# Suppression of $n = 1$ Tilt Instability by Magnetic Shaping Coils in Rotamak Plasmas

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Measurements from the array of Mirnov magnetic coils provide the first evidence for $n = 1$ tilt and radial shift instabilities in a 40 ms field-reversed configuration (FRC) driven by rotating magnetic field. External plasma-shaping magnetic coils are utilized to suppress the $n = 1$ instability modes. It is demonstrated that by energizing the middle shaping coil with 250–500 A current, the tilt mode is completely suppressed when a doublet FRC with an internal figure-of-eight separatrix is formed.

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Rotamak is a device that can form and sustain a field-reversed configuration (FRC) confined plasma with no or negligible toroidal magnetic field [1]. Such configuration is considered as a promising alternative for a fusion reactor because of its compact and simple design, natural divertor, and high plasma $\beta$ (ratio of plasma pressure to magnetic field pressure). In a rotamak, the toroidal plasma current is driven in a steady-state, noninductive fashion by means of an externally applied rotating magnetic field (RMF) [2,3]. With this method, a sufficiently large plasma current can be generated to reverse the external equilibrium magnetic field on the symmetry axis so that a FRC is formed [3–10].

The principal concern about an FRC plasma is its stability to low-$n$ ($n$ is the toroidal number) magnetohydrodynamic (MHD) modes. Among them, the $n = 1$ tilt or shift modes are predicted to be most dangerous for global stability. The tilt instability originates from the trend of the plasma current ring to flip in order to align its magnetic moment with the external magnetic field. The radial shift instability occurs when the magnitude of external magnetic field decreases with radius, causing the Lorentz forces on the opposite sides of the plasma current ring to become increasingly unbalanced if the current ring is shifted in the radial direction. Theoretical models predict the tilt mode to be unstable for elongations of plasma $E > 0.5$, where $E = Z_S/(R_S - R_i)$ is the ratio of the separatrix half-length to its radial extension, and the radial shift mode to be unstable for $E < 0.7$ [11]. Thus there is no range for $E$ where the plasma is expected to be free from $n = 1$ instabilities.

Although the instabilities are predicted to destroy plasma at a time scale of $Z_S/V_A$, where $V_A$ is the Alfvén speed, most of experiments to date demonstrate that FRCs remain grossly stable, with a lifetimes factor of hundreds longer than the Alfvén transit time. The stabilization of $n = 1$ modes is partially attributed to nearby passive conducting structures, to high viscosity of present FRC plasmas, and finite Larmor radius effects [11–13]. Experiments have shown that the amplitude of tilt mode is reduced by a factor of 10 when the value of $S_*/E$ is smaller than 3, for a range of $S_* \sim 8$–40 and $E \sim 3$–8 [14]. Kinetic parameter $S_* = R_S/(c/\omega_{\rm pi})$, which is the ratio of the separatrix radius to the ion skin depth, also counts the number of thermal ion gyroradii in case of $T_i \gg T_e$, when $c/\omega_{\rm pi}$ approximately equals the ion gyroradius in the external field.

But if this empirical stability criterion is extrapolated to fusion reactor plasmas, where the values of $S_* \sim 100$–200 are needed for achieving ignition [14], the elongation and length of an FRC becomes unrealistic. Therefore, the study of additional stabilizing techniques is still needed for future large-size FRCs.

The newly constructed Prairie View (PV) Rotamak is uniquely positioned for such study. The 40 ms long duration of plasma discharge allows for observing global MHD instability modes even if their growth time is much longer than the Alfvén transit time. In addition, a set of shaping magnetic coils permits plasmas to form with $E < 1$ (oblate FRC), $E > 1$ (prolate FRC), and doublet-shape configurations with two magnetic axes. The main questions for the study are: (1) What kind of MHD instability occurs in our rotamak FRC plasmas and which modes result in plasma disruption? (2) Can MHD instabilities be completely suppressed with shaping magnetic coils?

In this Letter, we report the first observation of the $n = 1$ tilt and radial shift modes in rotamak FRC plasmas. Experimental results demonstrate that in RMF FRC plasmas, the $n = 1$ tilt mode becomes saturated after a long growth time, which can be up to 10 ms (~2000 of Alfvén transit time). In contrast, when the $n = 1$ radial shift mode occurs, it leads to a fast destruction of plasma. As shown in this Letter, stability of modes is strongly related to plasma shape. Tilt mode is completely suppressed when a doublet-shape FRC with an internal figure-of-eight separatrix is formed by energizing the shaping magnetic coil at the midplane. We show that there is an optimum doublet-shape configuration that is completely free from both tilt and radial shift modes.

The Rotamak device [9] is illustrated schematically in Fig. 1. The glass chamber includes the center column covered with quartz tube; there are no conducting surfaces facing the plasma. Three shaping magnetic coils are wound over the chamber surface and connected to a programmable current source with current rise time of 1–2 ms. One middle coil (8 turns) is located at $Z = 0$, two side coils (4 turns each) at $Z = \pm 8$ cm. To characterize MHD insta-

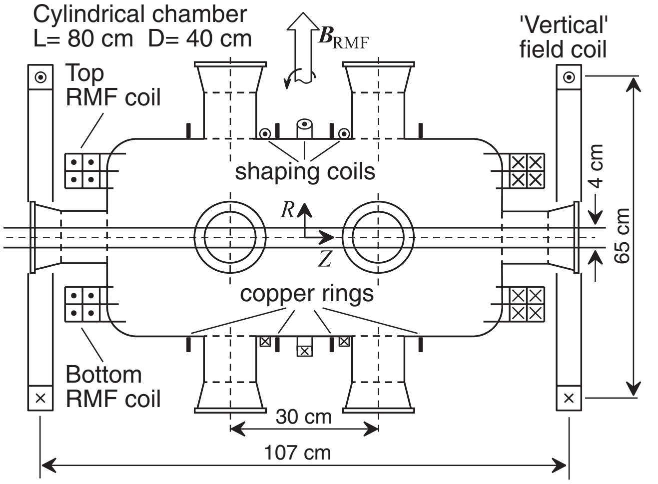

FIG. 1. The schematic of the PV Rotamak.

bility, two sets of Mirnov magnetic coils are located at $Z = \pm 4$ cm. At each $Z$ position, eight $B_R$-oriented coils are mounted around chamber surface at equal 45° $\theta$ intervals ($\theta$ is the toroidal angle). The "vertical" field coils provide the vacuum magnetic field $B_v = 25$ G at the chamber center. No external toroidal field is used in this study. Hydrogen gas is continuously fed through the chamber at filling pressure $p_f = 1.3$ m Torr. The rf power ($f_{\text{rmf}} = 500$ kHz) delivered to the plasma is 200 kW that produces RMF components $\tilde{B}_R \approx 10$ G and $\tilde{B}_\theta \approx 20$ G at the plasma edge. The plasma temperature and density are measured by double floating Langmuir probes. Typically, $T_e = 15$–$20$ eV and $n_e = 1.2$–$2.5 \times 10^{12}$ cm$^{-3}$.

It is seen that the plasma is weakly ionized: the filling pressure $p_f = 1.3$ m Torr corresponds to the neutral density $n_n = 4.5 \times 10^{13}$ cm$^{-3}$, about 20–30 times larger than the electron density. The collision rate of electrons with neutrals is $\nu_{\text{en}} = n_n \sigma_{\text{en}} V_{Te} \approx 2 \times 10^6$ s$^{-1}$, about 3 times higher than $\nu_{\text{ei}}$. The electron collision rate is still much smaller than their gyrofrequency $\omega_{\text{ce}}/2\pi = 7 \times 10^7$ s$^{-1}$. For ions, assuming $T_i = 5$ eV, the collision rates are $\nu_{\text{in}} = n_n \sigma_{\text{in}} V_{Ti} \approx 5 \times 10^5$ s$^{-1}$ (based on $\sigma_{\text{in}} = 5 \times 10^{-19}$ m$^2$) and $\nu_{\text{ii}} \approx 8 \times 10^4$ s$^{-1}$, while $\omega_{\text{ci}}/2\pi \approx 4 \times 10^4$ s$^{-1}$. Therefore, the ions are not magnetized and can be treated as an immobile background, while the electron fluid is described by Ohm's law

$$\mathbf{E} - (\mathbf{J} \times \mathbf{B} - \boldsymbol{\nabla} P_e)/n_e e = \eta \mathbf{J}. \quad (1)$$

The main difference from the ideal MHD model is the presence of the Hall current, which can provide additional stabilization for the tilt mode [15].

A two-component internal magnetic pickup probe was used to perform radial scans at axial positions $Z = 0$, $\pm 5$, $\pm 11$, $\pm 17$ and $\pm 22.5$ cm. After the 2D scan is completed, the field components are interpolated to intermediate radial and axial points. As seen in Fig. 2(a), when no shaping coils are energized, the closed magnetic surfaces are quite oblate, in spite of the fact that the RMF antennae extend to the end of the chamber. This plasma shape is confirmed by an electron density scan with a Langmuir probe [9]; the plasma is located mainly within the area of $|Z| < 15$ cm, the position of maximum density being at $Z = 0$ cm and $R \approx 15$ cm. The nearly spherical shape of the plasma might be the consequence of two factors: the high mirror ratio of the equilibrium vertical field, and the ponderomotive force originating from the inhomogeneous oscillating electromagnetic field (RMF) that pushes plasma towards the midplane [16]. From Fig. 2(a) we estimate that when no shaping coils are energized, the elongation of the plasma is $E \approx 0.9$.

For plasma discharges without shaping coils, the $n = 1$ mode often appears during discharge, as seen in Fig. 3. Judging by the small phase shift between signals from two sets of Mirnov coils ($Z = -4$ cm and $+4$ cm), we identified it as a tilt mode. This mode usually grows within a period of 1–2 ms, and then becomes saturated during the shot. Depending on vertical field and filling gas pressure, the oscillation frequency of the mode is between 15 and 19 kHz; the growth time can be up to 10 ms. This tilt mode is nondestructive, but it usually results in a plasma current drop by 10%. As indicated in Fig. 3, when $B_v$ is increased from 25 G to 28 G, the tilt mode starts later and grows more slowly, and its frequency and amplitude gradually becomes smaller with higher $B_v$. Measurements with poloidal flux loops show the elongation $E$ reduces from 0.9 to 0.7 when $B_v$ is increased from 25 G to 30 G. This result qualitatively agrees with theoretical prediction [11] that the growth rate for tilt mode drops for smaller elongation, and the mode is completely suppressed at $E < 0.5$. When $B_v$ is slightly

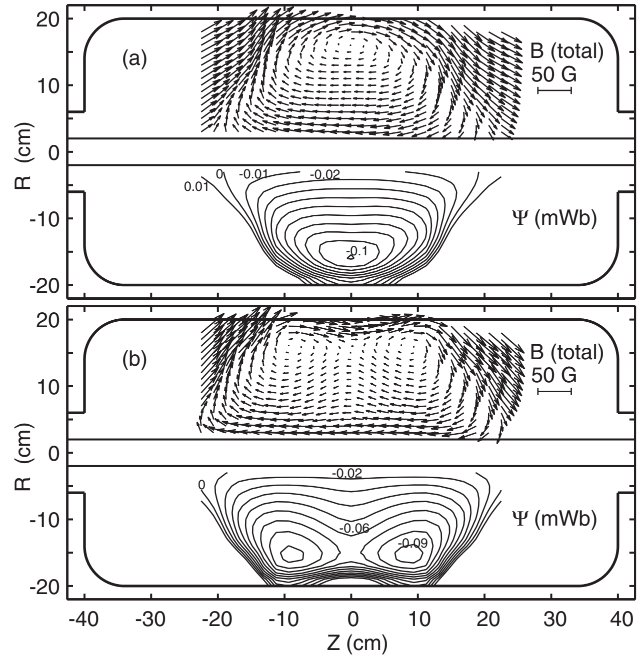

FIG. 2. Measured magnetic field and poloidal flux (a) before, and (b) after the middle shaping coil is energized.

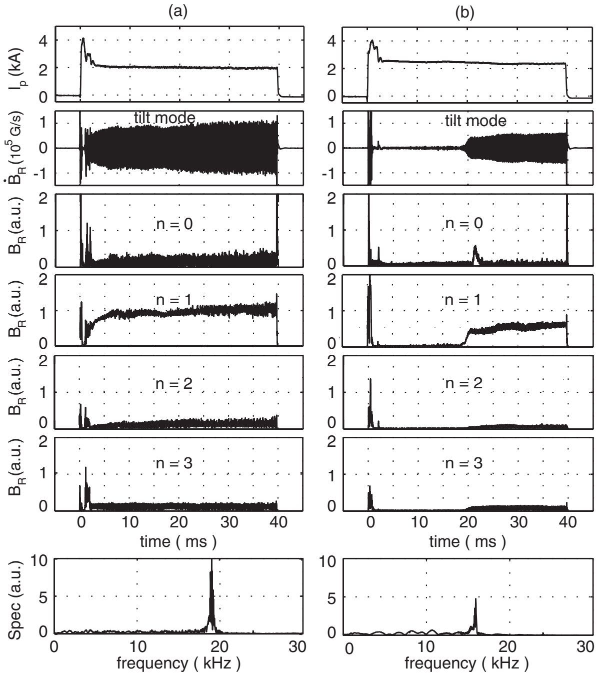

FIG. 3. Time evolution of plasma current, MHD oscillation signal, magnitude of $n = 0$–3 perturbations to $B_R$ and spectra for (a) $B_v = 25$ G; (b) $B_v = 28$ G.

larger than 30 G, tilt mode becomes very small or even disappears, but another mode with oscillation frequency around 10 kHz appears. It is identified as $n = 1$ radial shift mode (the phase shift between two sets of Mirnov coils is $\pi$). Usually excitation of radial shift mode results in plasma disruption within 1 ms. According to theoretical study [11], the radial shift mode is expected to become unstable at sufficiently small elongations ($E < 0.69$), in good agreement with our measurements.

From measurements, the tilt mode rotates in electron direction with angular speed $2\pi \times 16$ kHz $\approx 10^5$ rad/s. The two-fluid Hall MHD theory [15] yields the dispersion relation for the internal tilt mode ($E > 1$) that for the case of immobile ions can be written as

$$\hat{\omega}^2 - \hat{\omega}\frac{11\hat{\gamma}^2}{6S_*}\left(E + \frac{1}{E}\right) + \hat{\gamma}^2 = 0. \qquad (2)$$

Here, $\hat{\gamma}^2 = (15E^2 - 31)/(11 + 11E^2)$ corresponds to the growth rate in the ideal MHD limit normalized to $V_A/Z_S$, and $\hat{\omega} = \omega/(V_A/Z_S)$ is the normalized perturbation frequency. In our case of $S_* = R_S/(c/\omega_{\mathrm{pi}}) \approx 1$, the solution of Eq. (2) possesses $\mathrm{Re}(\omega) \approx 0.25V_A/Z_S \approx 6 \times 10^4$ rad/s for the range $0.6 < E < 1.2$, which is close to the measured angular speed of $10^5$ rad/s. The linear growth rate is given by $\mathrm{Im}(\omega)$ which is positive in a narrow range of elongation, $1.437 < E < 1.645$. The highest growth rate, $\mathrm{Im}(\omega) \approx 0.25V_A/Z_S \approx 6 \times 10^4$ rad/s, is reached at $E = 1.54$. It should be noted, however, that for $E \approx 1$ the mode evolves into external tilt, for which Eq. (2) is not accurate. In any case, due to $V_A/Z_S \ll 2\pi\nu_{\mathrm{in}}$, the collisions are expected to stabilize the tilt mode for both $E < 1$ and $E > 1$ in our plasmas.

In general, the tilt mode stability is strongly affected by plasma shape. Experiments with external field shaping coils [13] proved that the tilt mode can be stabilized in very oblate plasmas ($E < 0.5$).

We examined how the plasma shaping could affect stability of tilt mode in the range of $E \sim 0.7$–1.5. The typical value of $E = 0.9$ for our plasmas can be reduced to $E \sim 0.7$ by energizing side shaping coils only (at $Z = \pm 8$ cm). In this case the tilt mode is not suppressed; besides, the radial shift mode appears, which results in plasma disruption. When all three shaping coils are energized, the elongation can be increased up to $E \sim 1.5$. In such case the tilt mode is still present, although with a smaller amplitude (sometimes undetectable).

The most striking result is seen when only the middle shaping coil (at $Z = 0$) is energized, producing doublet-shape FRCs, as shown in Fig. 2(b). As seen in Fig. 4(a), the $n = 1$ tilt mode starts growing at 4th ms and saturates at 5th ms with its oscillation frequency $f \sim 17.5$ kHz; the excitation of this mode results in $I_p$ drop from 2.3 kA to 2.1 kA. In this shot, the middle shaping coil is energized with $I_m = 400$ A current during $t = 20$–40 ms. It is seen in Fig. 4(a) that after the middle coil is energized, the $n = 1$ tilt mode is completely suppressed. The change of elongation from $E \approx 0.92$ to $E \approx 1.37$ after the middle coil is energized cannot explain by itself the suppression of the tilt mode. As mentioned above, the tilt mode is still observed in elliptically shaped plasmas with $E \approx 1.5$, when all three shaping coils are energized.

Repeatable discharges demonstrate that the tilt mode can be completely suppressed by energizing the middle coil with current in the range of $I_m \sim 250$–500 A. When $I_m$ current is higher, 500–600 A, the tilt mode is suppressed, but the $n = 1$ radial shift mode is excited and plasma is disrupted within 2–3 ms after energizing the coil, as shown in Fig. 4(b). For $I_m > 600$ A, plasma is disrupted immediately as the coil is energized.

Plasma global stability to $n = 1$ tilt or shift modes can be examined with the rigid-body model [12,13] which approximates plasma current by several rigid current rings that can tilt or shift in unison with each other. The onset of tilt instability is determined by value of

$$N_{\mathrm{decay}} = -\frac{R}{B_z}\left[\frac{\partial B_z}{\partial R} - \frac{Z}{R^2}\frac{\partial}{\partial R}(2RB_R + ZB_z)\right], \qquad (3)$$

which characterizes the curvature of equilibrium magnetic field at the location of current rings. The torque on the plasma is due to the $\mathbf{J} \times \mathbf{B}$ force from equilibrium field onto tilted plasma current rings. The torque acts to restore the current ring to its equilibrium position if $N_{\mathrm{decay}} > 1$, or to increase the tilt if $N_{\mathrm{decay}} < 1$. For our conditions, before the middle coil is energized, $N_{\mathrm{decay}} < 0.4$, so the tilt instability should be present. This result is in agreement with experiment. When the middle shaping coil is energized, for

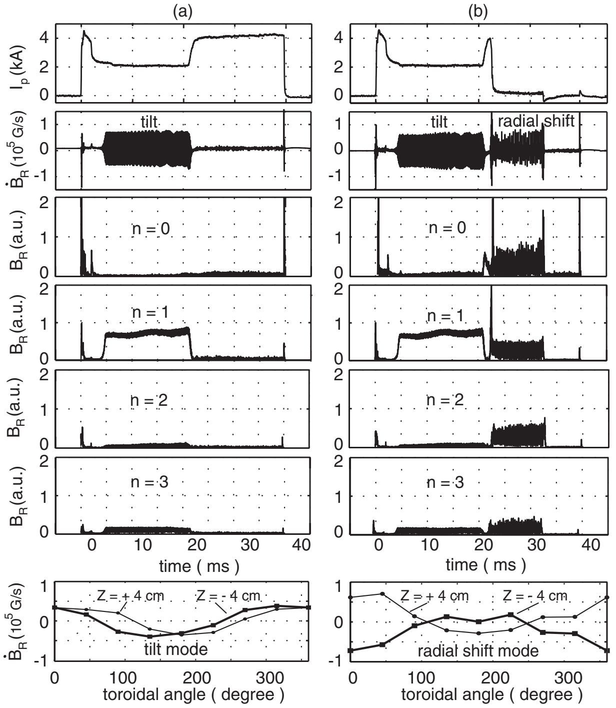

FIG. 4. Time evolution of plasma current, MHD oscillation signal, magnitude of $n = 0$–3 perturbations to $B_R$ and signals from two sets of Mirnov coils. $I_m$ current is fed to the middle coil during 20–40 ms (a) $I_m = 400$ A; (b) $I_m = 550$ A.

example, with $I_m = 360$ A current, $N_{\text{decay}}$ still remains lower than 0.5 in most area of plasma. However, in experiments, the tilt mode is fully suppressed.

The apparent absence of tilt mode could be attributed to the fact that in a doublet FRC the tilt perturbation becomes localized near the FRC ends, too far from magnetic probes. To verify the measurements, a following simulation is performed. The plasma current is represented by a set of 10 current rings with different radii and axial positions to fit experimental data on the boundary magnetic flux and the structure of magnetic field. In one simulation, the current rings are uniformly positioned within $-11.25 \text{ cm} \leq Z \leq 11.25$ cm. Each current ring is carrying $-200$ A current (totally $-2$ kA), while the two equilibrium coils at $Z = \pm 53.5$ cm carry 4.3 kA current each. When the current rings that represent the plasma current are tilted by $\pm 2.5°$, the calculated $B_R$ component of magnetic field at Mirnov coil's position is changed from $-6.8$ G to $-4.8$ G. Assuming that this change occurs during the half-period of the oscillating field, $\Delta t = T/2 = 0.5/18$ kHz, the amplitude of $dB_R/dt$ oscillations can be estimated as $|\Delta B_R/\Delta t| = 0.7 \times 10^5$ G/s, close to that shown in Fig. 4.

In another simulation, an additional coil with $I_m = 400$ A is added at the midplane. The plasma current ($-4$ kA) is represented by 10 current rings positioned within $-15 \text{ cm} \leq Z \leq 15$ cm, to reproduce the field structure shown in Fig. 2(b). The tilting angle is set to 2.5° for the current rings at $Z = \pm 15$ cm, but gradually reducing to zero for the current rings closer to the midplane. The calculated $B_R$ component of magnetic field at Mirnov coil's position is 10.2 G and 11.1 G, corresponding to a different sign of the tilt angle. Therefore, the amplitude of $dB_R/dt$ oscillations can be estimated as $|\Delta B_R/\Delta t| = 0.3 \times 10^5$ G/s. This value is still an order of magnitude higher than the noise level of $0.05 \times 10^5$ G/s seen in Fig. 4(a) for $t > 20$ ms, so the tilt mode could be detected if it were present.

Thus we are inclined to conclude that it is the doublet configuration that makes the plasma stable to the tilt mode. The improved stability of doublets against other types of MHD instabilities (pressure-driven high-$n$ interchange modes) has been predicted analytically [17]; the stability improves when $E$ gets smaller but indentation remains moderate. It was speculated that the doublet configuration should also be stable to the cointerchange (tilt) mode.

In summary, for the first time in rotamak FRC plasmas, the $n = 1$ tilt and radial shift modes have been clearly observed and identified. The tilt mode is partially stabilized by collisional and Hall-current effects. By using a middle shaping coil with a moderate current of 250–500 A, the doublet FRCs are formed which are completely free from both the tilt and radial shift modes. Experimental results suggest that doublet FRCs are more stable than standard oblate or prolate FRCs.

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