energy flow towards the ensemble of ultracold atoms. We estimate the photon absorption rate from the cavity into the atomic ensemble to be $\gamma_c/2\pi \sim g^2 N/(\gamma_a 2\pi) \sim 8.6$ MHz, assuming an upper state with a lifetime of $\gamma_a^{-1} \sim 1$ ms (lifetime of an untrapped upper state) which has to be compared to the heating rate $R \sim \kappa\bar{n}_T$. The suppression of the thermal photons is then given by $\kappa/(\gamma_c + \kappa)$. We can remove thermal photons from the mode as long as $\gamma_c \gg \kappa$, which can hold for the parameters given above for several tens of seconds.

Superradiance from a completely inverted atomic ensemble has been discussed in the microwave context [30] and led to extensive theoretical and experimental studies [31]. Here one can study an alternative implementation with magnetic coupling in a very clean form by preparing an almost perfectly inverted atomic system with all atoms in $F = 2$. The situation is very close to the original model for a superradiant system proposed by Dicke [30], where a highly excited atomic system is supposed to spontaneously emit coherent multiphoton pulses. In our realization, the lifetime of the excited state is much longer than all other relevant time scales, and thus the dominant decay will happen almost exclusively via the cavity mode. Instead of the single particle decay being enhanced by the Purcell effect of $\gamma_a^{\text{eff}} \approx g^2/\kappa \sim 1.5$ s^{-1}, we get an extra superradiant enhancement proportional to the atom number

$N \approx 10^6$. Hence, within a very short time, about 10^6 microwave photons will be collectively emitted into the microwave mode in a coherent pulse, which should be readily observable.

Comparing with other related cavity QED systems, the cooperativity for the collective qubit state is very large and even comes close to the values for a BEC coupled to a resonator on a strong optical transition [32–34]. Note that the involved transition frequency is much lower in our case. Hence one could even envisage reaching the regime of the quantum phase transition to a collective superradiant phase, predicted for $gN \approx \omega_\gamma$ in a classic paper by Hepp and Lieb [35].

Finally, the field in the cavity mode can exert forces on the cloud of ultracold atoms through microwave-induced dressed-state potentials (microwave ac-Stark shift) [36,37]. A coupling strength of $g/2\pi \sim 40$ Hz results in small modifications (dressed-state shifts) of the trapping potential, and forces are small on the few photon level [38]. Potential energies in the order of a typical chemical potential of a trapped 1D cloud ($V \sim 1$ kHz) appear for microwave fields of 1000 photons in the mode. The quantum fluctuations of the cavity field will strongly influence the trapped atoms. This may be used to measure and manipulate the quantum state of both the cavity field and the atomic ensemble.

In conclusion, we found that coherent strong coupling between a collective spin state in an atomic ensemble and a microwave photon from a CPWR is feasible by combining current state-of-the-art technology of atomchips and superconducting microwave resonators. Such a quantum interconnect will allow the transfer of a quantum state of a solid-state qubit into an atomic ensemble where the state can be stored. This could then be used as an interface for long distance quantum communication. In addition, this system is a realization of the microwave Tavis-Cummings model. Superradiance, cooling of the microwave mode of the CPWR, and strong light forces based on mesoscopic microwave fields can be studied. Finally, we want to point to that our calculations hold for any magnetic coupling between a spin ensemble [39] and microwave photons in a CPWR.

This work was supported by the European Union project MIDAS, the Austrian FWF Projects No. P20391 and No. Z118-N16, and SFB Foqus P13. J. V. acknowledges support from the Marie Curie Action MIEDFAM and J. M. from the Marie Curie Action HQS. Ch. K. acknowledges funding from the FunMat research alliance. We thank M. Trinker and A. Wallraff for stimulating discussions.

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