Non-Abelian Topological Order in s-Wave Superfluids of Ultracold Fermionic Atoms

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(Received 30 January 2009; revised manuscript received 8 June 2009; published 6 July 2009)

It is proposed that in s-wave superfluids of cold fermionic atoms with laser-field-generated effective spin-orbit interactions, a topological phase with gapless edge states and Majorana fermion quasiparticles obeying non-Abelian statistics is realized in the case with a large Zeeman magnetic field. Our scenario provides a promising approach to the realization of quantum computation based on the manipulation of non-Abelian anyons via an s-wave Feshbach resonance.

DOI: 10.1103/PhysRevLett.103.020401 PACS numbers: 05.30.Fk, 03.65.Vf, 03.75.Ss, 67.85.−d

Introduction.—Recently, there has been considerable interest in topological phases of quantum many-body systems, which are characterized by the following features [1–4]: (i) there are topologically protected gapless edge states on surface boundaries of the systems, which are stable against local perturbations, (ii) for two-dimensional (2D) systems, there are quasiparticles with fractional quantum numbers (e.g., fractional charges) termed "anyons." To this time, the possibility of realizing topological phases has been studied for various states realized in condensed matter systems, such as quantum (spin) Hall states [2], vortex states of $p + ip$ superconductors [5,6], and spin liquid states [3,7], and for cosmological systems such as axion strings [8]. The feature (ii) is particularly of interest in connection with the realization of fault-tolerant quantum computation based on the manipulation of non-Abelian anyons [3,7,9,10]. Since topological phases provide not only a novel paradigm of quantum ground states but also a potential breakthrough for technological advance, it is desirable to pursue various possible schemes for their realization.

In this Letter, we propose a scenario in which a topological phase, possessing gapless edge states and non-Abelian anyons, is realized in a BCS s-wave superfluid (SF) of ultracold fermionic atoms in an optical lattice with a laser-field-generated effective spin-orbit (SO) interaction. It is possible to generate an artificial SO interaction that acts on atoms by using spatially varying laser fields [11,12]. The effective SO interaction is a key factor in our scenario of the topological phase. Recently, topological phases in superconductors and SFs have been investigated by several authors [5,6,10,13–15]. The previous studies, however, focus on the p-wave pairing state [16]. BCS gaps of p-wave superconductors in solid state systems are typically very small, and it is difficult to utilize them for topological quantum computation, because topological phases are destroyed by thermal excitations beyond bulk energy gaps. For cold atoms, in principle, a p-wave SF with a large BCS gap can be produced via a p-wave Feshbach resonance [10]. However, unfortunately, this has not yet been realized because of huge loss [17]. Contrastingly, s-wave SFs of cold atoms with large BCS gaps have been realized via an s-wave Feshbach resonance [18]. Thus, our scenario based on s-wave SFs of cold atoms is deemed more advantageous for the realization of the topological order than that using p-wave SFs via a p-wave Feshbach resonance. Moreover, there is an important difference between the topological phase considered here and that of p-wave SFs. For chiral p-wave SFs, the non-Abelian anyons are vortices of the SF order parameter, which contain Majorana fermion modes. In striking contrast, in our system, the non-Abelian anyons are vortices of the SO interaction, i.e., the phase twist caused by the orbital motion accompanying spin flip. We propose an experimental scheme for generating and controlling vortices in the SO interaction, i.e., non-Abelian anyons, which are stabilized by use of a carefully designed laser setup rather than spontaneously formed macroscopic condensates. This scheme can be carried out by utilizing current sophisticated laser techniques. Thus, our proposal provides a promising approach to the realization of topological quantum computation based on the manipulation of non-Abelian anyons.

Model and analysis of the topological phase.—Let us consider an s-wave SF of neutral fermionic atoms in the 2D optical square lattice, which is described by the Hamiltonian $\mathcal{H} = \mathcal{H}_{\rm kin} + \mathcal{H}_{\rm SO} + \mathcal{H}_s$:

$$\begin{aligned}
\mathcal{H}_{\rm kin} &= -t\sum_{i\sigma}\sum_{\hat{\mu}=\hat{x},\hat{y}}(c^\dagger_{i+\hat{\mu}\sigma}c_{i\sigma} + c^\dagger_{i-\hat{\mu}\sigma}c_{i\sigma}) \\
&\quad - \mu\sum_{i\sigma}c^\dagger_{i\sigma}c_{i\sigma} - h\sum_i(c^\dagger_{i\uparrow}c_{i\uparrow} - c^\dagger_{i\downarrow}c_{i\downarrow}), \\
\mathcal{H}_{\rm SO} &= -\lambda\sum_i[(c^\dagger_{i-\hat{x}\downarrow}c_{i\uparrow} - c^\dagger_{i+\hat{x}\downarrow}c_{i\uparrow}) \\
&\quad + i(c^\dagger_{i-\hat{y}\downarrow}c_{i\uparrow} - c^\dagger_{i+\hat{y}\downarrow}c_{i\uparrow}) + \text{H.c.}], \\
\mathcal{H}_s &= -\sum_i\psi_s(c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} + \text{H.c.}),
\end{aligned} \tag{1}$$

where $c^\dagger_{i\sigma}$ ($c_{i\sigma}$) denotes a creation (annihilation) operator of the fermionic atom with pseudospin $\sigma = (\uparrow, \downarrow)$ at site $\boldsymbol{i} = (i_x, i_y)$, and ψ_s the gap function. $\hat{x}$ ($\hat{y}$) is a basic lattice vector along the x (y) axis. $\mathcal{H}_{\rm SO}$ is an effective Rashba type SO interaction [19]. We will discuss later the method of generating the Rashba SO interaction for neutral atoms via laser fields. We also introduce the chemical potential μ and the Zeeman term induced by a magnetic field h. In the momentum space, the Hamiltonian is recast into $\mathcal{H} = \frac{1}{2}\sum_{\boldsymbol{k}}(\boldsymbol{c}_{\boldsymbol{k}}^\dagger, \boldsymbol{c}_{-\boldsymbol{k}})\mathcal{H}(\boldsymbol{k})(\boldsymbol{c}_{\boldsymbol{k}}, \boldsymbol{c}_{-\boldsymbol{k}}^\dagger)^T$ with $\boldsymbol{c}_{\boldsymbol{k}}^\dagger = (1/\sqrt{V})\sum_{\boldsymbol{i}} e^{i\boldsymbol{k}\boldsymbol{i}}(c_{\boldsymbol{i}\uparrow}^\dagger, c_{\boldsymbol{i}\downarrow}^\dagger)$, and

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \epsilon_{\boldsymbol{k}} - h\sigma_z + \boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma} & i\psi_s\sigma_y \\ -i\psi_s\sigma_y & -\epsilon_{\boldsymbol{k}} + h\sigma_z + \boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}^* \end{pmatrix}, \tag{2}$$

where $\epsilon_{\boldsymbol{k}} = -2t(\cos k_x + \cos k_y) - \mu$, $\boldsymbol{g}_{\boldsymbol{k}} = 2\lambda(\sin k_y, -\sin k_x)$, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ the Pauli matrices.

As mentioned in the introduction, the non-Abelian topological order is characterized by the existence of gapless chiral edge states propagating only in one direction and the existence of the non-Abelian anyons [5]. The former is also associated with the nonzero Chern number [20]. In the following, we demonstrate that these features are indeed realized in the system (1) when a certain relation among μ, h, and ψ_s holds [Eq. (5) below].

A key observation of our analysis is that the Hamiltonian $\mathcal{H}(\boldsymbol{k})$ is unitary equivalent to the following "dual" Hamiltonian $\mathcal{H}^{\rm D}(\boldsymbol{k})$,

$$\mathcal{H}^{\rm D}(\boldsymbol{k}) = \begin{pmatrix} \psi_s - h\sigma_z & -i\epsilon_{\boldsymbol{k}}\sigma_y - i\boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}\sigma_y \\ i\epsilon_{\boldsymbol{k}}\sigma_y + i\boldsymbol{g}_{\boldsymbol{k}}\sigma_y\boldsymbol{\sigma} & -\psi_s + h\sigma_z \end{pmatrix}, \tag{3}$$

with the unitary transformation

$$\mathcal{H}^{\rm D}(\boldsymbol{k}) = D\mathcal{H}(\boldsymbol{k})D^\dagger, \qquad D = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i\sigma_y \\ i\sigma_y & 1 \end{pmatrix}. \tag{4}$$

From Eq. (3), it is found that the Rashba SO interaction $\boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}$ in the original Hamiltonian $\mathcal{H}(\boldsymbol{k})$ is formally transformed into a "p-wave SF gap" with the $\boldsymbol{d}$ vector, $\boldsymbol{d}_{\boldsymbol{k}}^{\rm D} \equiv -\boldsymbol{g}_{\boldsymbol{k}}$, in the dual Hamiltonian $\mathcal{H}^{\rm D}(\boldsymbol{k})$. However, this does not necessarily mean that the topological properties of $\mathcal{H}(\boldsymbol{k})$ are the same as those of a p-wave SF, since $\mathcal{H}^{\rm D}(\boldsymbol{k})$ has a nonstandard constant kinetic term $\epsilon_{\boldsymbol{k}}^{\rm D} \equiv \psi_s$. A similar p-wave SF state with a constant kinetic energy term was considered before in the context of the quantum-Hall effect (QHE) state [5]. An important feature of (3) is that the topological order emerges when μ, h, and ψ_s satisfy

$$\psi_s^2 + \epsilon(0,0)^2 < h^2 < \psi_s^2 + \epsilon(\pi,0)^2, \tag{5}$$

with $\epsilon(k_x, k_y) \equiv \epsilon_{\boldsymbol{k}}$. Here note that although the condition (5) implies the Zeeman energy larger than the BCS gap ψ_s, the superfluidity is stable when $\lambda \gg h$ [21]. This stability is specific to neutral atomic systems. For electron systems, such large magnetic fields usually destroy superconductivity via an orbital depairing effect.

Let us first examine edge states in our model. Figure 1 illustrates the energy bands obtained by diagonalizing the lattice Hamiltonian (1) with the open boundaries at $i_x = 0$, L for various h. Here we have taken the periodic boundary condition in the y direction, and $k_y \in [-\pi, \pi]$ is the lattice momentum in the y direction. By increasing h from zero adiabatically, it is found that the bulk energy gap closes at $h = \sqrt{\psi_s^2 + \epsilon(0,0)^2}$ [Fig. 1(b)], then, for h satisfying (5), a gapless edge mode with a linear dispersion $E \sim ck_y$ ($E \sim -ck_y$) localized on the one edge (the other edge) appears between the bulk energy gap [Fig. 1(c)]. This chiral edge state is stable against any weak local perturbations provided that there exists the nonzero Chern number; i.e., the topological number equivalent to the total number of gapless chiral edge modes, which was first introduced in the case of the QHE states [20]. We calculated the Chern number $\mathcal{Q}$ for $\mathcal{H}(\boldsymbol{k})$ or equivalently $\mathcal{H}^{\rm D}(\boldsymbol{k})$. [Since the Chern number is calculated from the Berry curvature in the momentum space, it is not affect by the unitary transformation D which is independent of $\boldsymbol{k}$, ensuring the topological equivalence between (2) and (3).] We found that $\mathcal{Q} = 1$ when the condition (5) is satisfied [22]. This is consistent with the numerical results for edge states shown above.

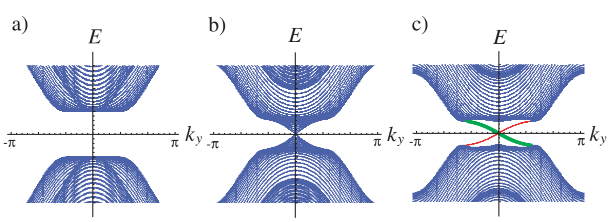

FIG. 1 (color online). The band energy of the lattice Hamiltonian (1) with edges at $i_x = 0$ and $i_x = 50$ ($= L$). Here $k_y \in [-\pi, \pi]$ denotes the momentum in the y direction. We set $t = 1$, $\mu = -4$, $\lambda = 0.5$, and $\psi = 0.5$. h are (a) $h = 0$, (b) $h = 0.5$, (c) $h = 0.8$. The red thin line indicates a gapless chiral edge mode localized on the one side and green thick line a gapless chiral edge mode on the other side. They appear for $\sqrt{\psi^2 + \epsilon(0,0)^2} < h < \sqrt{\psi^2 + \epsilon(0,\pi)^2}$.

We now demonstrate that there exist the non-Abelian anyons in our system. For this purpose, we solve the Bogoliubov–de Gennes (BdG) equation for a single vortex: If there exists a single Majorana fermion zero mode for each vortex, vortices obey the non-Abelian statistics [5,6]. We use the dual Hamiltonian $\mathcal{H}^{\rm D}$ to solve the BdG equation, then construct a solution in the original Hamiltonian $\mathcal{H}$ by using the duality transformation (4). For simplicity, we assume $\epsilon(0,0) = 0$ for the time being. Then, low-energy properties are governed by fermions on the Fermi surface, which is split into $|\boldsymbol{k}| \sim 0$ and $|\boldsymbol{k}| \sim \lambda/t$ by the SO interaction, but the larger Fermi surface ($|\boldsymbol{k}| \sim \lambda/t$) can be neglected for the zero mode [22]. Thus, we concentrate on fermions with $\boldsymbol{k} \approx (0,0)$, for which $\mathcal{H}^{\rm D}(\boldsymbol{k})$ is decomposed into the following two 2×2 matrices $\mathcal{H}_{+}^{\mathrm{D}}$ and $\mathcal{H}_{-}^{\mathrm{D}}$,

$$\mathcal{H}_{\pm}^{\mathrm{D}}(\boldsymbol{k}) = \begin{pmatrix} \psi_s \mp h & 2\lambda(\pm k_y + ik_x) \\ 2\lambda(\pm k_y - ik_x) & -\psi_s \pm h \end{pmatrix}. \quad (6)$$

We consider a single vortex of the "p-wave SF gap" $2\lambda(\pm k_y + ik_x)$. The BdG equations for $\mathcal{H}_{\pm}^{\mathrm{D}}$ with the single vortex can be solved by using the method developed in [5]. Then, we find a unique zero energy solution with a quasiparticle field $\gamma^{\dagger} = \int d\boldsymbol{r}[u_0\psi_{+}^{\dagger} + v_0\psi_{+}]$, where $u_0 = i(re^{i\theta})^{-1/2}e^{-(h-\psi_s)r/2\lambda}$, $v_0 = -i(re^{-i\theta})^{-1/2} \times e^{-(h-\psi_s)r/2\lambda}$ [22]. The solution is normalizable when (5) is satisfied. This is the Majorana zero energy mode; i.e., $\gamma^{\dagger} = \gamma$. Using the duality transformation (4), we found that a vortex in the original Hamiltonian has a single Majorana zero mode given by $D(u_0, 0, v_0, 0)^T$, which implies that the vortex is a non-Abelian anyon [5].

From the construction of the zero mode above, we notice that there is an important difference between a chiral p-wave SF and our system: While for a spinless chiral p-wave SF, a single Majorana zero mode exists in a vortex of the SF order parameter, for our noncentrosymmetric s-wave SF, a Majorana zero mode exists in a vortex twisting a phase of the SO interaction. This difference can be understood immediately from the duality (3) since a vortex in a gap function in the dual Hamiltonian is transformed into a vortex in the SO coupling. So far, we have assumed $\epsilon(0,0) = 0$, but even when $\epsilon(0,0) \neq 0$, the existence of the non-Abelian topological order is robust as long as h satisfies (6), because the topological character is not changed unless the bulk gap closes [14].

For the detection of the non-Abelian anyons, it is desirable that the zero energy state in a vortex is well separated from excited states, the interaction with which may cause decoherence. The excitation energy in the vortex core is due to the kinetic energy which stems from the derivative term of the BdG equation, $2\lambda(\mp i\partial_y + \partial_x)$. Since the above solution for (u_0, v_0) indicates that the size of the vortex core is $\sim 2\lambda/(h - \psi_s)$, the excitation energy is of the order $\sim 2\lambda/(2\lambda/(h - \psi_s)) \sim h - \psi_s$. It can be tuned to be relatively large, and thus the experimental detection of the non-Abelian anyons is quite feasible.

Possible realization in cold fermionic atoms.—We now propose an experimental scheme for the realization of the topological phase mentioned above in ultracold fermionic atoms. It was recently pointed out by several authors that effective gauge fields interacting with atoms can be generated by spatially varying laser fields [11,12]. These ideas can be utilized for our purpose. We consider fermionic atoms loaded in a 2D periodic optical lattice, where there is no hopping along the z direction [11]. The atoms occupy doubly degenerate Zeeman levels of the hyperfine ground state manifolds, which are, respectively, the "spin-up" state $|\uparrow\rangle$ and the "spin-down" state $|\downarrow\rangle$. We introduce the Zeeman field to lift the degeneracy. The Zeeman level split is denoted as E_Z. It is assumed that standard tunneling of atoms between sites due to kinetic energy is suppressed by the large depth of the optical lattice potential. Tunneling of atoms between neighboring sites along the ν direction ($\nu = x, y$) which conserves spins is caused by laser beams with the Rabi frequency $\Omega_{\nu 0}$ via optical Raman transitions as proposed in Refs. [11,23]; i.e. $\Omega_{\nu 0} = \Omega_1'\Omega_2'/2\Delta$ with $\Omega_{1,2}'$ the Rabi frequencies for the transition between the ground state and an excited state, and Δ detuning from the excited state. In addition, tunneling which accompanies spin flip is also driven by two Raman lasers [11]. In Fig. 2, we show the optical lattice setup. The laser with the Rabi frequency $\Omega_{\nu 1}$ ($\Omega_{\nu 2}$) is resonant for transition $|\uparrow\rangle \rightarrow |\downarrow\rangle$ for the tunneling between neighboring sites in the forward (backward) ν direction. As proposed in Ref. [11], the confining optical potential is tilted along both the x direction and the y direction to assure that the forward and backward tunneling processes are, respectively, induced by the lasers with the different Rabi frequencies $\Omega_{\nu 1}$ and $\Omega_{\nu 2}$, which are required for the realization of the Rashba spin-orbit interaction as discussed below. The energy shift between nearest neighbor sites due to the tilting potential is Δ_x for the x direction and Δ_y for the y direction. We impose the condition $\Delta_x \neq \Delta_y$ to prevent tunneling with spin flip along the $y(x)$ direction due to the lasers with $\Omega_{x(y)1,2}$. It is also assumed that the detuning from excited states for optical Raman transitions is much larger than $\Delta_{x(y)}$, and thus the spatial variation of the amplitudes of the Rabi frequencies due to the tilting potential is negligible. To realize the Rashba spin-orbit interaction for the two Zeeman levels, we choose the phases of the lasers as follows. The lasers are propagating along the z direction with an oscillating factor $e^{ik_z z}$. The Rabi frequency Ω_{x2} is expressed as $\Omega_{x2} = |\Omega_{x2}|e^{ik_z z}$. The phase of the laser Ω_{x1} is shifted by π from that of Ω_{x2}, and $\Omega_{x2} = -\Omega_{x1}$ holds. The phase of Ω_{y1} (Ω_{y2}) is shifted by $-\pi/2$ ($\pi/2$), and $\Omega_{y2} = -i\Omega_{x1}$, $\Omega_{y2} = -\Omega_{y1}$. Then, the laser-induced tunneling term which accompanies spin

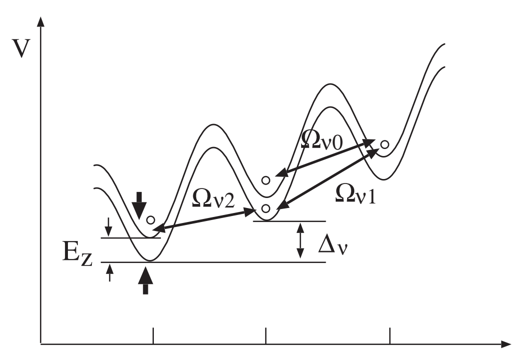

FIG. 2. Setup of the confining optical potential for the ground state. $\nu = x$ or y. Bold up and down arrows indicate, respectively, the spin-up and spin-down states. In this figure, excited levels which mediate the hopping via two Raman lasers are not shown explicitly.

flip is expressed by $\mathcal{H}_{\rm SO} = \sum_i [\lambda_x (c^\dagger_{i-\hat{x}\downarrow} c_{i\uparrow} - c^\dagger_{i+\hat{x}\downarrow} c_{i\uparrow}) + i\lambda_y (c^\dagger_{i-\hat{y}\downarrow} c_{i\uparrow} - c^\dagger_{i+\hat{y}\downarrow} c_{i\uparrow}) + \text{H.c.}]$ with $\lambda_\nu = c_\nu \int d\boldsymbol{r} \psi^*_\downarrow(\boldsymbol{r} - \boldsymbol{r}_{i-\hat{\mu}}) \Omega_{\nu 2}(\boldsymbol{r}) \psi_\uparrow(\boldsymbol{r} - \boldsymbol{r}_i)$, $\nu = x,\ y$, and $c_x = 1$, $c_y = -i$. Since we consider the 2D xy plane with $z = 0$, λ_ν is real. For $\lambda_x = \lambda_y$, $\mathcal{H}_{\rm SO}$ is the Rashba SO interaction.

To create vortices of the SO interaction, which are key ingredients for the realization of non-Abelian anyons, we replace the lasers that generate the SO interaction with those carrying orbital angular momentum parallel to the z axis. Such lasers can be prepared by using Laguerre-Gaussian beams [24]. Then, a ''vortex'' of the SO interaction is introduced: $\lambda_\nu \rightarrow \lambda_\nu e^{im\theta}$. The vorticity m is controlled by changing the configuration of the lasers. Furthermore, the spatially separated multiple vortices can be generated by using the following method. After introducing a vortex with vorticity m into the system, we switch the Laguerre-Gaussian beam to a Gaussian beam, i.e., a laser without angular momentum [24]. The vortex still exists in the system, because of the conservation of the total angular momentum. However, for $m > 1$, the vortex with higher charges become energetically unstable toward dissociation into m vortices with single vorticity, and thus spatially separated multiple vortices are created in the system [25]. This multiple vortex state allows the realization of the non-Abelian statistics of vortices.

We can use the Feshbach resonance in the s-wave channel for the formation of the s-wave Cooper pairs in this system [18]. Then, the topological phase described by the Hamiltonian (1) is realized. In this scheme, the role of the s-wave superfluidity is twofold. One is to suppress bulk gapless quasiparticles which are harmful for topological stability. The other one is to generate the superposition of particles and holes, which results in the Majorana quasiparticles in vortices and edge states. It is noted that the vortices in the SO interaction are stabilized by the carefully designed laser setup rather than by macroscopic condensates. It should be emphasized that this experimental scheme is feasible for currently accessible laser techniques. As mentioned before, the non-Abelian anyons are stable for sufficiently low energies $\ll \min\{h - \psi_s, \psi_s\}$. Since ψ_s can be tuned to be large, i.e., $\psi_s \sim E_F$, by using the s-wave Feshbach resonance, the realization of the non-Abelian anyons in this scheme is quite promising.

Summary.—We have proposed a feasible scheme for the realization of the non-Abelian topological phase in an s-wave superfluid of cold atoms in an optical lattice, in which the non-Abelian anyons exist, and opens a possible way to realize topological quantum computation.

The authors thank the organizers of the symposium, ''Topological Aspects of Solid State Physics,'' at YITP, Kyoto, where this work has been started. This work was partly supported by the Grant-in-Aids for the Global COE Program and Scientific Research (Grant No. 18540347, No. 19014009, No. 19052003) from MEXT Japan.

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