Influence of a Polarization Force on Dust Acoustic Waves

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The effect of the polarization force acting on the grains in a nonuniform plasma background on the propagation of low-frequency waves in complex (dusty) plasmas is analyzed. It is shown that polarization interaction leads to a renormalization (decrease) of the dust acoustic phase velocity. The effect becomes more pronounced as the grain size increases. Finally, there is a critical grain size above which the dust acoustic waves cannot propagate, but aperiodic (nonpropagating) perturbations form instead.

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A remarkable feature of complex (dusty) plasmas—systems consisting of charged micron-size particles (grains) in a neutralizing plasma background—is the ability to sustain low-frequency waves. These waves are associated with the motion of highly charged massive grains and as a result occur at relatively long time and space scales (typically ~ 0.1 s and ~ 1 cm, respectively). Dissipation caused by friction with the neutral background is usually weak, and therefore wave processes in complex plasmas can be easily observed at the kinetic level. Moreover, depending on the strength of intergrain interaction, complex plasmas can be in weakly coupled (gaseous-like) or strongly coupled (liquidlike or crystal-like) states. This provides a unique opportunity to investigate wave-related phenomena occurring in different phase states (linear and nonlinear waves, solitons and shocks, Mach cones, etc.) at the most fundamental particle level [1,2].

The focus of this Letter is on the linear waves in weakly coupled unmagnetized complex plasmas. In the long-wavelength limit these waves exhibit acousticlike dispersion and are therefore called “dust acoustic waves” (DAWs). The dispersion relation of DAWs was originally derived by Rao, Shukla, and Yu [3]. Then the DAWs, either self-sustained or excited externally, were observed in numerous experiments under quite different conditions, e.g., in an rf magnetron discharge [4], in a Q machine [5], in dc discharges [6–8], and in rf discharges [9–13]. Excitation of DAWs is known to be a powerful tool for complex plasma diagnostics since it allows us to estimate the grain charge by analyzing the experimentally measured dispersion relation [14,15]. Alternatively, the grain charge can be evaluated from the threshold condition of DAW instability [16].

In this Letter we discuss an effect which has not yet been considered in connection with the propagation of low-frequency waves in complex plasmas. It is associated with the polarization force acting on the grains when waves are propagating and the plasma background becomes locally nonuniform. We show that this interaction can modify considerably the DAW dispersion relation and discuss the consequences of this modification.

We start with a brief description of the polarization force. Hamaguchi and Farouki [17,18] demonstrated that the total force acting on a small charged grain in a nonuniform plasma with external electric field $\mathbf{E}$ is

$$\mathbf{F}_{\Sigma} = Q\mathbf{E} - \frac{Q^2}{2}\frac{\nabla\lambda_D}{\lambda_D^2}, \tag{1}$$

where Q is the grain charge, λ_D is the linearized Debye radius, $\lambda_D = \lambda_{Di}/\sqrt{1+(\lambda_{Di}/\lambda_{De})^2}$, $\lambda_{Di(e)} = \sqrt{T_{i(e)}/4\pi e^2 n_{i(e)}}$, and $T_{i(e)}$ and $n_{i(e)}$ are ion (electron) temperature and density, respectively. The first term in Eq. (1) is the conventional electric force, while the second term is the polarization force arising due to plasma polarization around the grain. The latter is derived from the linear Poisson equation of the form $\Delta\phi = \lambda^{-2}\phi + 4\pi Q\delta(\mathbf{r})$, where $\lambda^{-2} = \lambda_D^{-2} - 2\lambda_D^{-3}(\mathbf{r}\nabla\lambda_D)$ using the standard perturbation technique [17,18]. Here λ_D and $\nabla\lambda_D$ should be evaluated at the position of the grain $\mathbf{r} = 0$.

In most practical cases the polarization force is small as compared to other forces present in the system. Let us illustrate this. First, we consider an individual charged grain in the regime of ambipolar plasma diffusion. The plasma is quasineutral $n_e \simeq n_i \simeq n_0$ and weakly nonuniform with $\nabla n_e \simeq \nabla n_i \simeq -E(en_0/T_e)$. The polarization force in this case is $F_p = -(Qe/4\lambda_D T_e)QE$. Independently of the sign of the grain charge it is directed opposite to the electric field. The absolute ratio of the polarization and electric forces is thus $\frac{1}{4}(|Q|e/\lambda_D T_e) = \frac{1}{4}(a/\lambda_D)z$, where $z = |Q|e/aT_e$ is the dimensionless grain charge and a is the grain radius. In typical experiments in gas discharges $z \sim 1$ while $a \ll \lambda_D$, and therefore the polarization force is only a small fraction of the electric force [17]. Second, let us consider the situation when the neutral gas temperature is slightly nonuniform. This can be, for instance, related to heating/cooling of the electrodes in rf discharges in order to compensate for grain gravity in ground-based experiments [19]. Since the ion temperature is coupled to the neutral gas temperature, $T_i \simeq T_n$, the polarization force in this case is $F_p = -\frac{1}{4}(Q^2\lambda_D/\lambda_{Di}^2) \times (\nabla T_i/T_i)$, and it points in the direction of lower ion temperature. This should be compared with the thermophoretic force $F_{\rm th} = -C(a^2/\sigma_{nn})\nabla T_n$, where σ_{nn} is the neutral-

neutral collision cross section and C is a numerical coefficient of the order of a few (C is somewhat different in different models; see, e.g., Refs. [2,19]). Neglecting numerical coefficients of the order of unity the ratio of the polarization and thermophoretic forces is $F_p/F_{\rm th} \sim (T_e/T_i)(T_e\sigma_{nn}/e^2\lambda_D)$. For typical conditions $T_e/T_i = 100$, $T_e \sim 3$ eV, $\sigma_{nn} \sim 4\times 10^{-15}$ cm^2 (argon gas), $\lambda_D \sim 10^{-2}$ cm, the force ratio is $\sim 10^{-3}$; i.e., again the polarization force is negligibly small. Thus, in usual steady states the polarization force plays a negligible role. This might be the reason why this interaction received almost no attention so far. We will demonstrate, however, that in the context of low-frequency (dust) waves this interaction is quite important and certainly needs to be taken into account.

In order to make the physics behind the considered effect more transparent, let us derive a simple dispersion relation for the DAW in ideal unmagnetized complex (dusty) plasmas. In doing so we do not take into account collisions between the grain component and other complex plasma components, assume that both electrons and ions provide an equilibrium neutralizing background, and neglect fluctuations of the grain charges. We also assume that the grains are identical in size, mass, and (negative) charge. Then, the continuity and momentum equations for the dust density N_d and velocity $\mathbf{V}_d$ can be written in the following form:

$$\frac{\partial N_d}{\partial t} + \nabla(N_d\mathbf{V}_d) = 0, \tag{2}$$

$$\frac{\partial \mathbf{V}_d}{\partial t} + (\mathbf{V_d}\cdot\nabla)\mathbf{V_d} = \frac{\mathbf{F}_\Sigma}{M_d} - \frac{\nabla(N_dT_d)}{M_dN_d}, \tag{3}$$

where M_d is the grain mass, T_d is the grain kinetic temperature, and the force $\mathbf{F}_\Sigma$ is given by Eq. (1). The densities of electrons and ions satisfy the Boltzmann distribution $n_{e(i)} \simeq n_0 \exp(\pm e\phi/T_{i(e)})$. The system is closed by the Poisson equation

$$\Delta\phi = -4\pi e(n_i - n_e) - 4\pi Q N_d. \tag{4}$$

Taking into account that $\mathbf{E} = -\nabla\phi$ and in the considered case $\mathbf{F}_\Sigma = -Q\nabla\phi(1-\Re)$, where $\Re = \frac{1}{4}(|Q|e/\lambda_D T_i)\times(1-\frac{T_i}{T_e})$, the standard linearization procedure yields the dispersion relation of low-frequency waves

$$\frac{\omega^2}{k^2} = \gamma_d V_{T_d}^2 + \frac{(1-\Re)\omega_{\rm pd}^2\lambda_D^2}{1+k^2\lambda_D^2}, \tag{5}$$

where ω and k are the wave frequency and wave number, γ_d is the effective polytropic index for the grain component, $V_{T_d} = \sqrt{T_d/M_d}$ is the grain thermal velocity, and $\omega_{\rm pd} = \sqrt{4\pi Q^2 N_d/M_d}$ is the plasma frequency of the grain component. Note that in the limit $T_e \gg T_i$ the factor $\Re$ can be presented as $\Re = \frac{1}{4}\beta_T$, where β_T is the ratio of the Coulomb radius of interaction between thermal ions and the grain and the linear Debye radius. This is a natural measure of the nonlinearity in ion-grain interaction [20]. It is also known as the thermal scattering parameter since it proved to be very useful in describing ion scattering on the grain potential and the corresponding momentum transfer in ion-grain collisions [20].

The first term on the right-hand side in Eq. (5) represents the dust thermal mode, and the second one corresponds to the dust acoustic mode (DAW). The phase velocity of the DA mode in the long-wavelength limit is $C_{\rm DA} \simeq \omega_{\rm pd}\lambda_D\sqrt{1-\Re}$. The difference from the conventional dispersion relation of Rao, Shukla, and Yu [3] is the normalization factor $\sqrt{1-\Re}$. This renormalization is the result of taking into account the plasma polarization force which naturally arises due to plasma nonuniformity accompanying wave propagation. The effect is no longer negligible since usually the value of β_T is of the order of unity or larger [21]. For typical complex plasma parameters $a \sim 1$ μm, $Q \sim 10^3 e$, $\lambda_D \sim 10^{-2}$ cm, and $T_i \sim 0.03$ eV, we have $\beta_T \simeq 0.48$ and $\Re \simeq 0.12$. Since the grain charge is approximately proportional to the grain radius, the effect becomes more and more important when the grain size increases. Moreover, for relatively large grains we can expect $\Re > 1$. In this case the net force on the grains is no longer a restoring force, and then the dispersion relation (5) admits a transition from propagating DA waves to aperiodically growing perturbations. Such unstable perturbations appear to some extent similar to these described in Refs. [22–24]. However, the physical processes triggering these instabilities are completely different: In Refs. [22–24] the origin is the ion drag force, while in our case instability is driven by the polarization force.

It should be emphasized that the threshold condition for the occurrence of the nonpropagating unstable mode $\beta_T \simeq 4$ cannot be claimed accurate. The point is that the regime $\beta_T \gtrsim 1$ corresponds to the situation when the nonlinear region around the grain is large and the linear approximation used to derive Eq. (1) is not applicable. Detailed analysis of the nonlinear situation is beyond the scope of the present Letter. Fortunately, one can find simple arguments which allow us to gain some insight into the problem: A number of numerical calculations of the electric potential around a spherical floating particle in plasmas [25–27] demonstrated that in a broad range of distances (up to several Debye lengths from the grain) the Debye-Hückel (Yukawa) profile fits the numerical results reasonably well even in the highly nonlinear regime, provided an effective screening length, $\lambda_{\rm eff}$, is chosen appropriately. Actually, taking into account the tremendous nonlinearity near the grain surface, it remains a bit of a mystery as to why the Debye-Hückel form works so well. Nevertheless, this fact implies that the potential in the vicinity of the grain should satisfy the linear Poisson equation with λ_D replaced with $\lambda_{\rm eff}$. In turn, this suggests that the polarization force can be estimated as

$$F_p = -\frac{Q^2}{2}\frac{\nabla\lambda_{\rm eff}}{\lambda_{\rm eff}^2}. \tag{6}$$

In the linear regime ($\beta_T \lesssim 1$) the effective screening length $\lambda_{\rm eff}$ tends to λ_D [25,27], and therefore, the generalized Eq. (6) recovers the linear limit result of Eq. (1). In the nonlinear regime $\lambda_{\rm eff}$ can deviate considerably from λ_D [25–27]. The exact dependence of $\lambda_{\rm eff}$ on plasma and grain parameters is not known for the general case and can be found only through extensive numerical simulations. However, it has been suggested in Ref. [27] that for a collisionless quasi-isotropic plasma with $T_e \gg T_i$ (so that $\lambda_D \simeq \lambda_{Di}$), the ratio $\lambda_{\rm eff}/\lambda_{Di}$ is a function of the single parameter β_T. A fit based on the numerical results of Ref. [25] obtained in a wide range of β_T has been recently proposed in Ref. [28]. The corresponding expression is $\lambda_{\rm eff} = \lambda_{Di}(1 + 0.105\sqrt{\beta_T} + 0.013\beta_T)$. Using this fit we find

$$\Re \simeq \frac{\beta_T(1 + 0.053\sqrt{\beta_T})}{4(1 + 0.105\sqrt{\beta_T} + 0.013\beta_T)^2}.$$

Analysis of this expression shows that at small and moderate β_T the parameter $\Re$ grows almost linearly with β_T. The onset of the unstable aperiodic mode occurs at $\beta_T \simeq 6.4$. This corresponds to rather big grains, $a \gtrsim 10\ \mu$m under typical conditions. At very high β_T we have $\Re \propto 1/\sqrt{\beta_T}$; i.e., reentrant transition to a propagating DAW should occur. This happens, however, for values of $\beta_T \gtrsim 6 \times 10^3$ which are at least 2 orders of magnitude higher than those of any realistic complex plasmas investigated so far. Thus, the main conclusion is that under usual conditions polarization interaction slows down the DAW and sets up an upper limit of grain sizes allowing DAW propagation.

An important remaining question related to the problem investigated is, Could a similar effect affect the dispersion relation of the ion-acoustic waves in conventional electron-ion plasmas? The dispersion relation in this case is

$$\frac{\omega^2}{k^2} = \gamma_i v_{T_i}^2 + \frac{(1 - \Re_i)\omega_{\rm pi}^2\lambda_{De}^2}{1 + k^2\lambda_{De}^2},$$

where $\Re_i = e^2/4T_e\lambda_{De}$ and $\omega_{\rm pi}$ is the plasma-ion frequency. Since $\Re_i \sim (\lambda_{De}^3 n_e)^{-1} \equiv N_{De}^{-1} \ll 1$, where N_{De} is the number of electrons in the electron Debye sphere, the effect of polarization interaction on the ion-acoustic mode in conventional plasmas is negligibly small. This conclusion is related of course to the fact that electron-ion coupling in conventional plasmas is weak, while ion-grain coupling in complex plasmas is usually moderate or even strong [21].

To summarize, we have investigated the effect of the polarization force on the propagation of dust acoustic waves in complex (dusty) plasmas. It has been shown that polarization interactions affect the DAW dispersion relation considerably, leading to a renormalization of the dust-acoustic velocity. This renormalization should be especially important in the context of experiments aiming to determine the grain charge from the analysis of the DAW dispersion relation. In addition, we have found that there should exist a critical grain size such that for larger grains DAW cannot be sustained, but aperiodic growing perturbations are excited. Experimental verification of this prediction is desirable, but perhaps this would be only possible in experiments under microgravity conditions since rather big grains ($\gtrsim 10\ \mu$m) have to be used.

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