Determination of the Newtonian Gravitational Constant G with Time-of-Swing Method

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(Received 7 May 2006; revised manuscript received 2 March 2009; published 16 June 2009)

We present a new value of the Newtonian gravitational constant G by using the time-of-swing method. Several improvements greatly reduce the uncertainties: (1) measuring the anelasticity of the fiber directly; (2) using spherical source masses minimizes the effects of density inhomogeneity and eccentricities; (3) using a quartz block pendulum simplifies its vibration modes and minimizes the uncertainty of inertial moment; (4) setting the pendulum and source masses both in a vacuum chamber reduces the error of measuring the relative positions. By two individual experiments, we obtain $G = 6.673\,49(18) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ with a standard uncertainty of about 2.6 parts in 10^5.

DOI: 10.1103/PhysRevLett.102.240801 PACS numbers: 06.20.Jr, 04.80.Cc

The Newtonian gravitational constant G plays a key role in fields of gravitation, cosmology, geophysics, and astrophysics and is still the least precisely known. Based on the weighted mean of eight values obtained in the past few years [1], in 2006 the Committee on Data for Science and Technology (CODATA) recommended a value with a relative uncertainty of 100 ppm, in which the value exceeded the 1998 recommended value by a fractional amount of 192 ppm [2]. Although the situation of the measurements of G has improved considerably since 1998, the values in CODATA 2006 are still in poor agreement. Even for the five most precise values of G with assigning uncertainties less than 50 ppm [3], they are only consistent within about 200 ppm.

Here we report our improved measurement of the G value by means of the time-of-swing method with a flat plate torsion pendulum. Using the same method, a preliminary result was reported in 1998 [4]. In our following work, two systematic errors were found and corrected. The first was the eccentricity of the mass center from the geometric one of the two cylindrical source masses, and the second was the effect of the air buoyancy [5]. Even so, the corrected G value is still model dependent on the density distribution of the cylinders. In this experiment, spherical source masses with more homogeneous density and a quartz block pendulum having fewer vibration modes were used and both were set in a vacuum chamber, the configuration of which was remotely operated by using a step motor.

The time-of-swing method, developed by Heyl [6], is based on detecting the change of the angular oscillation frequencies for the source masses at two configurations, referred to as the "near" position (the equilibrium position of the pendulum is in line with the source masses) and the "far" position. The torque on the pendulum when rotated by an angle θ is the sum of a torque $-K\theta$ produced by the twisted fiber and a torque $\tau_{gn}(\theta)$ or $\tau_{gf}(\theta)$ from gravitational interaction with the source masses, where K is the torsional spring constant of the fiber and n and f represent near and far source mass positions, respectively. $\tau_{gn}(\theta)$ (and similarly $\tau_{gf}(\theta)$) may be expanded as

$$\tau_{gn}(\theta) = -K_{1gn}\theta - K_{3gn}\theta^3 + O(\theta^5), \tag{1}$$

where $K_{1gn} = \frac{\partial^2 V_g(\theta)}{\partial\theta^2}$ and $K_{3gn} = \frac{1}{6}\frac{\partial^4 V_g(\theta)}{\partial\theta^4}$ evaluated at $\theta = 0$ with $V_g(\theta)$ being the gravitational potential energy between the pendulum and the source masses. If K_{3g} and higher terms are neglected, we may treat K_{1gn} and K_{1gf} as the effective gravitational torsion constants. Then we may write these as GC_{gn} and GC_{gf}, respectively, where C_{gn} and C_{gf} are determined by the mass distributions of the pendulum and source masses in near and far positions. Hence the frequency squared of small oscillations of the pendulum is $\omega_{n,f}^2 = (K_{n,f} + GC_{gn,gf})/I$, where I is the moment of inertia of the pendulum. Then G can be determined by

$$G = \frac{I(\omega_n^2 - \omega_f^2) - (K_n - K_f)}{C_{gn} - C_{gf}} = \frac{I\Delta(\omega^2)}{\Delta C_g}\left[1 - \frac{\Delta K}{I\Delta(\omega^2)}\right], \tag{2}$$

where ΔK denotes the possible changes of spring constant of the fiber at two configurations if K is a function of oscillation frequency, namely, the anelasticity of the fiber, as proposed by Kuroda [7].

A schematic diagram of our experimental setup is shown in Fig. 1. The pendulum was a gold-coated rectangular quartz block with length of 91.465 46(13) mm, width of 12.014 71(5) mm, height of 26.216 18(7) mm, and mass of 63.383 88(21) g. The pendulum was suspended by an 890 mm long, 25 μm diameter annealed and thoriated tungsten fiber linked with a cylindrical aluminum clamp, which was adhered centrically on the pendulum with the departures of 16(4) μm in the X, Y directions. The upper end of the fiber was connected to a magnetic damper, which was used to suppress the simple pendulum motions [8]. The copper disk of the damper was suspended by a 7 cm long, 50 μm diameter, annealed tungsten prehanger fiber and connected to a rotational feedthrough fixed on the top of the electrical-grounded vacuum chamber.

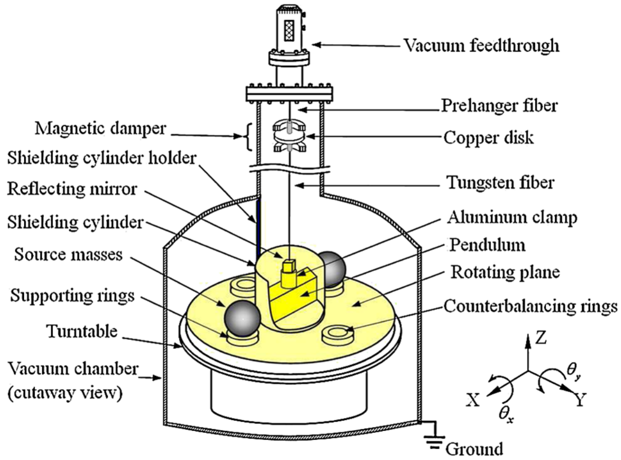

FIG. 1 (color online). Schematic view of the pendulum system used to measure G by time-of-swing method. The directions (X, Y, Z) used in text were also defined in this figure.

Four small identical Zerodur rings were symmetrically adhered on a Zerodur disk with a diameter of 240 mm and a thickness of 11 mm, which was fixed on a turntable. Two rings were used to support the source masses, and the others acted as counterbalances. Because of the extremely low thermal expansion coefficient for Zerodur of $(0 \pm 1) \times 10^{-7}/°\mathrm{C}$, the variations of the separation between the mass centers of the source masses caused by the temperature fluctuation (<0.2 °C during the measurement) was negligible. The rings and the plate were all gold coated to keep the source masses well grounded. The turntable was driven by a step motor with 0.0005°/step. The two source masses were SS316 stainless steel spheres with the vacuum masses of 778.1794(9) and 778.1763(9) g, and the diameters were 57.151 23(24) and 57.150 74(21) mm, respectively.

The distance between the geometric centers (GC) of the spheres was measured [9] to be 157.161 54(37) mm, and the uncertainty was mainly attributed to the roundness of the spheres, which was 0.23(3) and 0.27(3) μm, respectively. The attitude of the pendulum was measured by an autocollimator when the pendulum was twisted by 180° around the fiber, which gave $\theta_x = 4.06(5)$ mrad and $\theta_y = -1.91(2)$ mrad, as shown in Fig. 1. The turntable was adjusted to be coaxal with the torsion fiber within 13 μm. In the Z direction, the top surface of the rotating plane was chosen as the reference plane, and the centric height of the pendulum and the spheres were measured to be 34.250(8), 34.176(6), and 34.188(6) mm, respectively. The equilibrium position of the pendulum relative to the central line of the source masses (angle θ_0) was aligned with an uncertainty of ≤ 0.1 mrad. The creep of ~ 1 μrad/h of the torsion fiber was corrected when alternating the configurations every time.

A thinner hollow gold-coated aluminum cylinder was inserted between the pendulum and the source masses (as shown in Fig. 1). It was treated as the electrostatic shielding and evidently improved the stability of the pendulum's period. The initial oscillation amplitude of the pendulum, about 2 mrad, could be stimulated by applying a voltage of about 2 V to the shielding cylinder because of a tiny misalignment between the pendulum and the shielding cylinder. Once the pendulum was stimulated, the shielding cylinder was directly grounded.

The torsion pendulum, the source masses, and the turntable were all located inside the vacuum chamber with a pressure of $\sim 10^{-5}$ Pa maintained by an ion pump. The oscillation of the pendulum was monitored by an optical lever, and the output signal was sampled at a rate of 2 Hz with a frequency accuracy and stability of $\pm 5 \times 10^{-9}$ Hz and $\leq 3 \times 10^{-10}$/day, respectively. Six temperature sensors (four inside the chamber) were also recording continuously.

Ten sets of experimental data with the source masses in near and far positions were taken alternately, and the duration was about 3 days at each position. The oscillation periods of the pendulum were extracted from the angle-time data by a correlative method [10]. The period change of the pendulum at the two positions was ~ 3.23 s with a free oscillation of ~ 535.2 s. For each 3-day data segment, the pendulum's periods decreased at a rate from 0.2 to 0.7 ms/day, and the same phenomena was also observed when using a symmetric disk pendulum. Using the disk pendulum, the coefficients accounting for the thermoelastic [11] and nonlinear [12] properties of the fiber were measured to be $\alpha_K = -145(2) \times 10^{-6}/°\mathrm{C}$ and $\kappa_3 < 0.014\ \mathrm{rad}^2$, respectively.

For obtaining the proper period for every data segment, a precise time interval of 3 days was used as the window to match the whole data sequence, which utilizes the experimental data by the greatest extent to lower the external noises. During the period extraction, the gravitational nonlinearity [K_{3g} in Eq. (1)] of the source masses was synchronously corrected according to the pendulum's amplitudes with an accuracy of 20 μrad and angle θ_0 in every 3-day data segment, which provided a correction of 7.73(30) ppm to G. Over each data segment, the variation in temperature was less than 0.05 °C, while over the complete measurement of G it was less than 0.17 °C. The periods were all corrected to the same temperature of 20.20 °C (the average temperature during G measurement) by using the measured α_K, which yielded the average correction of $-39.83(1.52)$ ppm to G. The periods of ten data sets are shown in Fig. 2. It is obvious that a long-term drift of the periods, reducing at a uniform rate of ~ 0.5 ms/day for both positions, comes from the aging of the torsion fiber and the inhomogeneous background gravitational gradient. In order to deduce the effect of the drift, the "A-B-A" method (also shown in Fig. 2) was used to obtain the statistical average value of $\langle \Delta(\omega_{nf}^2) \rangle = 1.682\,275(29) \times 10^{-6}\ \mathrm{s}^{-2}$.

The correction accounting for the background gravitational effects, including the contributions from the inhomogeneous density of the Zerodur disk, the turntable, and

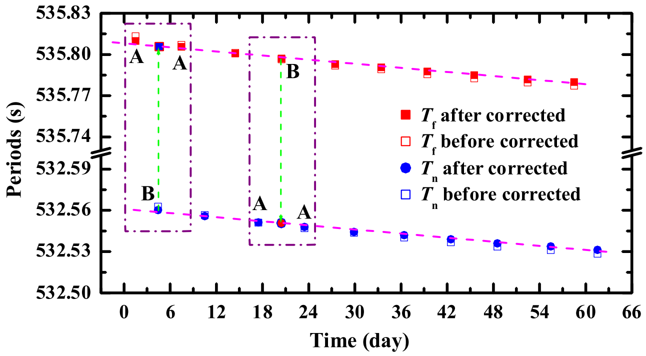

FIG. 2 (color online). Periods of the ten sets of experimental data extracted by the correlative method with uncertainty of ~0.01 ms. After being corrected by the thermoelastic and nonlinear properties of the fiber and the gravitational nonlinearity of the source masses, the slopes of the two dashed lines, representing the linear drifts of the pendulum's periods at near and far positions, are almost same. The "A-B-A" method is also shown in the figure.

the misaligned supporting rings, were measured by the pendulum itself with the above procedure but without the source masses. Six sets of background data, which showed almost the same drift in period series as before, yielded $\langle\Delta(\omega_b^2)\rangle = 30(12)\times 10^{-12}\ \mathrm{s}^{-2}$. Hence, the net value of the parameter in Eq. (2) was $\langle\Delta(\omega^2)\rangle = 1.682\,245(31)\times 10^{-6}\ \mathrm{s}^{-2}$. The moment of inertia of the pendulum was calculated to be $I = 4.505\,679(35)\times 10^{-5}\ \mathrm{kg\,m}^2$, and the ratio $\Delta C_g/I = 25\,202.85(28)\ \mathrm{kg\,m}^{-3}$ could be calculated according to the mass distributions of the pendulum and the source masses.

The Q factor is about 1700 for our pendulum system, and the anelasticity of the fiber should bring an upward bias about 200 ppm as predicted by Kuroda [7] and Newman and Bantel [13]. This bias was measured directly with two additional disk pendulums [14]. The free oscillation periods of the two disk pendulums were about 581.2 and 488.2 s (marked with subscripts "1" and "2," respectively), centered at the periods in our G measurement. By exchanging the pendulums, the downward correction to the G value due to the anelasticity, the last item in Eq. (2), could be expressed as

$$\frac{\Delta K}{I\Delta(\omega^2)} = \frac{I_1}{I[(\omega_2/\omega_1)^2 - 1]}\left[\left(\frac{I_2}{I_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2 - 1\right]. \quad (3)$$

After ten turns exchanging the pendulums, the ratio of squared frequencies for the two disk pendulums is yielded as $(\omega_2/\omega_1)^2 = 1.417\,172\,3(92)$. However, the precision measurements of the I_2 and I_1 of the two disk pendulums is very difficult. To solve this problem, a high Q ($\sim 3.36\times 10^5$) quartz fiber, whose anelasticity is negligible, was used to determined the ratio of the inertial moments by measuring the free oscillation periods of the two disk pendulums, which are about 29.629 and 35.271 s, respectively. From the ten sets of data, we give a statistical value of the ratio of the inertial moments $I_2/I_1 = 0.705\,683\,2(8)$. One of the disk pendulum's inertial moment is measured to be $I_1 = 5.3318(8)\times 10^{-5}\ \mathrm{kg\,m}^2$ with a relative lower precision. Finally, the resultant correction accounting for the anelasticity of the used tungsten fiber is $-211.80(18.69)$ ppm [15]. The details will be submitted separately.

The correction from the magnetic damper, combined as a two-stage vibration system with the pendulum, was $I_m K^2/IK_m^2 = 17.54(31)$ ppm, where $I_m = 2.220(18)\times 10^{-5}\ \mathrm{kg\,m}^2$ and $K_m = 1.048(9)\times 10^{-6}\ \mathrm{N\,m/rad}$ are the moment of inertia of the damper and the spring constant of the prehanger fiber, respectively. The effect of the local magnetic field was studied by applying a magnetic field at the location of the pendulum to 3.144(4) G in the Y direction, and the observed changes of the pendulum's period were 115(2) ms. It means that the effect that resulted from the fluctuations of 0.03 mG at the location of the pendulum due to the local magnetic field, after being shielded by the chamber, only brought an error of less than 0.40 ppm to the final G. The effect in the X direction was measured to be only one-third of that in the Y direction. For the SS316 source masses, a vector magnetometer was used to measure the differences of the magnetic field by placing the spheres nearby the probe with different orientations or without the spheres, and no net change was observed within 0.01 mG. The electrostatic effect was investigated by applying varied voltages on the shielding cylinder, and the observed period change due to the potential difference

TABLE I. One σ uncertainty budget (in units of ppm).

Error Sources	Corrections		$\Delta G/G$
Pendulum		5.07	
Dimensions			1.95
Attitude			0.13
Nonalignment with fiber			0.45
Flatness			0.34
Clamp			1.65
Density inhomogeneity			$\leq$0.21
Coating layer	−24.28		4.33
Edge flaw	−0.12		0.17
Source masses		10.68	
Masses			0.82
Distance of GC			9.64
Density inhomogeneity			4.50
XYZ positions			0.48
Fiber		18.76	
Nonlinearity			<0.70
Thermoelasticity	−39.83		1.52
Anelasticity	−211.80		18.69
Aging			<0.01
Gravitational nonlinearity	7.73	0.30	
Magnetic damper	17.54	0.31	
Magnetic field		0.40	
Electrostatic field		0.10	
Combined statistical $\Delta(\omega^2)$		14.18	
Total		26.33	

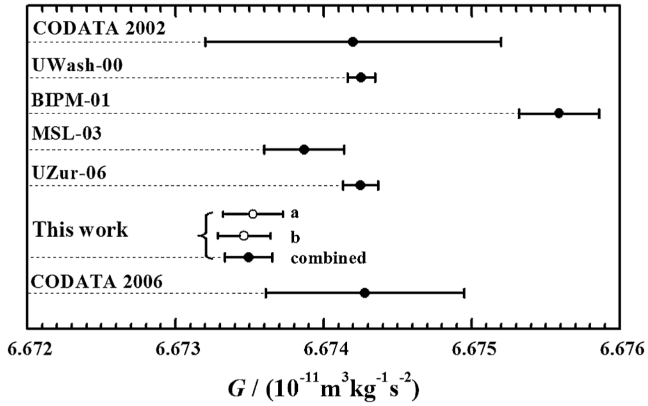

FIG. 3. A comparison of this work with others of uncertainties within 50 ppm [3] and the CODATA recommended values [1].

was $-53(4)$ ms/V^2. Therefore, a 0.10 ppm error was attributed to the electrostatic fluctuations of <1 mV.

The detailed uncertainty budget is listed in Table I. The density inhomogeneity of the pendulum was measured by an optical interference method [16], which provided an error of less than 0.21 ppm. In order to study the inhomogeneity of the coating layer, an identical pendulum was cut into 19 small blocks along the length direction, and the distribution of the coating layer was determined by measuring the mass changes of the blocks at different position on the pendulum before and after coating, which contributed a correction of $-24.28(4.33)$ ppm to G [17]. The main uncertainty coming from the source masses was the degree of roundness, which directly influenced the distance of the geometric centers of the source masses (0.37 μm corresponding to 9.64 ppm).

The uncertainty of G attributed to the density inhomogeneity of the source masses was considered by using different methods. First, one sphere with the same quality as the source masses was cut into several slices and scanned by scanning electron microscopy, in which the distributions of material components were determined by the backscattered electron images. The statistical results from 210 samples showed that the density inhomogeneity could contribute to G measurement with an uncertainty of less than 0.04 ppm [18]. Second, the mass center offset from the geometric center of the source masses was determined by using a beam balance [19], and the eccentricities were 0.34(8) and 0.24(8) μm, respectively, which should be mainly caused by the roundness of the source masses. Third, we directly exchanged the positions and also changed the orientations of the source masses on the supporting rings and repeated the G measurement. The second G value was 9.0 ppm smaller than the first one, half of which was hence chosen as the error due to the density inhomogeneity of the source masses.

By directly averaging the two G values of individual experiments, the combined final value of G was found to be (as shown in Fig. 3)

$$G = (6.673\,49 \pm 0.000\,18) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}. \quad (4)$$

The uncertainties of the two individual measurements were added quadratically, taking account of correlations.

In conclusion, our largest systematic uncertainty was due to the anelasticity measurement. For future improvement, using different torsion fibers with different Q factors to measure the anelasticity, elaborately compensating the background gravitational gradient, and further improving the stability of the pendulum's period to suppress the statistical uncertainty of $\Delta(\omega^2)$ will be carried out in our future G measurement.

We wish to thank R. D. Newman of UCI and V. Milyukov of MSU for their valuable discussions. This work is partly supported by the National Basic Research Program of China (Grant No. 2003CB716300) and the National Natural Science Foundation of China (Grants No. 10121503 and No. 10805021).

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