Quantum Information Transfer from Spin to Orbital Angular Momentum of Photons

Eleonora Nagali,[1] Fabio Sciarrino,[1,*] Francesco De Martini,[1,2] Lorenzo Marrucci,[3,4,†] Bruno Piccirillo,[3,5] Ebrahim Karimi,[3] and Enrico Santamato[3,5]

[1]*Dipartimento di Fisica dell'Università "La Sapienza" and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma 00185, Italy*

[2]*Accademia Nazionale dei Lincei, via della Lungara 10, Roma 00165, Italy*

[3]*Dipartimento di Scienze Fisiche, Università di Napoli "Federico II," Complesso di Monte S. Angelo, 80126 Napoli, Italy*

[4]*CNR-INFM Coherentia, Complesso di Monte S. Angelo, 80126 Napoli, Italy*

[5]*Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Napoli, Italy*

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The optical "spin-orbit" coupling occurring in a suitably patterned nonuniform birefringent plate known as a "q plate" allows entangling the polarization of a single photon with its orbital angular momentum (OAM). This process, in turn, can be exploited for building a bidirectional "spin-OAM interface," capable of transposing the quantum information from the spin to the OAM degree of freedom of photons and vice versa. Here, we experimentally demonstrate this process by single-photon quantum tomographic analysis. Moreover, we show that two-photon quantum correlations such as those resulting from coalescence interference can be successfully transferred into the OAM degree of freedom.

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In the past few decades, quantum optics has allowed the implementation of a variety of quantum-information protocols. However, the standard information encoding based on the two-dimensional quantum space of photon polarizations (or "spin" angular momentum) imposes significant limitations to the protocols that may be implemented. To overcome such limitations, more recently the orbital angular momentum (OAM) of light, related to the photon's transverse-mode spatial structure [1], has been recognized as a new promising resource, allowing the implementation of a higher-dimensional quantum space, or a "qudit," encoded in a single photon [2–4]. Thus far, the generation of OAM-entangled photon pairs has been carried out by exploiting the process of parametric down-conversion [5,6] and has also been utilized in few quantum-information protocols [7–10]. Despite these successes, the optical tools for generating and controlling the OAM photon states (computer-generated holograms, Dove's prisms, cylindrical lens, etc.) are rather limited. A convenient way to coherently "interface" the OAM degree of freedom of photons with the more easily manipulated spin or polarization one has been missing so far. In this context, the recent invention of an optical device, the so-called "q plate" (QP), that couples the photon spin to its orbital angular momentum opens up a number of new possibilities [11]. In this work we show, both theoretically and experimentally, that this optical "spin-orbit" coupling can be exploited as an effective coherent bidirectional interface between polarization and orbital angular momentum in the quantum regime. The QP also enables the efficient generation of single-photon states in which the OAM and polarization degrees of freedom are entangled. Furthermore, we show that photon-photon quantum correlations of a biphoton [12] can be transferred from the spin to the OAM degree of freedom.

The QP is a birefringent slab having a suitably patterned transverse optical axis, with a topological singularity at its center. Here, we consider a QP with "charge" of the singularity $q = 1$ and uniform birefringent retardation $\delta = \pi$. Such QP modifies the OAM quantum number l (in units of $\hbar$) of a light beam crossing it, imposing a variation $\Delta l = \pm 2$ whose sign depends on the input polarization, positive for left-circular and negative for right-circular. The helicity of the output circular polarization is also inverted; i.e., the optical spin is flipped [13]. Let us now rephrase this behavior in a quantum formalism suitable for describing multiphoton states. Let $\hat{a}^{\dagger}_{j,l}$ be the operator creating a photon in the polarization state $\vec{\pi}_j$ (where $\vec{\pi}_H$ and $\vec{\pi}_V$ stand for horizontal and vertical linear polarizations, respectively) and with the OAM value l. For simplicity, we omit here the radial quantum number which would be needed for a complete characterization of transverse orbital states, as it will play no significant role in the following [14]. The overall dynamics induced by the QP can then be described by the transformations $\hat{a}^{\dagger}_{L,0} \Rightarrow \hat{a}^{\dagger}_{R,2}$ and $\hat{a}^{\dagger}_{R,0} \Rightarrow \hat{a}^{\dagger}_{L,-2}$, with $\vec{\pi}_L = 2^{-1/2}(\vec{\pi}_H + i\vec{\pi}_V)$ and $\vec{\pi}_R = \vec{\pi}_L^{\perp}$ standing for left- and right-circular polarizations, respectively.

The previous relations describe the coupling of the OAM l and the polarization π degrees of freedom taking place in the QP. Interestingly, this property can be exploited to generate single-particle entanglement of π and l degrees of freedom:

$$\left.\begin{matrix} |H\rangle_\pi|0\rangle_l \\ |V\rangle_\pi|0\rangle_l \end{matrix}\right\} \xrightarrow{\text{QP}} \frac{1}{\sqrt{2}}(|L\rangle_\pi|-2\rangle_l \pm |R\rangle_\pi|+2\rangle_l). \quad (1)$$

This is an entangled state between two qubits encoded in

different degrees of freedom. In particular, $\{|+2\rangle_l, |-2\rangle_l\}$ is the basis for the OAM qubit which lies in the $|l| = 2$ subspace of the infinite dimensional Hilbert space of orbital angular momentum. As a first experimental step, we set out to verify how accurately the real QP device performs these transformations in the single-photon regime (see Fig. 1 for experimental details). First, the QP conversion efficiency η from the input TEM_{00} to the $l = \pm 2$ modes has been estimated through the coupling efficiency with the single-mode fiber. We find $\eta \simeq 85\%$, ascribed to light scattering, radial mode residual mismatch, and imperfect tuning of the QP birefringent retardation δ [11,15] (the unconverted component remains $l = 0$ and is therefore filtered out). Next, in order to assess the coherence of the transformations in Eq. (1), single photons in the states $|H\rangle_\pi|0\rangle_l$ or $|V\rangle_\pi|0\rangle_l$ were used as input in the QP. We analyzed the output state through a double-fork hologram and a circular-polarization analysis setup along the two diffracted modes: the intensity of the π_R (π_L) polarization component in the mode corresponding to $l = +2$ ($l = -2$) was measured to be equal to 99.8% (99.6%) of the total, with a high agreement with theory. To demonstrate the realization of the pure states given in Eq. (1), a complete single-photon two-qubit quantum state tomography has been carried out, performing measurements both in π and l degrees of freedom. Besides the normal $\{|+2\rangle_l, |-2\rangle_l\}$ OAM basis, measurements were carried out in the two superposition bases $\{|d_+\rangle_l, |d_-\rangle_l\}$ and $\{|d_R\rangle_l, |d_L\rangle_l\}$, where $|d_\pm\rangle_l = \frac{1}{\sqrt{2}}(|+2\rangle_l \pm |-2\rangle_l)$ and $|d_{L,R}\rangle_l = \frac{1}{\sqrt{2}} \times (|+2\rangle_l \pm i|-2\rangle_l)$. The OAM degree of freedom was analyzed in these bases by means of different computer-generated holograms, reported in the inset of Fig. 1 [16]. The experimental results are in high agreement with theory, as shown in Figs. 2(a) and 2(b).

Because of its peculiarities, the q plate provides a convenient way to "interface" the photon OAM with the more easily manipulated spin degree of freedom. Hence, as the next step we show that such an interface can be considered as a quantum "transferrer" device, which allows one to

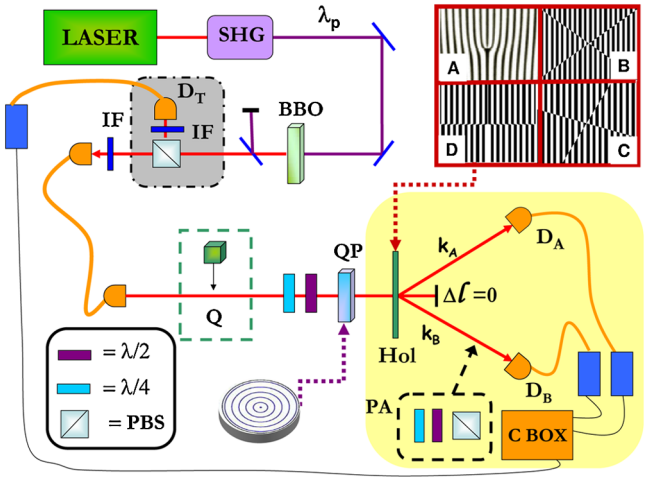

FIG. 1 (color online). A Ti:sapphire mode-locked laser converted by second harmonic generation (SHG) into a beam with wavelength $\lambda_p = 397.5$ nm. This field pumps a nonlinear crystal of β-barium borate (BBO) which emits a single-mode biphoton state with H and V polarizations and $\lambda = 795$ nm, filtered by the interference filter (IF) with $\Delta\lambda = 6$ nm and then coupled to a single-mode fiber [19]. The gray dot-dashed box has been optionally inserted to prepare a single-photon state triggered by detector D_T. Birefringent quartz crystals (Q) having different thicknesses were used to introduce a controlled temporal delay between the two photons. After setting the input polarization by means of a suitably oriented quarter wave plate, the photons were sent through the q plate (QP) and the output OAM states were analyzed with the help of a hologram (Hol) and a polarization analysis set (PA). In OAM-to-spin conversion experiments, Hol and QP were interchanged. To measure (or prepare) OAM states in the basis $l = \pm 2$, a double-fork hologram has been used [inset (A)], so that the OAM state of the first diffracted modes is shifted by $\Delta l = \pm 2$, while the undiffracted zero-order beam has $\Delta l = 0$. The photons on the first diffracted modes are then coupled to single-mode fibers which select output states with $l = 0$ and convey them to the detectors D_A and D_B. Hence, the detection of a photon in D_A (D_B) corresponds to a photon incident on the hologram with OAM $l = +2$ ($l = -2$). The first-order diffraction efficiency of the hologram was ~10%. The measurement (or preparation) of OAM in superposition states has been realized by adopting the other holograms shown in the inset. [The hologram (B) refers to $|d_+\rangle_l$, (C) to $|d_R\rangle_l$, (D) to $|d_-\rangle_l$. $|d_L\rangle_l$ was also analyzed by hologram (C) after reversing its orientation.]

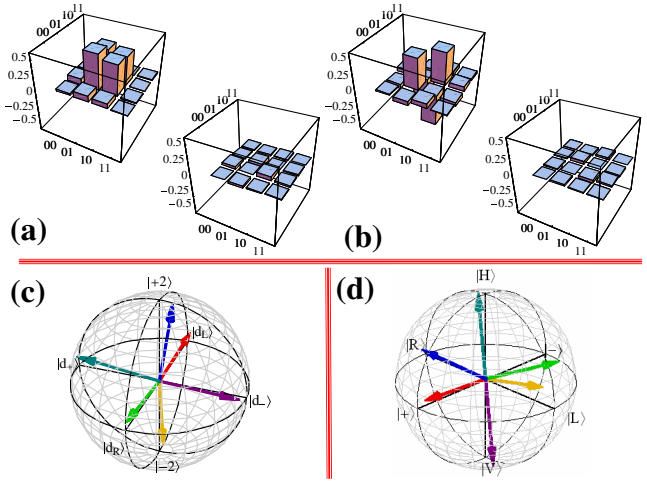

FIG. 2 (color online). (a),(b) Experimental density matrices for the single-photon entangled state. The computational values $\{0, 1\}$ are associated to the $\{|R\rangle, |L\rangle\}$ polarization states and to $\{|+2\rangle, |-2\rangle\}$ for the orbital angular momentum l for the first and the second qubit, respectively. The incoming state on the QP is (a) $|H\rangle_\pi|0\rangle_l$ and (b) $|V\rangle_\pi|0\rangle_l$. The average experimental concurrence is $C = (0.95 \pm 0.02)$. (c),(d) Experimental Poincaré sphere both for the OAM (c) and π (d) degrees of freedom obtained after the $\pi \rightarrow l$ and the $l \rightarrow \pi$ transferrer, respectively. Experimentally we carried out single-qubit tomography to determine the Stokes parameters for the π and the analogous parameters for the l degrees of freedom. The mean fidelity values are (c) $F = (98 \pm 1)\%$ and (d) $F = (97 \pm 1)\%$.

transfer coherently the quantum information from the polarization π to the OAM l degree of freedom, and vice versa. Such processes can be formally expressed as $|\varphi\rangle_\pi|0\rangle_l \leftrightarrow |H\rangle_\pi|\varphi\rangle_l$, where $|\varphi\rangle_\pi = (\alpha|H\rangle + \beta|V\rangle)_\pi$ and $|\varphi\rangle_l = (\alpha|+2\rangle + \beta|-2\rangle)_l$. Here we demonstrate a probabilistic conversion (with probability $p \cong 50\%$), since some output state contributions are discarded. By extending the present scheme, it is possible to achieve a complete deterministic information conversion [17].

(I) Transferrer $\pi \rightarrow l$.—Through a wave plate $\lambda/4$ and a QP, π and l become entangled [Fig. 3(Ia)]. Then, the information contained in the polarization is erased by inserting a polarizing beam splitter (PBS). This process has been experimentally verified for a set of maximally polarized states. Any π input state, represented by a vector in the Poincaré sphere, has been converted in another vector in the OAM "Poincaré sphere" [see Fig. 2(c)], determined by carrying out a quantum state tomography of the OAM state. The results demonstrate a high fidelity of the $\pi \rightarrow l$ transformation. An application of this transferrer is shown in Fig. 3(Ib). The initial information encoded in an input state $|\varphi\rangle_\pi|0\rangle_l$ is coherently transferred to the OAM. New information can then be stored in the polarization degree of freedom, thus allowing the encoding of two qubits of information in the same photon.

(II) Transferrer $l \rightarrow \pi$.—Let us show that the coupling between the spin and the orbital degree of freedom in the q plate is bidirectional [Fig. 3(IIa)]. The input state $|H\rangle_\pi|\varphi\rangle_l$ is sent through a QP and a $\lambda/4$ wave plate [Fig. 3(IIa)]. By inserting a single-mode fiber on the output, only the states with $l = 0$ are efficiently coupled, leading to a probabilistic process. After the fiber, the state $|\varphi\rangle_\pi|0\rangle_l$ has been obtained, which demonstrates the successful transfer

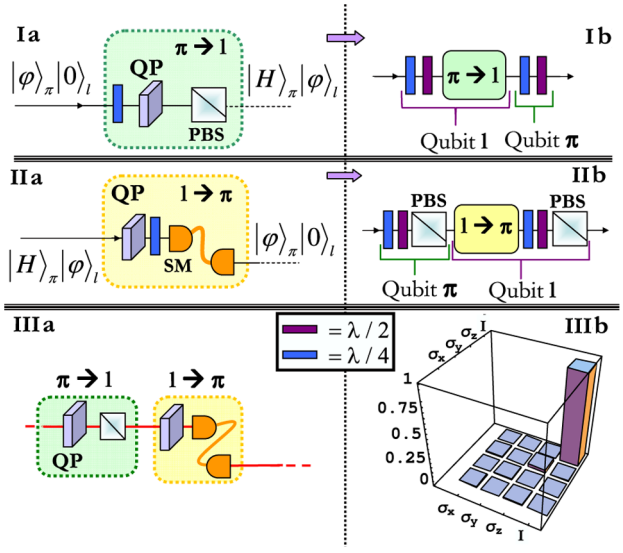

FIG. 3 (color online). (I),(II) Schematic representation of the two transferrer devices based on the q plate (QP). (III) (a) Double transfer $\pi \rightarrow l \rightarrow \pi$ and (b) experimental real part of the matrix χ represented in the Pauli operator basis, the imaginary elements are ≈ 0.

from the OAM degree of freedom to the polarization one. Analogously to the previous analysis, the experimental Poincaré sphere vectors obtained after the $l \rightarrow \pi$ conversion are reported in Fig. 2(d). Such transferrer can be, for example, exploited within a measurement apparatus for analyzing both π and l degrees of freedom without using holograms, with an advantage in terms of efficiency [Fig. 3(IIb)] and flexibility, since there is no need to change the hologram for each state to be analyzed. Finally, we proved the forward-backward double transfer $\pi \rightarrow l \rightarrow \pi$, by implementing both the previous transferrers together, as shown in Fig. 3(IIIa). We carried out the quantum process tomography [18] proving that the qubit state quantum information is coherently preserved in the whole process involving two separate QPs. Figure 3(IIIb) reports the reconstructed χ matrix representation of the overall process which exhibits a fidelity, i.e., overlap, with the identity map equal to $F = 0.950 \pm 0.015$. This bidirectional spin-OAM interface allows the extension to OAM of many protocols currently only possible with polarization. For example, the realization of a two-photon C-NOT gate for OAM states can be obtained by exploiting a C-NOT for π states. As a further consideration, the QP allows implementing a fast OAM switcher device, by modulating the polarization degree of freedom through a Pockels cell and then transferring such modulation to the OAM.

It is natural to ask if the QP is able to preserve the photon-photon correlations too. This would also be a crucial test of the QP potential in the quantum optics field, as multiphoton correlations are extremely sensitive to optical quality issues. Consider first the case of two independent linearly polarized photons, one horizontal and the other vertical, going through the QP: each will undergo the QP transformation given in Eq. (1), and the two outgoing photons will end up having opposite OAM values 50% of the times. If, however, the two photons are undistinguishable, except for their polarization, the field state at the QP input side is $|\Psi_{\text{in}}\rangle = |1\rangle_{H,0}|1\rangle_{V,0}$, where we are now using a photon number ket notation for our multiphoton quantum states. This input state can be rewritten in the circular-polarization basis as $|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}(|2\rangle_{R,0}|0\rangle_{L,0} - |0\rangle_{R,0}|2\rangle_{L,0})$. After the QP, this state evolves into

$$|\Psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}}(|2\rangle_{L,-2}|0\rangle_{R,2} - |0\rangle_{L,-2}|2\rangle_{R,2}), \qquad (2)$$

in which only photons carrying parallel OAM, either $+2$ or -2, are found. Hence, the QP action can again be interpreted as a mode converter, coherently transferring the two-photon quantum correlation from the spin degree of freedom to the OAM one. The QP is then able to generate a multiphoton state having nonclassical correlations in OAM within a single longitudinal mode. The experimental setup is shown in Fig. 1 (with the gray box removed). The coincidence between $[D_A, D_B]$ detects the state contribution of $|1\rangle_{l=2}|1\rangle_{l=-2}$. Because of the coalescence interference, for otherwise identical photons this component is

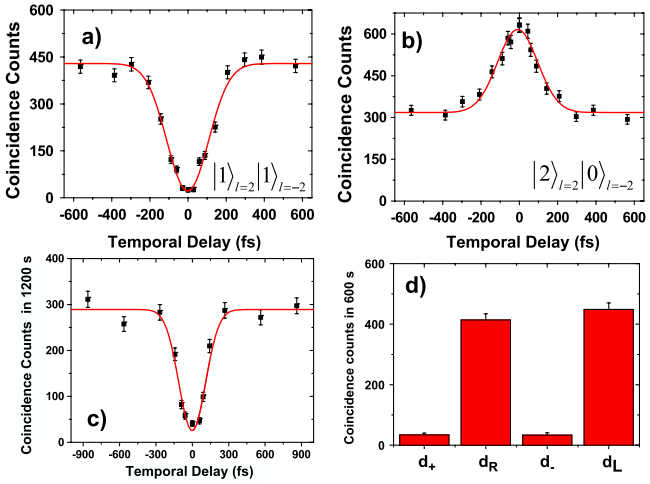

FIG. 4 (color online). (a) Coincidence counts between $[D_A, D_B]$ versus the temporal delay t_d, for the state $|\Psi_{\text{out}}\rangle$. The continuous line shows the best fit of the experimental data. (b) Coincidence counts $[D_A, D_{A'}]$ versus t_d. (c) Coincidence counts $[D_A, D_B]$ versus t_d $[V = (0.91 \pm 0.01)]$ for the state $|\Phi_{\text{out}}\rangle$. (d) Coincidence counts $[D_A, D_{A'}]$ in different OAM bases.

vanishing in the state outcoming from the QP. In order to study the transition from the case of classical behavior (independent photons) to the case of full quantum interference, a variable temporal delay t_d between the H and V polarizations in the state $|\Psi_{\text{in}}\rangle$ has been introduced (Q in Fig. 1). The experimental visibility of the quantum interference shown in Fig. 4(a) is $V_{\text{expt}} = (0.95 \pm 0.02)$. As a further confirmation, we have measured the contribution of two photons with $l = +2$ by recording the coincidence counts between two detectors $[D_A, D'_A]$ placed on the output modes of a fiber-based beam splitter inserted on the same k_A diffracted mode (not shown in the figure). Theoretically, the coalescence of the two photons should lead to a coincidence enhancement by a factor $\Gamma = 2$, and experimentally we found $\Gamma_{\text{expt}} = (1.94 \pm 0.02)$. For the sake of completeness, we verified that even after erasing all information still contained in the polarization degree of freedom, the final state is still coherent and exhibits the same nonclassical photon correlations in OAM. In order to do so, we let both of the two photons pass through a horizontal linear polarizer set in a common state H. We verified again the coalescence of the photons by a measurement similar to the previous one; Fig. 4(c). To verify that we really obtain a coherent state and not a statistical mixture having similar OAM correlation properties, we measured the coherence between the two contributions with opposite OAM states. This was accomplished by analyzing the photons in the other OAM bases already discussed above. Therefore, our coherence verification is actually turned into a measurement of two photons in the same OAM state ($|d_+\rangle$, $|d_-\rangle$, $|d_R\rangle$, $|d_L\rangle$). As expected, for ($|d_+\rangle$, $|d_-\rangle$) the events of two photons with the same orbital states are strongly suppressed, while for ($|d_R\rangle$, $|d_L\rangle$) they are doubled, with an overall correlation visibility $V = (0.86 \pm 0.02)$ [Fig. 4(d)]. This shows that QPs transfer not only single-photon information between polarization and OAM, but also multiphoton-encoded information (e.g., biphotons).

In conclusion, we have shown that the q-plate device can be used as a coherent and bidirectional quantum interface allowing the transfer of quantum information between the polarization and the orbital angular momentum degrees of freedom of the photon, both in the case of single-photon states and of two-photon correlated states. The results reported here show that this can be a useful tool for exploiting the OAM degree of freedom of photons, in combination with polarization, as a new resource to implement high-dimensional quantum-information protocols.

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*fabio.sciarrino@uniroma1.it
†lorenzo.marrucci@na.infn.it

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