Sound Produced by a Fast Parton in the Quark-Gluon Plasma is a "Crescendo"

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We calculate the total energy deposited into the medium per unit length by fast partons traversing a quark-gluon plasma. The medium excitation due to collisions is taken to be given by the well-known expression for the collisional drag force. The radiative energy loss of the parton contributes to the energy deposition because each radiated gluon acts as an additional source of collisional energy loss in the medium. We derive a differential equation which governs how the spectrum of radiated gluons is modified when this energy loss is taken into account. This modified spectrum is then used to calculate the additional energy deposition due to the interactions of radiated gluons with the medium. Numerical results are presented for the medium response for the case of two energetic back-to-back partons created in a hard interaction.

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Experimental results from the heavy-ion program at the Relativistic Heavy-Ion Collider (RHIC) indicate the creation of a new state of nuclear matter known as the quark-gluon plasma [1]. The results obtained so far indicate some fascinating properties of the quark-gluon plasma, not the least of which is the nearly perfect fluid behavior apparently exhibited by the medium [2]. Another significant observation [3] is that highly energetic partons which propagate through the medium rapidly lose energy and momentum as they interact with the surrounding matter in a process known as jet quenching [4]. An interesting question related to the above properties is: how does the quark-gluon plasma respond to fast partons as they propagate through it? This question has gained significance in light of measurements of hadron correlation functions that are consistent with a conical emission pattern [5–8]. These measurements suggest that fast partons may generate collective flow, such as a Mach-cone shaped shock wave, in the medium.

While jet quenching has been studied quite extensively [9–11], theoretical studies of the quark-gluon plasma response to fast partons are more recent [12–17]. The central challenge of this investigation is to calculate the energy deposited into the medium per unit length by an energetic parton. As the fast parton propagates through the medium, it loses energy through collisions and medium induced radiation. The energy deposited into the medium per unit length is then the sum of the collisional energy loss of the primary parton and of the radiated gluons.

In this work we calculate the total energy deposited into the medium per unit length by a fast parton traversing a quark-gluon plasma. The medium excitation due to collisions is taken to be given by the well-known expression for the collisional drag force [18]. To calculate the medium excitation due to radiation, we begin by deriving a differential equation which describes how the spectrum of radiated gluons is modified as the radiated gluons lose energy through collisions. This modified spectrum is then used to calculate the differential energy loss due to the interactions of radiated gluons with the medium, from which we find that the energy which goes into medium excitation is substantially less than the total radiative energy loss. The final result for the energy deposited into the medium per unit length, which is a sum of the primary and the secondary contributions, is then treated as the coefficient of a local hydrodynamic source term. Numerical results are presented for the medium response for the case of two fast, back-to-back partons created in an initial hard interaction.

We start by considering the collisional energy lost per unit length by a fast parton in the quark-gluon plasma. As mentioned above, for the collisional energy loss we will use the familiar expression [18]

$$\left(\frac{dE}{d\xi}\right)_C = \frac{\alpha_s C_2 m_D^2}{2} \ln\frac{2\sqrt{E_p T}}{m_D}, \tag{1}$$

where $\alpha_s = g^2/4\pi$ is the strong coupling, m_D is the Debye mass of the medium, which we take to be given by $m_D = gT$, E_p is the energy of the fast parton, T is the temperature of the medium, and C_2 is the eigenvalue of the quadratic Casimir operator of the color charge of the source parton, which is $4/3$ if the fast parton is a quark, and 3 for a gluon (we consider $N_c = 3$). In (1), the subscript C denotes collisional energy loss and ξ is the path length traveled by the source parton.

We are next interested in calculating the energy deposited, or gained by the medium, due to gluons radiated by the fast parton. We begin by defining the quantity $f(\omega, \xi) \equiv dI_M/d\omega$, which gives the spectrum of radiated gluons with energy ω in the medium. $f(\omega, \xi)$ is in general different than $dI/d\omega$, which is the spectrum of gluons emitted by the fast parton, because gluons, once emitted, lose energy in the medium due to collisions until they become part of the thermal bath. As a gluon with energy ω travels from ξ to $\xi + \Delta\xi$, it loses collisional energy

$\epsilon(\omega)\Delta\xi$, where $\epsilon(\omega)$ is obtained from (1) to be

$$\epsilon(\omega) = \frac{3}{2}\alpha_s m_D^2 \ln\frac{2\sqrt{\omega T}}{m_D}. \tag{2}$$

Thus, in order to find a gluon with energy ω at position $\xi + \Delta\xi$, there must be a gluon with energy $\omega + \epsilon(\omega)\Delta\xi$ at position ξ. Additionally, we require the total number of gluons, that is, $f d\omega$, to be invariant. This means that as $\omega \to \omega + \epsilon(\omega)\Delta\xi$ one has $d\omega \to d\omega(1 + \frac{\partial\epsilon(\omega)}{\partial\omega}\Delta\xi)$. In equation form, this is

$$f(\omega, \xi + \Delta\xi) = f[\omega + \epsilon(\omega)\Delta\xi, \xi]\left[1 + \frac{\partial\epsilon(\omega)}{\partial\omega}\Delta\xi\right]. \tag{3}$$

Furthermore, as the fast parton moves from ξ to $\xi + \Delta\xi$, it will emit additional gluons, $\Delta\xi dI/d\omega d\xi$, which add to $f(\omega, \xi + \Delta\xi)$. The evolution equation for $f(\omega, \xi)$ thus takes the form

$$\frac{\partial}{\partial\xi}f(\omega, \xi) - \frac{\partial}{\partial\omega}[\epsilon(\omega)f(\omega, \xi)] = \frac{dI}{d\omega d\xi}(\omega, \xi), \tag{4}$$

where we have taken the limit of $\Delta\xi \to 0$.

Equation (4) provides a partial differential equation through which one can determine the spectrum of radiated gluons in the medium, $f(\omega, \xi)$. In order to solve for f, it is necessary to specify $dI/d\omega d\xi$. We choose the spectrum calculated by Salgado and Wiedemann in the multiple soft scattering approximation [19]

$$\frac{dI}{d\omega d\xi} = -\frac{\sqrt{\hat{q}}\alpha_s C_2}{\pi}\,\mathrm{Re}\frac{(1+i)\tan[(1+i)\sqrt{\frac{\hat{q}}{\omega}\frac{\xi}{2}}]}{\omega^{3/2}}, \tag{5}$$

where we use [20]

$$\hat{q} = 2\alpha_s C_2 m_D^2 T \ln\frac{2\sqrt{E_p T}}{m_D}, \tag{6}$$

where the logarithm is consistent with (1).

$f(\omega, \xi)$ will in general consist of two components: energetic gluons which lose energy through collisions and low energy gluons which become a part of the medium. The total energy being dumped into the medium is then given by the sum of the collisional energy loss of high energy gluons and the energy absorbed by the medium in the form of low energy gluons. We make the distinction that gluons with $\omega > \bar{\omega} \equiv 2T$ are sources of collisional energy loss, while those with less energy are considered as immediately part of the medium. For $\omega > \bar{\omega}$ we solve for $f(\omega, \xi)$ numerically from Eq. (4) for a primary quark using the parameters: $\alpha_s = 1/\pi$, $T = 350$ MeV, and $E_p = 50$ GeV. The total energy deposited into the medium by the secondary gluons per unit length is then given by

$$\left(\frac{dE}{d\xi}\right)_R = \int_{\bar{\omega}}^{\omega_{\max}} d\omega\,\epsilon(\omega)f(\omega, \xi) + \int_0^{\bar{\omega}} d\omega\,\omega\frac{dI}{d\omega d\xi} + \bar{\omega}f(\bar{\omega}, \xi)\epsilon(\bar{\omega}), \tag{7}$$

where we set $\omega_{\max} = E_p/2$. The last term in Eq. (7) accounts for energetic gluons which have lost enough energy to become a part of the medium and thus deposit their entire remaining energy. When solving (4), we multiply the spectrum (5) by a factor of $1 - (\omega/\omega_{\max})^8$ to ensure it goes to zero at $\omega = \omega_{\max}$.

The result of (7) is shown as the dotted red line in Fig. 1(a) for the same parameters listed above, along with the differential collisional energy loss of the primary parton (solid black line) and its differential energy loss to radiation (dashed blue curve). One sees that the energy deposited by the radiation into the medium per unit length has an approximately linear growth in path length, which results from the steady increase in the number of gluons

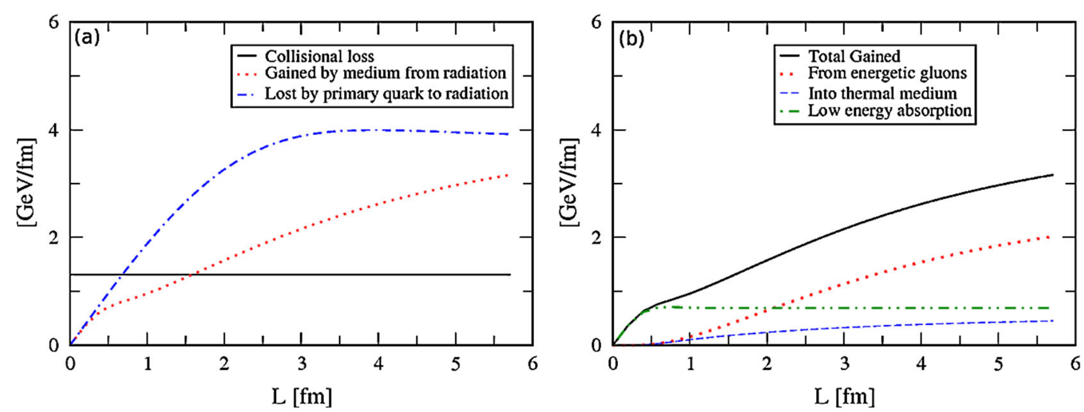

FIG. 1 (color online). Plot (a) shows the differential energy lost by the fast parton due to radiation and that gained by the medium as a function of path length, L, as well as the collisional energy loss (the specific parameters are discussed in the text). The differential energy deposition into the medium is the sum of the solid (black) and dotted (red) lines. Plot (b) shows the breakdown of various the contributions to the radiative energy gained by the medium. The dotted (red) line shows the collisional energy loss of the radiated gluons. The dash-dotted (green) line shows the energy radiated in gluons with energy below $\bar{\omega}$, which are assumed to immediately become part of the medium. The (blue) curve marked 'Into thermal medium' denotes the last term in (7).

that deposit energy in collisions. This linear growth is thus of a different origin than what is observed in the first few fm of the differential energy loss to radiation, which is caused by the energy dependent coherence length for radiated gluons. If one continues the curves shown in Fig. 1(a) out to a large enough path length, the dotted red line reaches a steady state solution and merges with the dashed blue curve. The individual contributions on the right-hand side of (7) are shown separately in Fig. 1(b), together with their sum.

The total energy deposited into the medium per unit length, or time, is given by the sum of (1) and (7), which we write as

$$\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_C + \left(\frac{dE}{dt}\right)_R. \tag{8}$$

We treat the fast parton as a point source of energy and momentum deposition in the medium, with velocity $\mathbf{u}$ and energy E_p. The hydrodynamic source term, denoted as J^ν, gives the energy and momentum density deposited in the medium per unit time. For a relativistic point source, J^ν takes the following form

$$J^\nu(x) = \frac{dE}{dt}\delta(\mathbf{x} - \mathbf{u}t)(1, \mathbf{u}), \tag{9}$$

where dE/dt is given by (8). To make the calculation more tractable, we fit the result of (7) to a linear function of time, from which we find

$$\left(\frac{dE}{dt}\right)_R \approx 0.474\frac{\mathrm{GeV}}{\mathrm{fm}} + 0.51t\frac{\mathrm{GeV}}{\mathrm{fm}^2}. \tag{10}$$

The fit slightly overestimates the energy deposition rate for $t < 0.5$ fm$/c$.

The source term couples to the hydrodynamic equations for the medium, $\partial_\mu T^{\mu\nu} = J^\nu$, where $T^{\mu\nu}$ is the energy-momentum tensor. For simplicity, we here consider the source to be coupled to an otherwise static medium at temperature T. Furthermore, we make the assumption that the energy and momentum density deposited by the fast parton is small compared to the equilibrium energy density of the medium, in which case the hydrodynamical equations can be linearized.

We are here interested in calculating the energy density perturbation excited in the medium, denoted as $\delta\varepsilon \equiv \delta T^{00}$. The equations of motion for a medium coupled to a source in linearized hydrodynamics are discussed in several places (for instance, [17]). We here write down the result for $\delta\varepsilon$ in Fourier space, which, to first order in the ratio of shear viscosity to entropy density, η/s, is given by

$$\delta\varepsilon(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \frac{ikJ_L(k) + J^0(k)(i\omega - \Gamma_s k^2)}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2}, \tag{11}$$

where c_s denotes the speed of sound, $\Gamma_s \equiv 4\eta/(3sT)$ is the sound attenuation length, and the source vector has been divided into transverse and longitudinal parts: $\mathbf{J} = \hat{\mathbf{k}}J_L + \mathbf{J}_T$, with $\hat{\mathbf{k}}$ denoting the unit vector in the direction of $\mathbf{k}$. We here consider two fast partons created in an initial hard interaction at time $t = 0$, which then propagate back to back for some time, t_M, before being absorbed by the medium. In this case (9) is modified to

$$J^\nu(x) = \frac{dE}{dt}[\Theta(t) - \Theta(t - t_M)] \times [\delta(\mathbf{x} - \mathbf{u}t)(1, \mathbf{u}) + \delta(\mathbf{x} + \mathbf{u}t)(1, -\mathbf{u})], \tag{12}$$

where $\Theta(t)$ is 1 if $t > 0$ and zero otherwise. The Fourier transform of (12) can be obtained in a straightforward way to obtain an expression for (11).

The final result for $\delta\varepsilon(x)$ is a sum of contributions from the collisional and radiative energy dumped into the me-

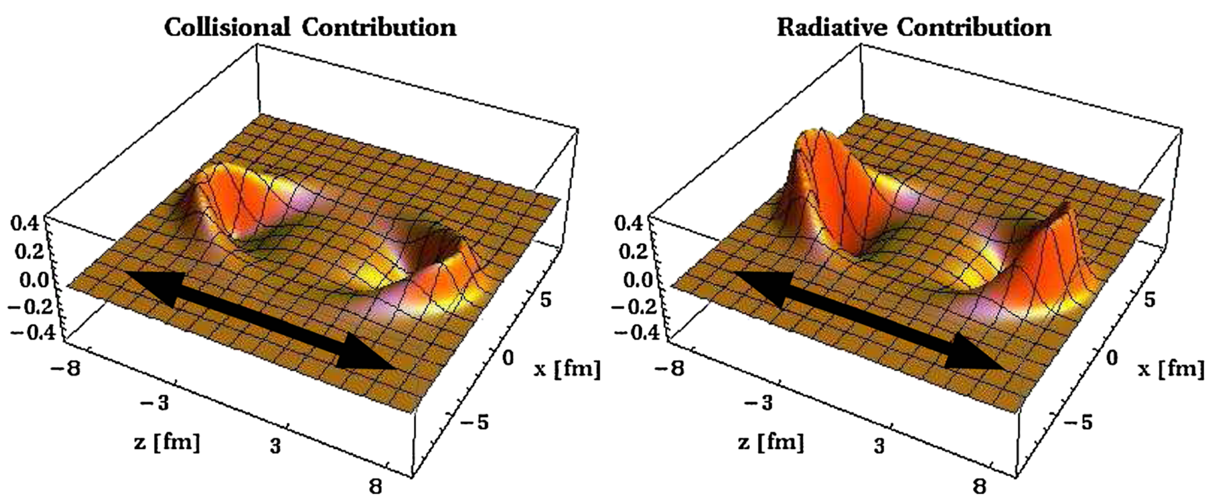

FIG. 2 (color online). Result for the energy density wave (GeV/fm^3) excited by back-to-back quarks propagating along the $\hat{z}$ axis (as indicated by the black arrows). The plots, which show the collisional and radiative contributions separately, are shown in the x-z plane, however, the results are cylindrically symmetric about the $\hat{z}$ axis. The Mach-cone formation is visible in both contributions. The radiative induced excitation leads to a t growth in the source strength, which can be seen from the plot.

dium. We calculate the medium response at a time $t = 8$ fm for back-to-back quarks which are created at $t = 0$ and $\mathbf{x} = 0$, and propagate along the $\hat{z}$ axis until $t = 6$ fm. Additionally, each quark is assumed to have an energy $E_p = 50$ GeV. The result from the collisional and radiative contributions are presented separately in Fig. 2, for a medium with the same parameters used above as well as $\eta/s = 0.2$ and $c_s/c = 0.57$. The plot shows the energy density wave excited by the source quarks in the x-z plane, however, the results are cylindrically symmetric about the $\hat{z}$ axis. The Mach cone formation is visible in both the collisional and radiative contributions. The radiative contribution to the source term, (12), exhibits a linear growth with time, which is reflected in the shape of the resulting energy density wave.

In summary, we have derived an expression for the total energy deposited into the medium per unit length (or time) by a fast parton propagating in the quark-gluon plasma, including both collisional and radiative energy deposition. We have shown that the contribution of gluon radiation to the medium excitation grows with path length. Our result is reminiscent of, but less dramatic than, the increase of the energy deposition by a light quark obtained in the strongly coupled supersymmetric Yang-Mills theory [21].

In our formalism, the magnitude of the wave depends on the specific value (6) of $\hat{q}$, as well as the form of the collisional energy loss (1). It is possible that in the quark-gluon plasma produced at RHIC either one or both of these has a larger value than that assumed here (see Bass *et al.* [22] for a range of values of $\hat{q}$ compatible with experimental data). A general feature of our result is that the perturbation created in the QCD medium by a fast parton will be dominated by the last stage of the phase in which color charges are deconfined and highly mobile, i.e., just before bulk hadronization. This observation may explain why the measured peak emission angle of secondary hadrons in the backward direction [6] corresponds to a small sound velocity $c_s/c \approx 0.3$ consistent with values deduced from lattice QCD for $T \approx T_c$.

One may ask if the rapid expansion of the medium in a heavy-ion collision may result in decreasing rate of collisional energy loss that counteracts the growth in time of the energy deposition found here. The collisional energy loss (1) is proportional to $g(T)^2 m_D^2$. An examination of lattice results for m_D [23] shows that this quantity is almost independent of T in the range relevant to the RHIC experiments. This suggests that the expansion of the medium in a heavy-ion collision will not substantially alter our conclusions.

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Note added.—We draw attention to recent work [24], in which Qin *et al.* treated the same problem in a different formalism and obtained similar results.

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