

Relativistic Shock Waves in Viscous Gluon Matter

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We solve the relativistic Riemann problem in viscous gluon matter employing a microscopic parton cascade. We demonstrate the transition from ideal to viscous shock waves by varying the shear viscosity to entropy density ratio η/s from zero to infinity. We show that an η/s ratio larger than 0.2 prevents the development of well-defined shock waves on time scales typical for ultrarelativistic heavy-ion collisions. Comparisons with viscous hydrodynamic calculations confirm our findings.

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In the 1970s, shock waves were theoretically predicted to occur in collisions of heavy nuclei [1]. This phenomenon has been experimentally investigated [2] and subsequently observed [3]. Recently, jet quenching [4] has been discovered in heavy-ion collisions at Brookhaven National Laboratory's Relativistic Heavy-Ion Collider (RHIC). In this context, very exciting jet-associated particle correlations [5] have been observed, which indicates the formation of shock waves in the form of Mach cones [6] induced by supersonic partons moving through the quark-gluon plasma (QGP). If true, it could give a direct access to the equation of state of the QGP, because the Mach cone angle is given by $\alpha = \arccos(c_s/v_{\text{jet}})$, where c_s is the velocity of sound of the QGP. The velocity of sound is related to the equation of state via $c_s^2 = dP/de$, where P is the pressure and e the energy density.

Shock waves can form and propagate only if matter behaves like a fluid. The large measured elliptic flow coefficient v_2 [7] indicates that the QGP created at the RHIC could even be a nearly perfect fluid. This is confirmed by recent calculations within viscous hydrodynamics [8] and microscopic transport theory [9] which estimate the shear viscosity to entropy density ratio η/s to be less than 0.4 in order to not spoil the agreement with the v_2 data. However, it is an important question whether the η/s value deduced from v_2 data is sufficiently small to allow for the formation of shock waves.

In this Letter, we make an effort to answer this question by considering the relativistic Riemann problem in viscous gluon matter. Using the BAMPS microscopic transport model (BAMPS denotes the Boltzmann approach of multiparton scattering) [10], we demonstrate the transition from ideal shock waves with zero width, to viscous shock waves with nonzero width, to free diffusion by varying the shear viscosity to entropy density ratio η/s from zero to infinity. We estimate the upper limit of the η/s ratio, for which shocks can still be observed experimentally on the time scale of an ultrarelativistic heavy-ion collision.

The initial condition for the relativistic Riemann problem consists of two regions of thermodynamically equi-

brated matter with different constant pressure separated by a membrane at $z = 0$, which is removed at $t = 0$. Matter is assumed to be homogeneous in the transverse (x, y) direction so that the further evolution of matter becomes (1 + 1)-dimensional. For a perfect fluid, i.e., $\eta = 0$, this problem has an analytical solution [11], given by the solid curves in Fig. 1. For a thermodynamically normal medium, there is a shock wave traveling into matter with the lower pressure (on the right) with velocity v_{shock} larger than the velocity of sound c_s . The region of constant pressure be-

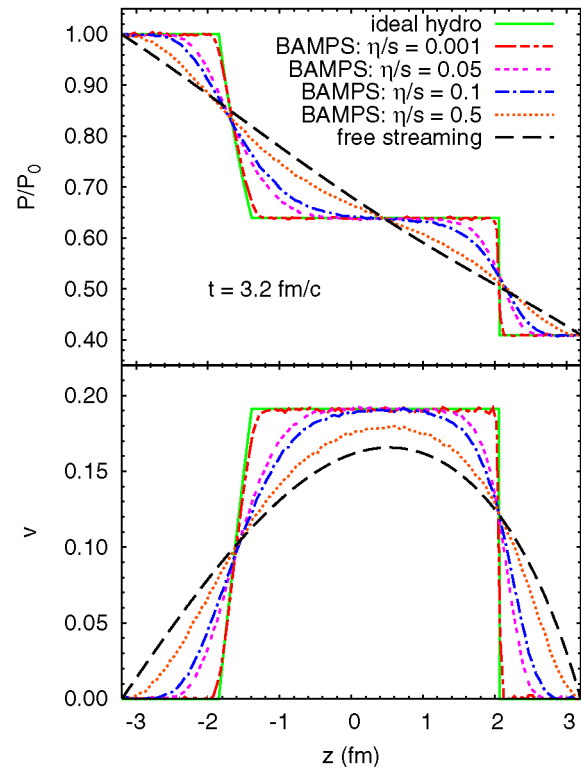


FIG. 1 (color online). The solution of the Riemann problem. At $t = 0$, the pressure is $P_0 = 5.43 \text{ GeV fm}^{-3}$ for $z < 0$ and $P_4 = 2.22 \text{ GeV fm}^{-3}$ for $z > 0$. The upper panel shows the pressure and the lower panel the velocity at time $t = 3.2 \text{ fm}/c$.

hind the shock wave is the so-called shock plateau. Here, matter moves collectively with a constant velocity v_{plat} as shown in the bottom panel in Fig. 1. Simultaneously with the creation of the shock wave, a rarefaction wave travels with velocity c_s into matter with the larger pressure (on the left). For an ideal fluid, the solution of the Riemann problem is self-similar in the variable $\zeta = z/t$; i.e., the solution as a function of ζ does not change with time.

The velocity of the shock front is determined from the relativistic Rankine-Hugoniot-Taub equations [12] and is given by [11]

$$v_{\text{shock}} = \left[\frac{(P_3 - P_4)(e_3 + P_4)}{(e_4 + P_3)(e_3 - e_4)} \right]^{1/2}, \quad (1)$$

where P_3 and e_3 (P_4 and e_4) are the pressure and energy density, respectively, on the left (right) side of the shock front. For noninteracting ultrarelativistic gluon matter, the equation of state is $e = 3P$. For the situation depicted in Fig. 1, $v_{\text{shock}} = 0.645$.

The extreme opposite of an ideal fluid (i.e., $\eta = 0$) is a gas of free-streaming particles (i.e., $\eta = \infty$). The solution of the Riemann problem for free-streaming particles is given by the short-dashed lines in Fig. 1, which can also be calculated analytically [13]. From the solution for the single-particle distribution function $f(x, p)$, one computes the energy-momentum tensor $T^{\mu\nu} = \int (d^3 p/E) p^\mu p^\nu f(x, p)$, where $p^\mu = (E, \mathbf{p})$ is the four-momentum. The energy density defined as $e = u_\mu T^{\mu\nu} u_\nu$ is the largest eigenvalue of T^μ_ν . The eigenvector u^μ is then the four velocity in Landau's definition [14]. In our case $u^\mu = \gamma(1, 0, 0, v)$, where $v = T^{03}/(e + T^{33})$ and $\gamma = (1 - v^2)^{-1/2}$. The pressure is defined as $P = -\Delta_{\mu\nu} T^{\mu\nu}/3$, where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ and $g^{\mu\nu} = (1, -1, -1, -1)$ is the metric tensor. For systems of massless particles, T^μ_ν is traceless and thus $P = e/3$. In the free-streaming case the characteristic structure of the solution of the Riemann problem for an ideal fluid is completely washed out, and a clear distinction between the shock wave and the rarefaction fan is no longer possible.

In the following, we study the influence of the η/s ratio on the formation and evolution of shock waves by solving the Riemann problem with the parton cascade BAMPS [10]. BAMPS is a microscopic transport model which solves the Boltzmann equation $p^\mu \partial_\mu f(x, p) = C(x, p)$ for on-shell gluons with the collision integral $C(x, p)$. In this study, we consider only binary gluon scattering processes with an isotropic cross section. We remark that, although perturbative QCD (pQCD) favors small angles in binary gluon scatterings, the use of an isotropic cross section effectively implements pQCD gluon bremsstrahlung, which has a broader angular distribution due to the suppression of soft collinear radiation [10].

In our calculations we use a constant η/s value. In order to achieve this, we have to locally adjust the cross section σ , since the particle density n is not constant. The shear viscosity η is given by $\eta = 4e/(15R^{\text{tr}})$ [15], where the

transport collision rate $R^{\text{tr}} = n\sigma^{\text{tr}} = 2n\sigma/3$ for isotropic scattering processes [16]. n is calculated via $n = N^\mu u_\mu$, where $N^\mu = \int (d^3 p/E) p^\mu f(x, p)$. We obtain

$$\eta = \frac{2}{5} e \lambda_{\text{mfp}}, \quad (2)$$

where $\lambda_{\text{mfp}} = 1/(n\sigma)$ is the gluon mean-free path. Binary collisions imply that we cannot maintain chemical equilibrium. In this case, in kinetic equilibrium the entropy density is calculated approximately via $s = 4n - n \ln \lambda$, where $\lambda = n/n_{\text{eq}}$ is the gluon fugacity measuring the deviation from chemical equilibrium. For a nonvanishing shear viscosity, the system will deviate from initial kinetic and chemical equilibrium during the evolution [17]. Gluons are regarded as Boltzmann particles, so that the number density in thermal equilibrium is given by $n_{\text{eq}} = d_G T^3/\pi^2$, with $d_G = 16$ for gluons and $T = e/(3n)$ denotes the temperature. By sending η/s to zero, the cross section will be unphysically large. However, it serves as a useful test of the parton cascade in the ideal hydrodynamical limit.

In Fig. 1, we show the results for various η/s values as computed with BAMPS, demonstrating the gradual transition from the ideal hydrodynamical limit to free streaming of particles. Remarkably, the ideal solution is reproduced to very high precision by the BAMPS calculation for a small $\eta/s = 0.001$. In this sense, BAMPS can compare with state-of-the-art numerical algorithms used to solve the ideal hydrodynamical equations [11]. A larger η/s value results in a finite transition layer where the quantities change smoothly rather than discontinuously as in the case of a perfect fluid. The width of the shock front is proportional to the shear viscosity [18].

As seen in Fig. 1, a nonzero viscosity smears the pressure and velocity profiles and impedes a clean separation of the shock front from the rarefaction fan. A criterion for a clear separation, and thus the observability of a shock wave, is the formation of a shock plateau, as in the ideal-fluid case. The formation of a shock plateau takes a certain amount of time, as demonstrated in Fig. 2, where we show the pressure and velocity profiles at different times for $\eta/s = 0.1$. Formally, we define the time of formation of the shock plateau when the maximum of the velocity distribution $v(z)$ reaches the value v_{plat} of the ideal-fluid solution. From the bottom part of Fig. 2, we see that this happens at $t = 3.2$ fm/c. This agrees with what we obtained in the bottom part of Fig. 1. From this figure we also infer that for $\eta/s > 0.1$ a shock plateau has not yet developed at $t = 3.2$ fm/c, whereas for $\eta/s < 0.1$ it has already fully formed.

In order to understand the time scale of the formation of a shock wave, we define the quantity [16,17,19]

$$K = \frac{\lambda_{\text{mfp}}}{L}, \quad (3)$$

where $L \equiv t(v_{\text{shock}} + c_s)$ is the width of the region bounded by the rarefaction wave traveling to the left and the shock front moving to the right (in the ideal-fluid case).

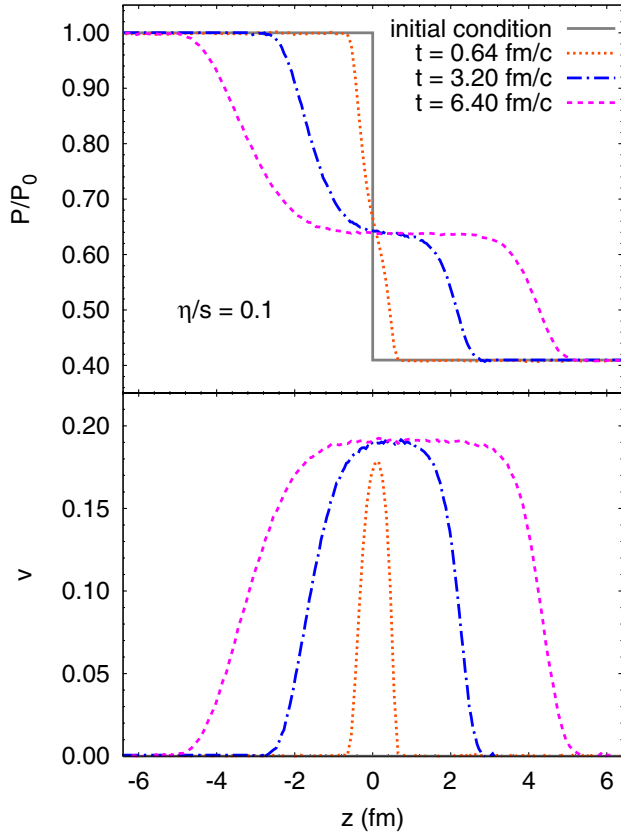


FIG. 2 (color online). The time evolution of the Riemann problem for $\eta/s = 0.1$ and the initial condition of Fig. 1.

Note that λ_{mfp} is not constant; we approximate it by its maximum value which is assumed on the low-pressure side (the undisturbed medium) in front of the shock wave. The quantity K can be viewed as a “global” Knudsen number for the Riemann problem. K goes to zero at late times, which implies that the medium behaves more and more like an ideal system.

We find a scaling behavior for the solution of the Riemann problem: The pressure profile $P(z, t; \eta/s)/P_0$ (and also the velocity profile) is only a function of $\zeta = z/t$ and K , i.e., $P(z, t; \eta/s)/P_0 = F(\zeta; K)$, for a given ratio P_4/P_0 . The value of K at which the shock wave forms, called K_f , is universal for a given P_4/P_0 . With Eq. (2) we obtain $\lambda_{\text{mfp}} = 10/(3T_4)\eta/s$ in the undisturbed medium with the lower pressure. Thus, using the observation from Fig. 2 that the shock wave forms at $t = 3.2$ fm/c for $\eta/s = 0.1$, we find that $K_f = 0.053$.

By inserting $\lambda_{\text{mfp}} = 10/(3T)\eta/s$ into Eq. (3), the formation time of shock waves is given by

$$t_f = \frac{10}{3} \frac{1}{K_f(v_{\text{shock}} + c_s)T} \frac{\eta}{s}. \quad (4)$$

Figure 3 shows the relation (4) with $T = 350$ MeV and for various initial pressure discontinuities P_4/P_0 . The difference in slopes reflects the dependence of K_f and v_{shock} on the ratio P_4/P_0 . For $\eta/s = 0.2$ a shock will not be visible

until 6–7.2 fm/c, which most likely exceeds the lifetime of the QGP at the RHIC. Vice versa, if shock phenomena are discovered at the RHIC, this could be an indication that the QGP has a small η/s ratio, probably smaller than 0.2. For a more viscous QGP, no shock waves and thus no Mach cones will be formed. In a relativistic heavy-ion collision, the temperature is decreasing during the expansion. Thus, according to Eq. (4), t_f is even larger and shock waves may be even harder to observe.

To confirm our results calculated with BAMPS, we solve the Riemann problem within relativistic dissipative fluid dynamics. To this end, we use the Israel-Stewart (IS) equations [20] (see Ref. [21] for different approaches). We neglect heat conductivity and bulk viscosity (thus assuming local chemical equilibrium). In 1 + 1 dimensions, the IS equations reduce to

$$\partial_t T^{00} + \partial_z(vT^{00}) = -\partial_z(vP + v\tilde{\pi}), \quad (5)$$

$$\partial_t T^{0z} + \partial_z(vT^{0z}) = -\partial_z(P + \tilde{\pi}), \quad (6)$$

$$\gamma\partial_t \tilde{\pi} + \gamma v\partial_z \tilde{\pi} = \frac{1}{\tau_\pi}(\pi_{\text{NS}} - \tilde{\pi}) - \frac{\tilde{\pi}}{2}\left(\theta + D \ln \frac{\beta_2}{T}\right), \quad (7)$$

where $\theta \equiv \partial_\mu u^\mu$ and $D \equiv u^\mu \partial_\mu$. The laboratory frame energy and momentum density are given by $T^{00} = (e + P + \tilde{\pi})\gamma^2 - (P + \tilde{\pi})$ and $T^{0z} = (e + P + \tilde{\pi})\gamma^2 v$, respectively, while the system is closed by a massless gluon equation of state. For vanishing shear viscous pressure $\tilde{\pi}$, Eqs. (5) and (6) reduce to the equations of ideal hydrodynamics.

The first-order theory of relativistic dissipative fluid dynamics defines the viscous pressure algebraically by the relativistic Navier-Stokes value $\pi_{\text{NS}} = -(4/3)\eta\theta$ [14,22]. In the IS equations, the local shear pressure relaxes to the Navier-Stokes value on a time scale comparable to the mean-free time between collisions, given by the relaxation time $\tau_\pi = 2\eta\beta_2$, where $\beta_2 = 3/(4P)$ [20].

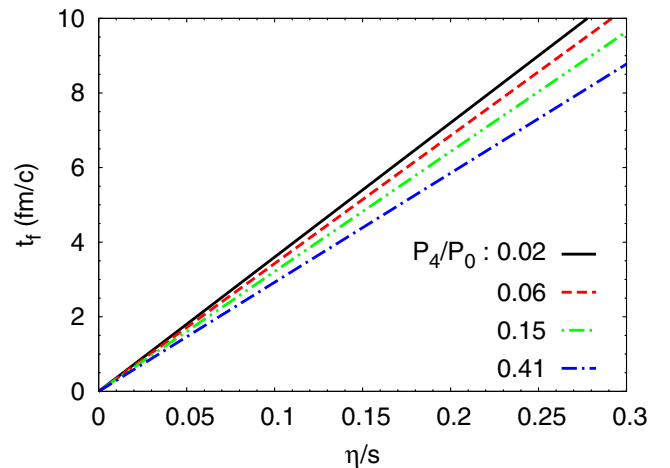


FIG. 3 (color online). Formation time of shock waves as a function of η/s .

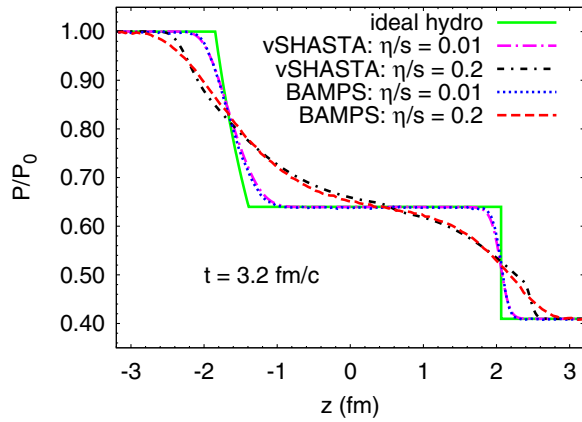


FIG. 4 (color online). The same as Fig. 1. Results obtained from vSHASTA and BAMPS calculations are compared.

The IS equations (5)–(7) are solved numerically using the viscous sharp and smooth transport algorithm (vSHASTA) approach [23]. Figure 4 shows comparisons between the results from BAMPS and vSHASTA calculations for $\eta/s = 0.01$ and 0.2 . We see perfect agreement for $\eta/s = 0.01$, whereas for the larger value of $\eta/s = 0.2$ small deviations in the region around the shock front and the rarefaction fan are found. The reason can be understood considering the Knudsen number $K_\theta \equiv \lambda_{\text{mfp}}\theta$. The IS equations contain terms of second order in K_θ [24]. However, at the shock front, the macroscopic scale over which the fluid velocity varies is comparable to the microscopic scale λ_{mfp} , such that $K_\theta \sim 1$, and one would need higher powers of K_θ in the IS equations to improve the description. Wherever K_θ is large, the applicability of the IS theory is questionable. Microscopic transport calculations do not suffer from this drawback.

In summary, employing the parton cascade BAMPS, we have solved the relativistic Riemann problem. The transition from ideal-fluid behavior to free streaming is demonstrated. Numerical results from BAMPS agree well with those obtained from viscous hydrodynamical calculations. We found that the formation of shock waves in gluon matter with $\eta/s > 0.2$ probably takes longer than the lifetime of the QGP at the RHIC. Whether Mach cones from jets [25] can be observed within nuclear collisions will be studied in the future within the BAMPS approach.

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