Theory of Heterotic Superconductor-Insulator-Superconductor Josephson Junctions between Single- and Multiple-Gap Superconductors

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Using the functional integral method, we construct a theory of heterotic superconductor-insulator-superconductor Josephson junctions between one- and two-gap superconductors. The theory predicts the presence of in-phase and out-of-phase collective oscillation modes of superconducting phases. The former corresponds to the Josephson plasma mode whose frequency is drastically reduced for $\pm$ s-wave symmetry, and the latter is a counterpart of Leggett's mode in Josephson junctions. We also reveal that the critical current and the Fraunhofer pattern strongly depend on the symmetry type of the two-gap superconductor.

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The Josephson effect is one of the most drastic phenomena in superconductivity [1]. Cooper pairs can tunnel through an insulating barrier in a nondissipative manner. This particular feature has attracted tremendous attention of not only physicists but also device engineers.

Very recently, multigap superconductors have been revisited since the discovery of an iron-based high-T_c superconductor [2–4]. In contrast to cuprate high-T_c superconductors, $3d$ electrons on the iron atom form multibands whose Cooper pairs condense into a multigap superconducting state. Angle resolved photoemission spectroscopy has reported that each of the multiple disconnected Fermi surfaces is fully gapped [5], and other experiments have also supported the gapful features [6]. On the contrary, nuclear magnetic resonance has shown typical gapless features [7].

In order to compromise the controversy, the presence of $\pm$ s-wave gaps on the disconnected Fermi surfaces has been proposed [8–10]. The essence of the $\pm$ s-wave symmetry is a sign change between different s-wave order parameters. This is expected to bring about novel behaviors in phase interference effects. In particular, Josephson effects in the superconductor-insulator-superconductor (SIS) junction between the single- and the $\pm$ s-wave multigap superconductors as schematically illustrated in Fig. 1 drastically reflect the sign change. In this Letter, we focus on such a heterotic SIS junction and clarify peculiar Josephson effects. We have three main results, i.e., the drastic reduction of (i) the Josephson plasma frequency, (ii) the critical current, and (iii) the Fraunhofer pattern visibility. The $\pm$ s-wave symmetry leads to a cancellation between the two Josephson currents which arise from the two tunneling channels in this system.

In the proposed junction as shown in Fig. 1, the left (right) electrode is a one- (two-) gap superconductor with the width s_L (s_R). The insulator width and the dielectric constant are d and ϵ, respectively. The current and the magnetic field are applied along the z and the y direction, respectively. Similar situations were also examined from other viewpoints [11].

The system's Hamiltonian $\hat{H} = \int_R d^3\boldsymbol{r}(\hat{\mathcal{H}}_R^{(1)} + \hat{\mathcal{H}}_R^{(2)} + \hat{\mathcal{H}}_R^{\rm pair}) + \int_L d^3\boldsymbol{r}(\hat{\mathcal{H}}_L^{s} + \hat{\mathcal{H}}_L^{\rm pair}) + \hat{H}_T$, where $\hat{\mathcal{H}}_R^{(i)}$ describes the kinetic energy of the ith band electrons $\hat{\psi}_\sigma^{(i)}$ in the right electrode. The pairing term in the right electrode $\hat{\mathcal{H}}_R^{\rm pair} = -g_1\hat{\psi}_\uparrow^{(1)\dagger}\hat{\psi}_\downarrow^{(1)\dagger}\hat{\psi}_\downarrow^{(1)}\hat{\psi}_\uparrow^{(1)} - g_2\hat{\psi}_\uparrow^{(2)\dagger}\hat{\psi}_\downarrow^{(2)\dagger}\hat{\psi}_\downarrow^{(2)}\hat{\psi}_\uparrow^{(2)} - g_{12}(\hat{\psi}_\uparrow^{(1)\dagger}\hat{\psi}_\downarrow^{(1)\dagger}\hat{\psi}_\downarrow^{(2)}\hat{\psi}_\uparrow^{(2)} + \text{H.c.})$ [12], where $g_i > 0$, and the interband interaction can be either attractive (i.e., $g_{12} > 0$) or repulsive (i.e., $g_{12} < 0$). The left electrode is described by $\hat{\mathcal{H}}_L^{s} + \hat{\mathcal{H}}_L^{\rm pair}$, where $\hat{\mathcal{H}}_L^{s}$ is the kinetic energy and $\hat{\mathcal{H}}_L^{\rm pair} = -g_s\hat{\psi}_\uparrow^{s\dagger}\hat{\psi}_\downarrow^{s\dagger}\hat{\psi}_\downarrow^{s}\hat{\psi}_\uparrow^{s}$, in which $g_s > 0$. The tunneling Hamiltonian $\hat{H}_T = \hat{H}_T^{(1)} + \hat{H}_T^{(2)}$, where $\hat{H}_T^{(i)}$ means the tunneling between the electrons in the left side and the ith band electrons in the right side. Using the imaginary time functional integral method [13,14], the effective action with respect to the order parameters $\Delta^{(i)}$ and Δ^{s} is given

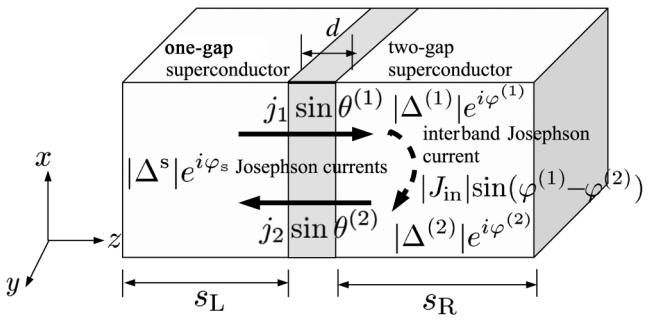

FIG. 1. A schematic figure of the present heterotic junction system. The left electrode is a one-gap superconductor, and the right electrode a two-gap superconductor.

by $S_{\rm eff} = \int_0^{\hbar\beta} d\tau[\int_R d^3\boldsymbol{r}(g_2|\Delta^{(1)}|^2/g + g_1|\Delta^{(2)}|^2/g + V_{\rm in}) + \int_L d^3\boldsymbol{r}|\Delta^s|^2/g_s] - {\rm Tr}\ln\hat{G}_0 - {\rm Tr}\ln\hat{G}^{-1}$, where β is the inverse temperature and $V_{\rm in} = -g_{12}(\Delta^{(1)*}\Delta^{(2)} + {\rm c.c.})/g$. We assume here that $g \equiv g_1 g_2 - g_{12}^2 > 0$ [12]. The Green functions for the noninteracting system $\hat{G}_0$ and the total system $\hat{G}$ are 6×6 matrices. We do not write their explicit expressions here, but those consist of 4×4 for the two-gap and 2×2 for the one-gap superconductors [13,14]. The interband Josephson coupling term $V_{\rm in}$ is rewritten as $V_{\rm in} = -2(g_{12}/g)|\Delta^{(1)}||\Delta^{(2)}|\cos(\varphi^{(1)} - \varphi^{(2)})$, in which $\Delta^{(i)} = |\Delta^{(i)}|e^{i\varphi^{(i)}}$.

Based on the standard procedure [13,14], the effective Lagrangian density of the superconducting phases on the zx plane in the real time formalism is given by

$$\mathcal{L}_{\rm eff} = \frac{s_L}{8\pi\mu^2}a_0^2 + \sum_{i=1}^{2}\frac{s_R}{8\pi\mu_{(i)2}}a_{0(i)}^2 - \frac{s_L}{8\pi\lambda^2}a_x^2 - \sum_{i=1}^{2}\frac{s_R}{8\pi\lambda^{(i)2}}a_{x(i)}^2 - V_J + \mathcal{L}_{\rm EM}, \quad (1)$$

where

$$V_J = -\frac{\hbar j_1}{e^*}\cos\theta^{(1)} - \frac{\hbar j_2}{e^*}\cos\theta^{(2)} - \frac{\hbar J_{\rm in}}{e^*}\cos\varphi, \quad (2)$$

$$\mathcal{L}_{\rm EM} = \frac{\epsilon d}{8\pi}E_{\rm RL}^{z2} - \frac{d}{8\pi}B_{\rm RL}^{y2}, \quad (3)$$

$$\theta^{(i)} = \varphi^{(i)} - \varphi_s - \frac{e^* d}{\hbar c}A_{\rm RL}^z, \quad (4)$$

$$\varphi = \varphi^{(1)} - \varphi^{(2)} = \theta^{(1)} - \theta^{(2)}, \quad (5)$$

and note that $a_0 = (\hbar/e^*)\partial_t\varphi_s + A_L^0$, $a_x = (\hbar c/e^*)\partial_x\varphi_s - A_L^x$, $a_{0(i)} = (\hbar/e^*)\partial_t\varphi^{(i)} + A_R^0$, $a_{x(i)} = (\hbar c/e^*)\partial_x\varphi^{(i)} - A_R^x$, and $e^* = 2e$. The phase φ_s is defined as $\Delta^s = |\Delta^s|e^{i\varphi_s}$, and j_i is the Josephson critical current between the ith and single-band Cooper pairs. The charge screening length and the penetration depth on the left (right) electrode are μ ($\mu^{(i)}$) and λ ($\lambda^{(i)}$), respectively. The last term in the gauge-invariant phase difference (4) is the z component of the spatial averaged vector potential in the insulator, defined as $A_{\rm RL}^z = d^{-1}\int_{-d/2}^{d/2}A^z(z)dz$. The electric and the magnetic fields in the insulator are defined as $E_{\rm RL}^z = -c^{-1}\partial_t A_{\rm RL}^z - d^{-1}(A_R^0 - A_L^0)$ and $B_{\rm RL}^y = d^{-1}(A_R^x - A_L^x) - \partial_x A_{\rm RL}^z$, respectively. Here let us focus on the Josephson coupling energy (2). The first and the second terms are the ordinary Josephson coupling terms, while the third term corresponds to the interband Josephson coupling energy and $|J_{\rm in}|$ is proportional to $|(g_{12}/g)s_R|$. One finds that $J_{\rm in}$ is positive (negative) if $g_{12} > 0$ ($g_{12} < 0$). If s_L and s_R are much larger than d, then it allows us to regard $|J_{\rm in}| \gg j_1, j_2$.

From Eq. (1), we have the Euler-Lagrange equations with respect to A_l^0 and A_l^x as follows:

$$\frac{\bar{\alpha}}{\alpha_1}\partial_t\theta^{(1)} + \frac{\bar{\alpha}}{\alpha_2}\partial_t\theta^{(2)} = C\frac{e^* d}{\hbar}E_{\rm RL}^z, \quad (6)$$

$$\frac{\bar{\eta}}{\eta_1}\partial_x\theta^{(1)} + \frac{\bar{\eta}}{\eta_2}\partial_x\theta^{(2)} = L\frac{e^* d}{\hbar c}B_{\rm RL}^y, \quad (7)$$

where the dimensionless parameters in each electrode are defined as $\alpha = \epsilon\mu^2/s_L d$, $\alpha_i = \epsilon\mu_i^2/s_R d$, $\bar{\alpha}^{-1} = \alpha_1^{-1} + \alpha_2^{-1}$, $\eta = \lambda^2/s_L d$, $\eta_i = \lambda^{(i)2}/s_R d$, and $\bar{\eta}^{-1} = \eta_1^{-1} + \eta_2^{-1}$. The magnitude of the electric (magnetic) field coupling is characterized by α and α_i (η and η_i) [15]. The constants C and L are defined as $C = 1 + \alpha + \bar{\alpha}$ and $L = 1 + \eta + \bar{\eta}$, respectively. Equations (6) and (7) correspond to the generalized Josephson relations [16]. The Euler-Lagrange equation with respect to $A_{\rm RL}^z$ gives the Maxwell equation

$$\frac{e^* d}{\hbar c}\partial_x B_{\rm RL}^y = \sum_{i=1}^{2}\frac{1}{\lambda_{Ji}^2}\sin\theta^{(i)} + \frac{\epsilon}{c^2}\frac{e^* d}{\hbar}\partial_t E_{\rm RL}^z, \quad (8)$$

where $\lambda_{Ji}^{-2} = 4\pi e^* d j_i/\hbar c^2$. The first term on the right-hand side of Eq. (8) is the summation of the Josephson current terms [Fig. 1]. Using Eqs. (6)–(8), we obtain

$$\sum_{i=1}^{2}\frac{C\bar{\eta}}{\eta_i}\partial_x^2\theta^{(i)} = \sum_{i=1}^{2}\frac{CL}{\lambda_{Ji}^2}\sin\theta^{(i)} + \sum_{i=1}^{2}\frac{L\bar{\alpha}}{\alpha_i}\partial_t^2\theta^{(i)}. \quad (9)$$

Next, from the Euler-Lagrange equations about φ_s and $\varphi^{(i)}$, we have

$$\frac{\epsilon}{c^2}\sum_{i=1}^{2}\left[(-1)^{i+1}\frac{CL}{\alpha_i}\partial_t^2\theta^{(i)} + \frac{(1+\alpha)\xi L}{\alpha_i}\partial_t^2\theta^{(i)}\right] = \sum_{i=1}^{2}\left[(-1)^{i+1}\frac{CL}{\eta_i}\partial_x^2\theta^{(i)} + \frac{(1+\eta)\zeta C}{\eta_i}\partial_x^2\theta^{(i)}\right] - 2CL\frac{4\pi e^* d}{\hbar c^2}\frac{\partial V_J}{\partial\varphi}, \quad (10)$$

where the parameter ξ (ζ) means the difference of the magnitude of the electric (magnetic) field coupling between the different superconducting bands as $\xi = (\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2)$ and $\zeta = (\eta_1 - \eta_2)/(\eta_1 + \eta_2)$.

The present description is valid from very thin electrode junctions ($s_L \sim \mu$ and $s_R \sim \mu^{(i)}$) to conventional thick ones. In the latter case ($s_L \gg \mu$ and $s_R \gg \mu^{(i)}$), we can take an approximate treatment: $\alpha \to 0$ and $\alpha_i \to 0$. Remark that the present Letter highlight, i.e., particular features due to the $\pm$ s wave, is unchanged in this limit.

Now let us examine the collective modes involved in Eqs. (9) and (10). For this purpose, we linearize them around a stable point of V. First, we focus on $J_{\rm in} > 0$. Then a stable point for $(\theta^{(1)}, \theta^{(2)})$ is $(0, 0)$, and the dispersion relations $\omega_\pm^2(k_x)$ is given by

$$\omega_\pm^2(J_{\rm in} > 0) = \frac{X(\omega_P^2 + \omega_L^2) - 2\bar{\alpha}\xi C^{-1}D \pm \sqrt{R}}{2(1-\xi^2)}, \quad (11)$$

where

$$\omega_P^2 = C(\omega_{p1}^2 + \omega_{p2}^2)[1 + L^{-1}(k_x/K)^2],$$
$$\omega_L^2 = \bar{\alpha}X^{-1}\{4\nu_{\rm in}^2 + (\omega_{p1}^2 + \omega_{p2}^2)[1 + \bar{\eta}^{-1}Y(k_x/K)^2]\}.$$

The Josephson plasma frequency associated with the Josephson current for $\theta^{(i)}$, $\omega_{pi} = c/\sqrt{\epsilon}\lambda_{Ji}$, while the pseudo-Josephson-plasma frequency associated with the interband Josephson current $\nu_{\rm in} = \sqrt{4\pi e^* d|J_{\rm in}|/\epsilon\hbar}$. Note that $K^2 = c^{-2}\epsilon(\omega_{p1}^2 + \omega_{p2}^2)$, the dimensionless parameters X and Y are defined as $X = 1 - \xi^2 C^{-1}(1 + \alpha)$ and $Y = 1 - \zeta^2 L^{-1}(1 + \eta)$, respectively, and the quantities D and R are, respectively, $D = C(\omega_{p1}^2 + \omega_{p2}^2) \times [-\delta + \zeta L^{-1}(k_x/K)^2]$ and $R = X\{(1 - \xi^2)(\omega_P^2 - \omega_L^2)^2 + 4\bar{\alpha}C^{-1}[D - \xi(\omega_P^2 + \omega_L^2)/2]^2\}$, where $\delta = (j_1 - j_2)/(j_1 + j_2)$. When $\xi = \zeta = \delta = 0$ (i.e., the superconducting characters are perfectly equivalent between the two bands), we find that $\omega_+ = \omega_P$ and $\omega_- = \omega_L$. We then notice that no term related to the interband Josephson coupling is involved in the expression of ω_P. It indicates that the origin of ω_P is irrelevant to the motion of the relative phase φ. Then ω_+ corresponds to the in-phase motion for $\theta^{(1)}$ and $\theta^{(2)}$. On the other hand, the origin of ω_- is the out-of-phase motion for $\theta^{(1)}$ and $\theta^{(2)}$. Next, we study another case of $J_{\rm in} < 0$, in which $(\pi, 0)$ is a stable point since $|J_{\rm in}| \gg j_1, j_2$. Expanding V around $(\pi, 0)$, we have

$$\omega_\pm^2(J_{\rm in} < 0) = \frac{X(\omega_P'^2 + \omega_L'^2) - 2\bar{\alpha}\xi C^{-1}D' \pm \sqrt{R'}}{2(1 - \xi^2)}, \quad (12)$$

where

$$\omega_P'^2 = C(\omega_{p1}^2 + \omega_{p2}^2)[|\delta| + L^{-1}(k_x/K)^2],$$
$$\omega_L'^2 = \bar{\alpha}X^{-1}[4\nu_{\rm in}^2 + (\omega_{p1}^2 + \omega_{p2}^2)(|\delta| + \bar{\eta}^{-1}Y(k_x/K)^2].$$

The quantities D' and R' are $D' = C(\omega_{p1}^2 + \omega_{p2}^2) \times [1 + \zeta L^{-1}(k_x/K)^2]$ and $R' = X\{(1 - \xi^2)(\omega_P'^2 - \omega_L'^2)^2 + 4\bar{\alpha}C^{-1}[D' - \xi(\omega_P'^2 - \omega_L'^2)/2]^2\}$, respectively. Figures 2(a) and 2(b) show the typical dispersion relations for $J_{\rm in} > 0$ and $J_{\rm in} < 0$, respectively. For both cases, the frequency of the out-of-phase mode ω_- is found to be lower than the in-phase mode ω_+ for an arbitrary value of k. Here we take the limit $k_x \to 0$ in Eqs. (11) and (12) to explicitly evaluate the gap frequency for these modes. As for $J_{\rm in} > 0$, the leading order terms are, respectively, given as $\omega_+ \simeq (\omega_{p1}^2 + \omega_{p2}^2)^{1/2}$ and $\omega_- \simeq (\alpha_1 + \alpha_2)^{1/2}\nu_{\rm in}$ by regarding $\bar{\alpha}$ and α to be small. However, remark that we keep the term $\bar{\alpha}\nu_{\rm in}^2(\omega_{p1}^2 + \omega_{p2}^2)^{-1}$ in the above evaluation, because $|J_{\rm in}| > j_1, j_2$ even though $\bar{\alpha}$ is small. Similarly, when $J_{\rm in} < 0$, we have $\omega_+ \simeq |\omega_{p1}^2 - \omega_{p2}^2|^{1/2}$ and $\omega_- \simeq (\alpha_1 + \alpha_2)^{1/2}\nu_{\rm in}$. The gap of ω_+ is characterized by a superposition ($J_{\rm in} > 0$) or subtraction ($J_{\rm in} < 0$) between ω_{p1} and ω_{p2}. Thus, we refer ω_+ to the Josephson plasma mode. We emphasize that the signature of the $\pm$ s wave is the reduction of plasma frequency [Fig. 2]. On the other hand, since the gap of ω_- is characterized by $\nu_{\rm in}$ and α_i, we find

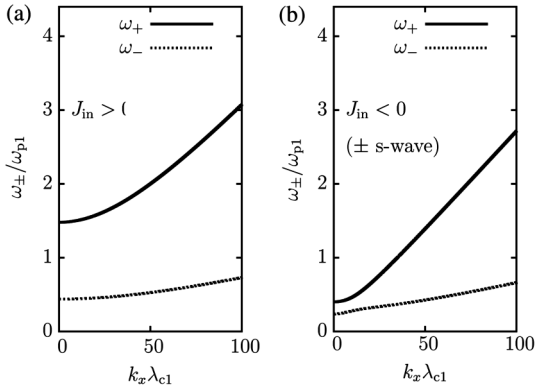

FIG. 2. The dispersion relations for $\omega_\pm$. The solid (dotted) line is for ω_+ (ω_-). The parameters are set as follows: $\alpha = \alpha_1 = 10^{-3}$, $\alpha_2/\alpha_1 = 1.4$, $\eta = \eta_1 = 10^3$, $\eta_2/\eta_1 = 1.4$, $j_2/j_1 = 0.9$, and $|J_{\rm in}|/j_1 = 40.0$. The reduction of ω_+ is observed for $J_{\rm in} < 0$. (a) $J_{\rm in} > 0$. (b) $J_{\rm in} < 0$.

that it corresponds to the gap of the Leggett's mode, which was derived as a collective mode generated by the density fluctuation between two superfluidities [17]. Thus, it should be called the Josephson-Leggett mode. The interband Josephson coupling and the charge density fluctuation create the mode. Conventionally, the first and the second terms in Eq. (1) are fixed to be zero ($\alpha_i \to 0$) because the charge screening length is much smaller than the electrode size. Then the effective Lagrangian density gives the standard Josephson relation $\partial_t\theta^{(i)} = (e^* d/\hbar)E_{\rm RL}^z$, resulting in $\partial_t(\theta^{(1)} - \theta^{(2)}) = 0$. It means that the Josephson-Leggett mode becomes a gapless mode. In contrast, the mode ω_P can still remain massive in $\alpha_i \to 0$. The retainment of nonzero α_i is responsible for the finite gap frequency of the Josephson-Leggett mode. The bulk Leggett's mode is normally embedded in the quasiparticle excitation continuum [18], while the Josephson-Leggett mode is more clearly and easily observable because the mode lies far beneath the gap energy.

The remaining part of this Letter is devoted to basic Josephson effects. First, let us discuss the Josephson critical current j_c. The bias current is assumed to be uniformly applied without the external magnetic field. Namely, $\theta^{(1)}$ and $\theta^{(2)}$ are assumed to be uniform along the x axis in Fig. 1. The bias current I is added to the right-hand side of Eq. (9) with the elimination of the spatial dependent terms, and j_c can be derived by estimating the maximum threshold of I under keeping a stationary solution. The condition is given by

$$I = j_1 \sin\theta^{(1)} + j_2 \sin\theta^{(2)}, \quad (13)$$

$$0 = -j_1 \sin\theta^{(1)} + j_2 \sin\theta^{(2)} - J_{\rm in} \sin\varphi. \quad (14)$$

Equation (13) means that I coincides with the sum of two Josephson currents between the electrodes, while Eq. (14) is an internal current conservation law, which gives a significant constraint on the critical current. When $J_{\rm in} > 0$, the preferable choice of φ is 0. Equation (14) implies

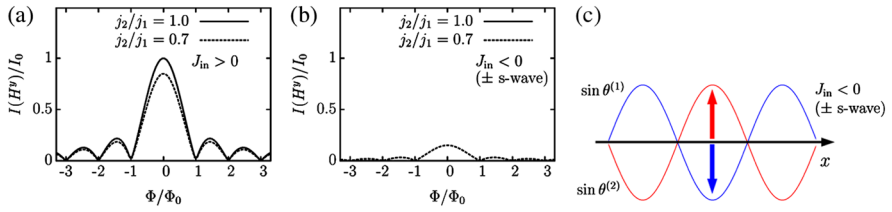

FIG. 3 (color online). The current vs the magnetic flux. The value of I_0 is $I(H^y = 0)$ for $\varphi = 0$. (a) $J_{in} > 0$ and $\varphi = 0$. (b) $J_{in} < 0$ and $\varphi = \pi$. No current is observed when $j_1 = j_2$. (c) The demonstration of the cancellation between two Josephson currents.

$(j_1 - j_2)\sin\theta^{(1)} = 0$, because $\theta^{(1)} = \theta^{(2)}$. This is always satisfied if $j_1 = j_2$. Then, since $\theta^{(1)}$ can vary from 0 to 2π, $j_c = j_1 + j_2$. If $j_1 \neq j_2$, then φ can deviate from 0 and $j_c \leq j_1 + j_2$. On the other hand, when $J_{in} < 0$, φ should be π. Equation (14) implies $(j_1 + j_2)\sin\theta^{(1)} = 0$. The only possible solution is $\theta^{(1)} = 0$ and $\theta^{(2)} = -\pi$, because $j_1 + j_2 \neq 0$. Thus, we find that the value of j_c is drastically reduced compared to the case of $J_{in} > 0$, e.g., $j_c = 0$ for the case of perfectly identical $\pm$ s-wave two-gap superconductivity.

Next, we consider the Josephson effects in the presence of the external magnetic field H^y. We focus on stationary solutions; i.e., we drop the temporal terms of $\theta^{(i)}$. According to Eq. (7), $\theta^{(1)}(x) = kx + \theta_0 + (\bar{\eta}/\eta_2)\varphi(x)$ and $\theta^{(2)}(x) = kx + \theta_0 - (\bar{\eta}/\eta_1)\varphi(x)$, where $k = L(e^*d/\hbar c)H^y$ and $\theta_0 \in [0, 2\pi)$ is an integral constant. The observed current is then given by $I(H^y, \theta_0) = \int_{-L_x/2}^{L_x/2}[j_1\sin\theta^{(1)}(x) + j_2\sin\theta^{(2)}(x)]dx$. Hereafter, we assume that $\varphi(x)$ is spatially uniform. Taking account of $|J_{in}| > j_1, j_2$, we should have $0 \approx -J_{in}\sin\varphi$ from Eq. (10). When $J_{in} > 0$ (i.e., $\varphi = 0$), the magnetic field dependence of the current is given by $I(H^y, \theta_0) = L_x\sin\theta_0[(j_1 + j_2) \times (\Phi_0/\pi\Phi)\sin(\pi\Phi/\Phi_0)]$, where $\Phi_0 = 2\pi\hbar c/e^*$ and $\Phi = LH^y dL_x$. Then $I(H^y) \equiv \max_{\theta_0}|I(H^y, \theta_0)| = L_x(j_1 + j_2)|(\Phi_0/\pi\Phi)\sin(\pi\Phi/\Phi_0)|$. As a result, we obtain the ordinary Fraunhofer diffraction pattern as a function of the magnetic flux Φ [Fig. 3(a)]. The maximum value of $I(H^y)$ is the sum of two Josephson currents as $I(0) = L_x(j_1 + j_2)$. We also observe that the net current conventionally vanishes when $\Phi = \kappa\Phi_0$ ($\kappa \in \mathbb{N}$). In contrast, as for $J_{in} < 0$ (i.e., $\pm$ s wave), the current is given by $I(H^y) = L_x|j_1 - j_2||(\Phi_0/\pi\Phi)\sin(\pi\Phi/\Phi_0)|$. If $j_1 = j_2$, the Fraunhofer diffraction pattern completely disappears. When $j_1 \neq j_2$, the pattern is observable except for $\Phi = \kappa\Phi_0$, but the maximum value becomes unexpectedly small [Fig. 3(b)]. The situation at $j_1 = j_2$ is schematically displayed in Fig. 3(c). The Josephson currents for $\theta^{(1)}$ and $\theta^{(2)}$ cancel out each other.

Finally, let us discuss how to experimentally confirm the theoretical predictions. We point out that the maximum Josephson current can be estimated from the normal state resistance based on the Ambegaokar-Baratoff relation [13] under an assumption $J_{in} > 0$. If the measured j_c is significantly reduced from the one estimated above, then $J_{in} < 0$, i.e., $\pm$ s-wave symmetry is concluded.

In summary, we microscopically derived an effective Lagrangian density of the SIS Josephson junction between one- and two-gap superconductors and examined the collective modes, the critical current, and the Fraunhofer pattern. We found that these properties are considerably affected by the type of the pairing symmetry of the two-gap superconductor. We conclude that the heterotic junction is useful to identify directly a symmetry of two-gap superconductors.

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