

Quantum Liquid Crystals in an Imbalanced Fermi Gas: Fluctuations and Fractional Vortices in Larkin-Ovchinnikov States

Leo Radzihovsky and Ashvin Vishwanath

Department of Physics, University of Colorado, Boulder, Colorado 80309, USA and Department of Physics, University of California, Berkeley, California 94703, USA

(Received 20 December 2008; published 2 July 2009)

We develop a low-energy model of an unidirectional Larkin-Ovchinnikov (LO) state. Because the underlying rotational and translational symmetries are broken spontaneously, this gapless superfluid is a smectic liquid crystal, that exhibits fluctuations that are qualitatively stronger than in a conventional superfluid, thus requiring a fully nonlinear description of its Goldstone modes. Consequently, at nonzero temperature the LO superfluid is an algebraic phase even in 3D. It exhibits half-integer vortex-dislocation defects, whose unbinding leads to transitions to a superfluid nematic and other phases. In 2D at nonzero temperature, the LO state is always unstable to a nematic superfluid. We expect this superfluid liquid-crystal phenomenology to be realizable in imbalanced resonant Fermi gases trapped isotropically.

DOI: 10.1103/PhysRevLett.103.010404

PACS numbers: 03.75.Hh, 03.75.Ss, 67.85.-d, 74.20.-z

The tunability of interactions through Feshbach resonances has led to a realization of an s -wave paired superfluidity and BCS to Bose-Einstein condensation (BEC) crossover [1,2], as well as promises of more exotic states such as gapless p -wave [3] and periodic Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) [4,5] superfluidity in strongly correlated degenerate alkali gases. The latter enigmatic state has been thoroughly explored within BCS mean-field studies [6–8] and is expected in a population-imbalanced (polarized) Feshbach resonant Fermi gas [9,10]. While experiments [11,12] have confirmed much of the predicted phenomenology of phase separation [10,13,14], the FFLO states have so far eluded definitive observation.

The simplest mean-field treatments [4,5,10] find that the FFLO type states are quite fragile, confined to a narrow range of polarization on the BCS side. However, motivated by earlier studies [6,7] and based on the finding of a negative domain-wall energy in an otherwise uniform singlet BCS superfluid [8,15], a more general periodic superfluid state may be significantly more stable. Much like a type-II superconductor undergoes a transition into a vortex state at a lower-critical field H_{c1} , here too, a Zeeman-field driven domain-wall nucleation (with the density increasing above the lower-critical h_{c1} field) allows a continuous mechanism for a transition from a singlet paired superfluid to a LO-like periodic state [6–8,15].

In this scenario the SF-LO transition is of a commensurate-incommensurate type as can be explicitly shown in one dimension (1D) [6,16]. The imposed species imbalance can be continuously accommodated by the sub-gap states localized on the self-consistently induced domain walls. Such LO state can also be thought of as a periodically ordered *micro*-phase separation between the normal and BCS states, that thus naturally replaces the *macro*-phase separation ubiquitously found in the BEC-BCS detuning-polarization phase diagram [10] (Fig. 1).

With this motivation in mind, here we report on our study that is complementary to these microscopic mean-field investigations. Namely, assuming that the LO state is indeed energetically favorable over a region of a phase diagram, we explore its stability to low-energy fluctuations and the resulting phenomenology.

We demonstrate that the low-energy model of the LO state is that of two coupled smectics, whose moduli we derive from the BCS theory. Thus a resonant imbalanced Fermi gas is a natural realization of a quantum (superfluid) liquid crystal, that unlike the solid state analogs [17–20] is

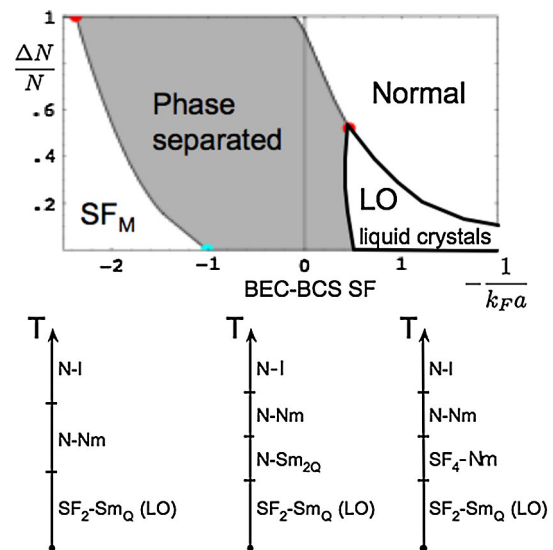


FIG. 1 (color online). Polarization $\Delta N/N$ vs $1/(k_F a)$ schematic phase diagram, showing LO liquid-crystal phases replacing phase-separated (PS) regime [10]. 3D transition scenarios as a function of temperature to the normal-nematic (N-Nm), normal-isotropic (N-I), normal-smectic (N-Sm_{2Q}), and charge-4 superfluid-nematic (SF₄-Nm) phases are illustrated in the lower panel.

not plagued by the underlying lattice potential that explicitly breaks continuous spatial symmetries.

We find that while it is stable to quantum fluctuations, in 3D a long-ranged LO order is marginally unstable at any nonzero T . The resulting superfluid state is an *algebraic* phase with universal *quasi*-Bragg peaks and correlations that admit an exact description [21]. In contrast, crystalline LO phases with multiple noncollinear ordering wave vectors are stable against thermal fluctuations.

As with earlier mean-field treatments [19,22,23], we also find an unusual topological excitation—a half vortex bound to a half dislocation—in addition to integer vortices and dislocations, in this algebraic LO phase. Because our conclusions are based on general symmetry principles, supported by detailed calculations, they are generic and robust to variations in microscopic details.

In 2D and nonzero T , the state exhibits universal power-law phonon correlations, controlled by an exactly calculable [24] fixed point. It displays short-range positional order with Lorentzian structure function peaks, and is thus unstable to proliferation of dislocations. The state is either a “charge”-4 (four-fermion) superfluid or a non-superfluid nematic, depending on the relative energetics of integer and half-integer vortex-dislocation defects. The latter normal nematic state is a (complimentarily described [20]) deformed Fermi surface state [25,26].

Model.—We begin with a Ginzburg-Landau theory that captures the system’s tendency to order into a finite wave vector Q_0 paired state, with a spontaneously chosen direction. The free-energy density

$$\mathcal{H} = J[|\nabla^2 \Delta|^2 - 2Q_0^2 |\nabla \Delta|^2] + r|\Delta|^2 + \frac{v_1}{2} |\Delta|^4 + \frac{v_2}{2} \mathbf{j}^2 + \dots \quad (1)$$

can be derived from a microscopic BCS model [5,10,27] near the upper-critical chemical potential difference, h_{c2} , with

$$J \approx \frac{0.61n}{\epsilon_F Q_0^4}, \quad Q_0 \approx \frac{1.81 \Delta_{\text{BCS}}}{\hbar v_F}, \quad r \approx \frac{3n}{4\epsilon_F} \ln \left[\frac{9h}{4h_{c2}} \right], \quad (2)$$

$$h_{c2} \approx \frac{3}{4} \Delta_{\text{BCS}}, \quad v_1 \approx \frac{3n}{4\epsilon_F \Delta_{\text{BCS}}^2}, \quad v_2 \approx \frac{1.83 \text{ nm}^2}{\epsilon_F \Delta_{\text{BCS}}^2 Q_0^2},$$

that can be more generally taken as phenomenological parameters to be determined experimentally. n , ϵ_F , v_F , Δ_{BCS} , and m are the atomic density, Fermi energy and velocity, BCS ($h = 0$) gap and atomic mass, respectively. Near the lower-critical Zeeman field, h_{c1} , $Q_0(h)$ is expected to vanish with the species imbalance, as the system continuously transitions into a uniform singlet superfluid [6–8], with this and other moduli’s dependences derivable via fluctuating domain-walls methods [27]. Above, \mathbf{j} is the supercurrent and the last term is crucial for getting a non-zero transverse (to \mathbf{Q}) superfluid stiffness in the LO state. From the first two terms it is clear that the dominant instability is at a wave vector Q_0 . Thus, for $h < h_{c2}$, $r <$

$JQ_0^4 = 0.61n/\epsilon_F$ and the system develops a pairing order parameter $\Delta(\mathbf{x}) = \sum_{\mathbf{Q}_n} \Delta_{\mathbf{Q}_n} e^{i\mathbf{Q}_n \cdot \mathbf{x}}$.

As with other crystallization problems, the choice of the set of \mathbf{Q}_n ’s is determined by the details of interactions and will not be addressed here. Motivated by LO findings [5], we focus on the unidirectional order characterized by a collinear set of \mathbf{Q}_n ’s. These fall into two, LO and FF universality classes. The LO (FF) states are characterized by breaking (preserving) translational and preserving (breaking) time-reversal symmetries. Low-energy properties of such states can be well captured with a single $\pm \mathbf{Q}$ pair (LO) and a single \mathbf{Q} wave vector (FF) approximations.

We focus on the more stable periodic LO state [5–8,15], only commenting on the homogeneous FF state. Within the LO approximation the pairing function is given by $\Delta_{\text{LO}}(\mathbf{x}) = \Delta_+(\mathbf{x})e^{i\mathbf{Q} \cdot \mathbf{x}} + \Delta_-(\mathbf{x})e^{-i\mathbf{Q} \cdot \mathbf{x}}$, where $\Delta_{\pm} = \Delta_Q e^{i\theta_{\pm}(\mathbf{x})}$ are the leading complex order parameters, whose amplitudes deep in the ordered LO state can be taken to be equal and constant, $\Delta_Q^2 \approx c\Delta_{\text{BCS}}^2 \ln(h_{c2}/h)$, thereby focusing on the two Goldstone modes $\theta_{\pm}(\mathbf{x})$. A rearranged form of the LO order parameter clarifies its physical interpretation

$$\Delta_{\text{LO}}(\mathbf{x}) = 2\Delta_Q e^{i\theta_{\text{sc}}(\mathbf{x})} \cos[\mathbf{Q} \cdot \mathbf{x} + \theta_{\text{sm}}(\mathbf{x})], \quad (3)$$

showing that it is a product of a superfluid order parameter and a unidirectional, spontaneously oriented (along \mathbf{Q}) Cooper-pair density wave, i.e., simultaneously exhibiting the superfluid and smectic orders. The low-energy properties are characterized by two Goldstone modes, the superconducting phase $\theta_{\text{sc}} \equiv \frac{1}{2}(\theta_+ + \theta_-)$ and the phonon $u = \theta_{\text{sm}}/Q \equiv \frac{1}{2}(\theta_+ - \theta_-)/Q$. The uniform FF state is characterized by a single Δ_Q amplitude and a Goldstone mode θ_Q .

Substituting $\Delta_{\text{LO}}(x)$ into \mathcal{H} we obtain a Hamiltonian density for the bosonic Goldstone modes of a generic LO state:

$$\mathcal{H}_{\text{LO}} = \sum_{\alpha=\pm} \left[\frac{K}{4} (\nabla^2 u_{\alpha})^2 + \frac{B}{4} \left(\partial u_{\alpha} + \frac{1}{2} (\nabla u_{\alpha})^2 \right)^2 \right] + \frac{\gamma}{2} (\nabla u_+ - \nabla u_-)^2, \quad (4)$$

$$\approx \frac{K}{2} (\nabla^2 u)^2 + \frac{B}{2} \left(\partial u + \frac{1}{2} (\nabla u)^2 \right)^2 + \frac{\rho_s^i}{2} (\nabla_i \theta_{\text{sc}})^2,$$

where we dropped constant and fast oscillating parts, chose $\mathbf{Q} = Q_0 \hat{z}$, and defined phonon fields $u_{\pm} = \pm \theta_{\pm}/Q_0$ and the bend ($K = 4JQ_0^2 \Delta_Q^2 \approx 2.4n\Delta_Q^2/(\epsilon_F Q_0^2)$) and compressional ($B = 16JQ_0^4 \Delta_Q^2 \approx 9.8n\Delta_Q^2/\epsilon_F$) elastic moduli.

This form (valid beyond above weak-coupling microscopic derivation) is familiar from studies of conventional smectic liquid crystals [28], with rotational invariance encoded in two ways. First, for a vanishing γ the gradient elasticity in u_{\pm} (and u) only appears *along* \mathbf{Q} , namely $\partial \equiv \hat{\mathbf{Q}} \cdot \nabla$ (compression), with elasticity transverse to \mathbf{Q} of a “softer” Laplacian (curvature) type. Second, the elastic

energy is an expansion in a strain tensor $u_{Q0}^{\pm} = \partial u_{\pm} + \frac{1}{2}(\nabla u_{\pm})^2$, whose nonlinearities in u_{\pm} ensure that it is fully rotationally invariant even for large reorientations $Q_0 \hat{z} \rightarrow \mathbf{Q}$ of the LO ground state.

A nonzero $\gamma \equiv v_2 Q_0^2 \Delta_Q^4 / m^2 \approx 1.8n \Delta_Q^4 / (\epsilon_F \Delta_{\text{BCS}}^2)$ coupling (minimized by a vanishing supercurrent $\nabla \theta_+ + \nabla \theta_-$) removes the two independent rotational symmetries, orientationally locking the two smectics. This leads to the superconducting phase $\theta_{\text{sc}} = \frac{1}{2}(\theta_+ + \theta_-)$ to be of a conventional XY type. It is characterized by parallel ($\rho_s^{i=\parallel} \approx B/Q_0^2$) and transverse ($\rho_s^{i=\perp} \equiv 4\gamma/Q_0^2$) superfluid stiffnesses, appearing in the second form of \mathcal{H}_{LO} .

Physically, ρ_s^{\parallel} and ρ_s^{\perp} are superfluid stiffnesses for the supercurrent $\mathbf{j} = (\mathbf{j}_+ + \mathbf{j}_-)/2$ produced by the imbalance in the left (\mathbf{j}_+) and right (\mathbf{j}_-) supercurrent magnitudes and directions, respectively. We thus find that the LO state is a highly anisotropic superfluid, with

$$\rho_s^{\perp} / \rho_s^{\parallel} = \frac{3}{4} (\Delta_Q / \Delta_{\text{BCS}})^2 \approx \ln(h_{c2}/h) \ll 1, \quad (5)$$

a ratio that vanishes for $h \rightarrow h_{c2}^-$. The FF state is even more exotic, with an identically vanishing transverse superfluid stiffness, a reflection of the rotational invariance under spontaneous current reorientation.

Fluctuations.—The thermodynamics can be obtained through a coherent path integral. Although there are nontrivial issues of the interplay between the fermionic quasiparticles and the Goldstone modes, we can show that at $T = 0$ the superfluid and smectic orders (and thus the LO state) are stable to quantum fluctuations in $d > 1$ [27].

For $T > 0$, θ_{sc} (θ_{sm}) fluctuations diverge and superfluid (smectic) order is destroyed for $d \leq 2$ ($d \leq 3$). The LO state is unstable to thermal fluctuations, displaying quasi-Bragg (Lorentzian) peaks in 3D (2D) in its structure function. Thus in both cases the LO order parameter, (3) vanishes and the state is qualitatively distinct from its mean-field form, at low T characterized by a charge-4 superfluid order parameter $\Delta_{\text{sc}}^{(4)} \sim \Delta^2 \approx \frac{1}{2} \Delta_Q^2 e^{i2\theta_{\text{sc}}}$.

In the presence of these divergent thermal fluctuations phonon nonlinearities in \mathcal{H}_{LO} qualitatively modify correlations on scales longer than $\xi_{\text{NL}} \sim [K^{3/2}/(B^{1/2}T)]^{1/(3-d)} \sim k_F^{-1} [\Delta_Q^2 \epsilon_F / (\Delta_{\text{BCS}}^3 T)]^{1/(3-d)}$ (on shorter scales the harmonic description above applies), giving universal power laws, e.g., $\langle u(z, x)u(0, 0) \rangle^{1/2} \sim \text{Max}[x^{\alpha}, z^{\beta}]$, controlled by a nontrivial low T (order $3-d$) fixed point [21], that has an exact description in 2D with $\alpha = 1/2$, $\beta = 1/3$ [24]. In 3D, $\xi_{\text{NL}} \sim e^{cK^{3/2}/(B^{1/2}T)}$ and phonon correlations grow as a universal power of a logarithm, a result that is asymptotically exact. These elastic results only hold as long as dislocations remain bound or on scales shorter than the dislocation unbinding scale.

Defects.—We now discuss topological defects and phases accessible by their unbinding. With two compact Goldstone modes θ_{sc} , u (equivalently, $\theta_{\pm} = \pm 2\pi u_{\pm}/a$), defects are labeled by vortex and dislocation charges

($2\pi n_v, an_d$). Ordinary vortex, $(2\pi, 0)$ and dislocation $(0, a)$ are clearly allowed, and in terms of the two smectic displacements these respectively correspond to the opposite and same signs of integer dislocations in u_{\pm} . When proliferated they destroy the superfluid phase coherence and smectic periodicity, and either one is sufficient to suppress the conventional LO order, Δ_{LO} , (3).

However, because a sign change in Δ_{LO} due to a $a/2$ dislocation in u can be compensated by a π vortex in θ_{sc} $1/2$ -charge defects in θ_{sc} and u are also allowed, but are confined into $(\pm\pi, \pm a/2)$ pairs [19,22,23]. In terms of the smectic fields, u_+ , u_- these correspond to an integer dislocation in one and no dislocation in the other.

Transitions.—There are many paths of continuous transitions out of the LO (SF₂-Sm_Q) state. One is through an unbinding of integer $(0, a)$ dislocations in u . This melts the smectic order in favor of a nematic, but retains a superfluid order, thereby transforming the LO state to a nematic charge-4 superfluid (SF₄-Nm). Another path, is by unbinding integer $(2\pi, 0)$ vortices in θ_{sc} . This destroys the superfluid order and converts the smectic order Q to $2Q$ (N-Sm_{2Q}). A third route out of the LO superfluid is through a proliferation of $(\pi, \pm a/2)$ fractional vortex-dislocation pairs, that destroy both smectic and superfluid orders, inducing a transition to a nonsuperfluid nematic (N-Nm). For 3D these possibilities, determined by the relative energetics of these defects are illustrated in Fig. 1. In 2D, the dislocation energy is finite and the LO state is necessarily destabilized by thermal fluctuations to a charge-4 superfluid nematic, SF₄-Nm. Upon rotation the resulting nematic superfluid will display π vortices ($\oint \mathbf{v} \cdot d\mathbf{l} = h/4m$), that (due to nematic order) will form a uniaxially distorted lattice. This rich fluctuations-driven phase behavior contrasts sharply with a direct LO-N transition (described by $U(1) \times U(1)$ Landau theory $H_{\text{mft}} = r(|\Delta_+|^2 + |\Delta_-|^2) + \lambda_1(|\Delta_+|^4 + |\Delta_-|^4) + \lambda_2|\Delta_+|^2|\Delta_-|^2$) found in mean-field theory.

Fermions.—We now turn to a discussion of the fermionic sector that we have so far ignored. Near h_{c2} a single harmonic (Q for FF and $\pm Q$ for LO states) approximation is sufficient. Unlike the simpler FF case (that can be diagonalized exactly with a two-component Nambu spinor [10]), the LO state involves a three-component spinor $\hat{\Psi}_{\mathbf{k}} \equiv (\hat{c}_{-\mathbf{k}+\mathbf{Q}\uparrow}, \hat{c}_{\mathbf{k}\uparrow}^{\dagger}, \hat{c}_{-\mathbf{k}-\mathbf{Q}\uparrow})$. Neglecting sparse off-resonant coupling between \mathbf{k} and $\mathbf{k} + 2\mathbf{Q}$ Cooper pairs and noting that only two of the three components in $\hat{\Psi}_{\mathbf{k}}$ are resonant at any one \mathbf{k} , the approximate spectrum is given by $E_{\mathbf{k},\sigma,\pm\mathbf{Q}} = (\epsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{Q}}^2)^{1/2} - \sigma(h \pm \frac{\mathbf{k} \cdot \mathbf{Q}}{2m})$, with $\sigma = \pm 1$, $\epsilon_{\mathbf{k}} = \frac{k^2}{2m} - \mu + \frac{Q^2}{8m}$, $\mu = \frac{1}{2}(\mu_{\uparrow} + \mu_{\downarrow})$, and $h = \frac{1}{2}(\mu_{\uparrow} - \mu_{\downarrow})$. The regions of \mathbf{k} where $E_{\mathbf{k},\sigma,\pm\mathbf{Q}}$ is negative corresponds to a Fermi sea (rather than the usual vacuum) of Bogoliubov quasiparticles in the BCS “vacuum”, and therefore leads to a Fermi surface of gapless fermionic excitations. These are nothing but the unpaired fraction of the majority atoms. From the spectrum above it is clear

that two distinct LO states are possible. One, LO1 exhibits a single (majority) species, fully polarized Fermi surface pockets. The other, LO2 is characterized by both majority and minority fermion flavor Fermi surface pockets. The Fermi surface volume difference is proportional to the species imbalance and the anisotropy encodes the long-range orientational order of the LO state. Because the polarization is conserved and Goldstone mode fluctuations are finite at zero temperature, we expect the LO1-LO2 quantum transition to be of a simple band filling type.

In the complementary regime near h_{c1} , the excess majority atoms occupy additional states localized on domain walls [6–8,15]. Because the atoms can freely move along and tunnel between adjacent domain walls, near h_{c1} they exhibit a “metallic”, but highly anisotropic dispersion. The resulting fermionic spectrum is qualitatively consistent with that near h_{c2} , $E_{\mathbf{k},\sigma,\pm\mathbf{Q}}$.

The interactions between the Goldstone modes and unpaired majority fermionic atoms ψ must now be included and are given by

$$\begin{aligned} \mathcal{H}_{j,s,j} &\sim i\nabla\theta \cdot \psi^\dagger \nabla\psi + \text{H.c.}, & \mathcal{H}_{j,s,n} &\sim (\nabla\theta)^2 \psi^\dagger \psi, \\ \mathcal{H}_{a-p} &\sim \left[\partial_z u + \frac{1}{2}(\nabla u)^2 \right] \psi^\dagger \psi + i\nabla u \cdot \psi^\dagger \nabla\psi + \text{H.c.} \end{aligned} \quad (6)$$

As with other analogous problems [25], we expect these to lead to Landau-like damping of the Goldstone modes θ_{sc} , u , and a finite fermionic quasiparticle lifetime. We leave the study of these and other affects on the properties of the LO states to the future.

Trap effects.—Since near h_{c2} the LO period $\lambda_Q = 2\pi/Q_0$ (2) is bounded by the coherence length (that near unitarity can be as short as $\sim R/N^{1/3}$, where R is the trapped condensate radius and N is the total number of atoms), and thus $\ll R$, in this regime the trap can be treated via a local density approximation (LDA). For $\lambda_Q \ll R$, LDA predicts weak pinning of the LO smectic, that can be estimated via finite size scaling, with trap size R cutting off $\langle u^2 \rangle \sim \eta \log(R/\lambda_Q)$, leading to $\langle \Delta_{LO} \rangle \sim (\lambda_Q/R)^\eta \ll 1$ that no longer truly vanishes, but is still strongly suppressed. We thus expect the predicted strong fluctuations effects to be experimentally accessible. We note, for example, that Kosterlitz-Thouless phase fluctuation physics has been reported in 2D trapped superfluids [29], despite the finite trap size. However, a more detailed analysis of the trap effects is necessary, particularly near h_{c1} , for a quantitative comparison with experiments.

To summarize, we studied fluctuation phenomena in a LO state, expected to be realizable in imbalanced resonant Fermi gases. The LO state is a superfluid smectic liquid crystal, whose elastic moduli and superfluid stiffness we derived near h_{c2} . It is extremely sensitive to thermal fluctuations that destroy its long-range positional order even in 3D, replacing it by an algebraic phase, that exhibits vortex fractionalization, where the basic superfluid vortex is half the strength of a vortex in a regular paired condensate. This

should be observable via a doubling of a vortex density in a rotated state. Also under rotation, the high superfluid anisotropy (5) leads to an imbalance-tunable strongly anisotropic vortex core and a lattice highly stretched along \mathbf{Q} . Bragg peaks in the time-of-flight images can distinguish the periodic SF₂-Sm_Q (superfluid smectic) state from the homogeneous SF₄-Nm (superfluid nematic), which are in turn distinguished from the N-Sm_{2Q} and N-Nm (normal smectic and nematic) by their superfluid properties, periodicity, collective modes, quantized vortices, and condensate peaks. Thermodynamic signatures will identify corresponding phase transitions.

We acknowledge support from the NSF DMR-0321848 (L.R.), DMR-0645691 (A.V.), the Miller Foundation and CU Faculty Fellowships (L.R.), and thank S. Kivelson for helpful comments on the manuscript. L.R. thanks Berkeley Physics Department for its hospitality.

-
- [1] C. A. Regal *et al.*, Phys. Rev. Lett. **92**, 040403 (2004).
 - [2] M. W. Zwierlein *et al.*, Phys. Rev. Lett. **92**, 120403 (2004).
 - [3] See for example V. Gurarie and L. Radzihovsky, Ann. Phys. (N.Y.) **322**, 2 (2007), and references therein.
 - [4] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).
 - [5] A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965).
 - [6] K. Machida and H. Nakanishi, Phys. Rev. B **30**, 122 (1984).
 - [7] H. Burkhardt and D. Rainer, Ann. Phys. (Leipzig) **3**, 181 (1994).
 - [8] S. Matsuo *et al.* J. Phys. Soc. Jpn. **67**, 280 (1998).
 - [9] T. Mizushima *et al.*, Phys. Rev. Lett. **94**, 060404 (2005).
 - [10] D. Sheehy and L. Radzihovsky, Phys. Rev. Lett. **96**, 060401 (2006); Ann. Phys. (N.Y.) **322**, 1790 (2007).
 - [11] M. W. Zwierlein *et al.*, Science **311**, 492 (2006).
 - [12] G. B. Partridge *et al.*, Science **311**, 503 (2006).
 - [13] R. Combescot, Europhys. Lett. **55**, 150 (2001).
 - [14] P. F. Bedaque *et al.*, Phys. Rev. Lett. **91**, 247002 (2003).
 - [15] N. Yoshida and S.-K. Yip, Phys. Rev. A **75**, 063601 (2007).
 - [16] K. Yang, Phys. Rev. B **63**, 140511 (2001).
 - [17] M. M. Fogler *et al.*, Phys. Rev. B **54**, 1853 (1996).
 - [18] S. A. Kivelson *et al.*, Nature (London) **393**, 550 (1998).
 - [19] E. Berg *et al.*, Phys. Rev. B **79**, 064515 (2009).
 - [20] L. Radzihovsky and A. T. Dorsey, Phys. Rev. Lett. **88**, 216802 (2002).
 - [21] G. Grinstein and R. Pelcovits, Phys. Rev. Lett. **47**, 856 (1981).
 - [22] E. Berg *et al.*, Phys. Rev. Lett. **99**, 127003 (2007).
 - [23] D. F. Agterberg and H. Tsunetsugu, Nature Phys. **4**, 639 (2008).
 - [24] L. Golubovic and Z. Wang, Phys. Rev. Lett. **69**, 2535 (1992).
 - [25] V. Oganesyan *et al.*, Phys. Rev. B **64**, 195109 (2001).
 - [26] A. Sedrakian *et al.*, Phys. Rev. A **72**, 013613 (2005).
 - [27] L. Radzihovsky *et al.* (unpublished).
 - [28] P. deGennes and J. Prost, *The Physics of Liquid Crystals* (Clarendon Press, Oxford, 1993).
 - [29] Z. Hadzibabic *et al.*, Nature (London) **441**, 1118 (2006).