Laser Beam Self-Focusing in the Atmosphere

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We propose to exploit a self-focusing effect in the atmosphere to assist delivering powerful laser beams from orbit to the ground. We demonstrate through numerical modeling that when the self-focusing length is comparable with the atmosphere height the spot size on the ground can be reduced well below the diffraction limits without beam quality degradation. The density variation suppresses beam filamentation and provides the self-focusing of the beam as a whole. The use of light self-focusing in the atmosphere can greatly relax the requirements for the orbital optics and ground receivers.

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The abundance of solar energy outside Earth makes electricity production in space attractive. Harnessing and accumulation of solar energy at orbit stations for further wireless power transportation to Earth is one of the global concepts of renewable energy sources [1,2]. The accumulated solar power can be used to produce focussed electromagnetic beams of either microwave waves or laser radiation. Despite many attractive features and the first scientific developments, [1–4] this technology is far from being practical with many technical and conceptual issues yet to be resolved before it becomes a real competitor to the existing power technologies. Initial proposals have been based on energy transportation by microwaves. However, recent progress in laser science has stimulated research into the feasibility of using laser-based orbit systems in which laser radiation is utilized for transport of converted solar energy to the ground. The use of lasers instead of microwaves would allow greatly reduced transmitter and receiver sizes. In this Letter we focus on one of the key problems of reducing the size of the transmission and receiving facilities. Indeed, even for a diffraction limited beam, large precise focusing optics in space and a large receiving facility on the ground would be required. We propose a general approach that reduces these size requirements by exploiting a self-focusing effect to deliver pulsed powerful laser beams to the ground. The idea to use self-focused beams for energy transportation was suggested just after the discovery of steady state channels where the self-focusing is compensated by diffraction [5]. Soon after it was shown that steady state propagation is always unstable [6]. For beams with power over a threshold critical power, filamentation, collapse and uncontrolled ionization result in breaking of the beams. However, the situation can be different for propagation from orbit through Earth's inhomogeneous atmosphere. We demonstrate that when the self-focusing length is comparable with the atmosphere height, the catastrophic self-focusing can be greatly suppressed and a smooth compression of the whole beam is possible. Numerical modeling shows the controlled beam compression is well below the diffraction limited spot size permitting size reductions in receivers on the ground and focusing optics in space.

To illustrate the idea, without loss of generality, consider vertical laser beam propagation. The full analysis based on using a detailed model taking into account all important physical aspects affecting beam propagation will be presented elsewhere, but here we focus only on the key physical effects that are relevant to the proposed concept. The key features of the beam evolution (diffraction and Kerr nonlinearity) can be described using the standard paraxial approximation for the envelope of the electric field—the nonlinear Schrödinger equation:

$$i\frac{\partial A}{\partial z} + \frac{1}{2n_0k_0}\Delta_\perp A + n_2k_0|A|^2A = 0. \qquad (1)$$

Here z is the direction of propagation, with $z = 0$ the sea level, and the propagation from the orbit to the ground. The initial condition for this Cauchy problem, beam profile and phase waveform at the height of the orbit, $z = F$ is assumed to be,

$$A(z = F, r) = \sqrt{\frac{P}{\pi R_0^2}}\exp\left[-\frac{(1 + iC_0)}{2R_0^2}r^2\right].$$

Here $D = 2R_0$ is the mirror diameter, $C_0 = k_0R_0^2/F$, and F is the focal distance or height of the orbit. In the limit $C_0 \gg 1$, when the focal spot is much smaller then the focusing mirror the beam has a Gaussian nonfocused shape on the ground with the radius

$$R_{\min} = R_{\text{ground}} = \frac{\lambda_0 F}{\pi D}. \qquad (2)$$

Typically, when the Rayleigh length is comparable to or longer than the atmosphere height, the beam size entering the atmosphere R_{atm} is close to its footprint on the ground $R_{\text{atm}} \approx R_{\text{ground}}$. Expression (2) relates the focusing mirror size laser wavelength, laser footprint on the ground R_{ground} and the orbit height F. It is seen from Eq. (2) that the beam spot size at the ground is inversely proportional to the orbital mirror diameter imposing some requirements on the orbital mirror size. Evidently, technical design issues related to the orbit mirror size depend on many factors and it is likely that different solutions for reduction of its scale can be proposed. Here we will show that under certain conditions a nonlinear lens effect provided by the self-focusing might reduce the beam footprint on the ground allowing for a larger initial size of a beam entering atmosphere $R_{\rm atm}$, and consequently smaller and lighter transmitter mirrors could be used in orbit.

It should be noted that the linear theory textbook formula (2) is valid only when the focal spot $R_{\min}$ is much smaller than the mirror radius R_0. In our case of long distance focusing, $R_{\min}$ can be comparable with R_0 and this must be taken into account. In the general case, the minimal beam waist is modified to be,

$$R'_{\min} = \frac{R_{\min}}{\sqrt{1 + (\frac{R_{\min}}{R_0})^2}} = \frac{\lambda_0 F/(\pi D)}{\sqrt{1 + (\frac{\lambda_0 F/\pi D}{R_0})^2}} \qquad (3)$$

and the location of the minimum (waist) is shifted toward the mirror with a new focus at

$$F' = \frac{F}{1 + (\frac{R_{\min}}{R_0})^2} = \frac{F}{1 + (\frac{\lambda_0 F/\pi D}{R_0})^2}. \qquad (4)$$

It is easily seen that in the linear theory (3) and (4) under conditions of maximal compression at the ground, the radius of the mirror R_0 cannot be less than a certain minimal value given by $R_0 \geq R_0^{(1)} = \sqrt{L\lambda_0/\pi}$. The minimum possible mirror radius corresponds to $F = 2L$ (and $C_0 = 1$). In physical terms, this means that at small mirror size the beam cannot be focused at all, because F in this case becomes comparable with the Rayleigh length. Note also that a linear compression factor of the laser beam with such a "minimal-diameter orbital mirror" is limited by the square root of 2: $R_{\min}^{(1)} = R_0^{(1)}/\sqrt{2} = \sqrt{L\lambda_0/(2\pi)}$.

The nonlinearity in (1) is a function of altitude. The nonlinear refractive index is proportional to density which can be interpolated as exponential (isothermal atmosphere) and the height $h \sim 6$ km. Introducing the density at sea level ρ_0, the corresponding nonlinear refractive index as a function of z is,

$$n_2(z) = n_2(0)\frac{\rho}{\rho_0}; \qquad \rho = \rho_0 e^{-(z/h)}.$$

Here the refractive index on the ground is $n_2(0) = 5.6 \times 10^{-19}$ cm^2/W. Note that at large distances from the ground the contribution of the nonlinear term in (1) is negligible, but it becomes more and more important when the beams approach sea level.

Self-focusing in homogeneous media starts when the beam power exceeds the critical value, $P_{\rm cr} = 11.68\lambda_0^2/(8\pi^2 n_0 n_2) = 0.93\lambda_0^2/(2\pi n_0 n_2)$. At $z = h = 6$ km, $P_{\rm cr} = 4.6$ GW for 0.8 μm light. In homogeneous media the beam experiences some compression even at power levels below $P_{\rm cr}$, above approximately $P \approx 0.7P_{\rm cr}$. Gaussian beams compress as a whole up to a radius of $\sim$0.5 of the diffraction limited one. At higher powers the central filament becomes important, forming a singularity when P approaches the critical value [7]. When P is above the critical value, the beam collapses to a singularity at a distance L, which for the unfocused beam with a radius is given by, $L \sim \frac{ka^2}{\sqrt{P/P_{\rm cr}-1}}$. For powers well above $P_{\rm cr}$, the beam breaks up in multiple filaments, each filament having $P \sim P_{\rm cr}$ and collapsing independently of the other filaments. In practical terms, compression up to 0.5 of the diffraction limited spot at $P \leq 0.7P_{\rm cr}$ is possible without strict restriction on the beam parameter's stability [8].

In what follows, it is convenient to use as a reference the $P_{\rm cr}$ at $z = h$. For P well below $P_{\rm cr}$, the self-focusing length exceeds the atmosphere thickness and the propagation is linear. When L is much smaller than h, self-focusing of the beam takes place in what is an essentially homogeneous media. For a beam diameter of $\sim$10 cm and $P \cong P_{\rm cr}$, the self-focusing length is about the scale of the density variation of the atmosphere. In this case one can expect that the density variation will suppress the beam filamentation and provide the self-focusing of the beam as a whole. Below we present the results of numerical modeling (1) supporting these arguments.

Consider energy transport from a low earth orbit in which a Gaussian beam is focused by a mirror and propagated to the ground from a height of 500 km in all modeling below. The two key parameters we vary in our simulations are the mirror diameter and the beam power. It was demonstrated in early studies of self-focusing that the filamentation threshold in the axis-symmetric case (formation of the ring structures) takes place at approximately the same place as filamentation within the complete model of (1). Therefore, without loss of generality, we consider the axis-symmetric problem to prove the concept. For $P \sim P_{\rm cr}$ the variation of atmospheric refractive index starts to affect the beam propagation below approximately 20 km. Our modeling results clearly demonstrate that in an inhomogeneous medium up to some power level the compression of the whole beam is possible without splitting up into multiple sub-beams. Figure 1 shows the intensity distribution on the ground for a mirror radius $R_0 = 1$ m and for several beam powers. One can observe a strong (factor of 5) beam compression in comparison with linear propagation without any indication of filamentation.

The compressed beam radius at the ground (radius at half maximum intensity) as a function of initial power for different mirror radii is presented in Fig. 2. It is seen that for the $R_0 = 1$ m mirror up to $P/P_{\rm cr} \sim 2$ the compression is not very sensitive to power variations. The beam experiences self-focusing with an average radial compression of 13.95 from 10.6 cm to 0.76 cm at $P/P_{\rm cr} = 2$. Further power increases result in a beam filamentation and splitting near the ground. We note that the results of calculations shown here are robust, being nonsensitive to the numerical

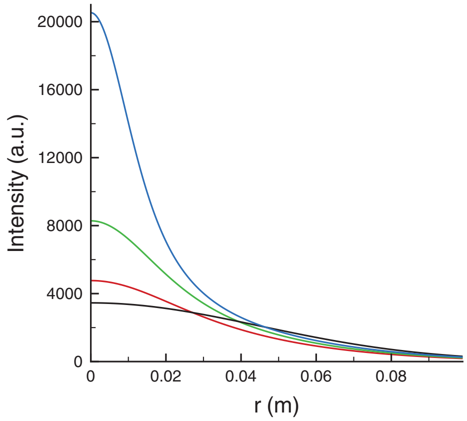

FIG. 1 (color). Beam intensity distribution on the ground for $R_0 = 1$ m, $C_0 = 15.7$: $P/P_{\rm cr} = 1.0$ (red line), $P/P_{\rm cr} = 1.29$ (green line), $P/P_{\rm cr} = 1.66$ (blue line). The black line corresponds to the linear diffraction.

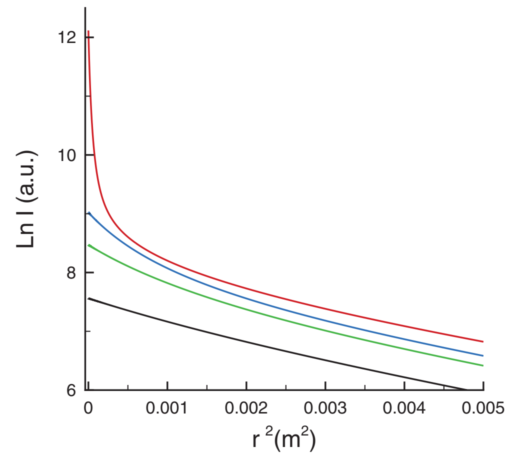

FIG. 3 (color). Logarithm of the field intensity $\ln I$ versus r^2 for $R_0 = 1$ m, $C_0 = 15.7$: $P/P_{\rm cr} = 0.55$ (black line), $P/P_{\rm cr} = 1.0$ (green line), $P/P_{\rm cr} = 1.29$ (blue line), $P/P_{\rm cr} = 2.0$ (red line).

noise and small variations of initial conditions. For the case of $R_0 = 0.5$ m filamentation can be suppressed for powers up to $P/P_{\rm cr} \sim 12$. For $R_0 = 0.357$ m filamentation starts with powers as high as $\frac{P}{P_{\rm cr}} = 134.6$. Note that at $P/P_{\rm cr} \sim 2$ the value of a beam power at the ground measured in units of critical power at the ground is $P/P_{\rm cr}(0) \sim 5.5$ for $R_0 = 1$ m indicating the self-focusing suppression by density inhomogeneity, the central result of this investigation. The effect is even more pronounced for $R_0 = 0.357$ m where the beam power at the ground measured in units of critical power at the ground is as high as $\frac{P}{P_{\rm cr}(0)} = 365.9$.

Figure 3 shows a beam structure near the ground in a logarithmic plot of the field intensity as a function of the square of a transversal coordinate radius—so that a Gaussian beam is presented here by a descending straight line. It is seen that up to some power level a laser beam is focused preserving its Gaussian shape. However, at higher powers the distribution acquires two distinctive scales: the central peak and the Gaussian background, with the central core containing most of the beam energy. Figure 4 presents results of modeling of the beam average radius evolution in the atmosphere for different mirror radii. For larger mirror radii the size of the beam entering the atmosphere is smaller. As the self-focusing length is approximately proportional to the square of the input beam radius, for smaller spot size beams the self-focusing length is short and the beam might collapse before it reaches the ground. Of course, physical singularity formation is stopped by higher order effects not included in Eq. (1). For the small mirrors, the beam size entering atmosphere is large and the characteristic self-focusing length might be longer than the atmospheric propagation distance and so higher power is required to compress the beam on the ground. In this case one can observe noticeable beam compression without loss of beam quality.

Note that for a fixed orbit height F the system design parameters are the beam power and focusing mirror diameter D. Using results mentioned above we can relate the self-focusing length L, the orbit height and the mirror diameter to the input beam power corresponding to the maximal compression without filamentation. The condi-

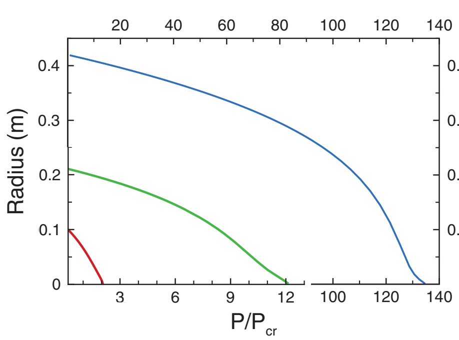

FIG. 2 (color). The beam radius at the ground versus power for the mirror radius. $R_0 = 1$ m, $C_0 = 15.7$ (red line) and $R_0 = 0.5$ m, $C_0 = 3.9$ (green line) and $R_0 = 0.357$ m, $C_0 = 1$ (blue line).

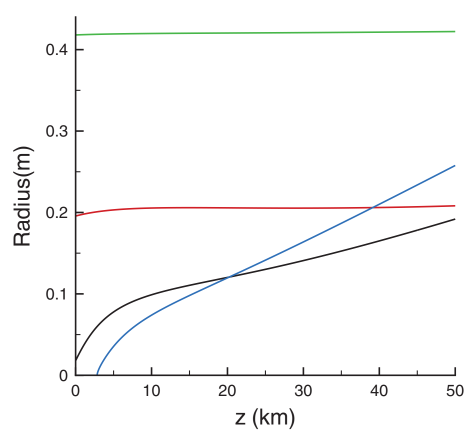

FIG. 4 (color). The beam radius versus distance z for mirror radius $R_0 = 1.5$ m, $C_0 = 35.3$ (blue line) and $R_0 = 1$ m, $C_0 = 15.7$ (black line) and $R_0 = 0.5$ m, $C_0 = 3.93$ (red line) and $R_0 = 0.357$ m, $C_0 = 1$ (green line).

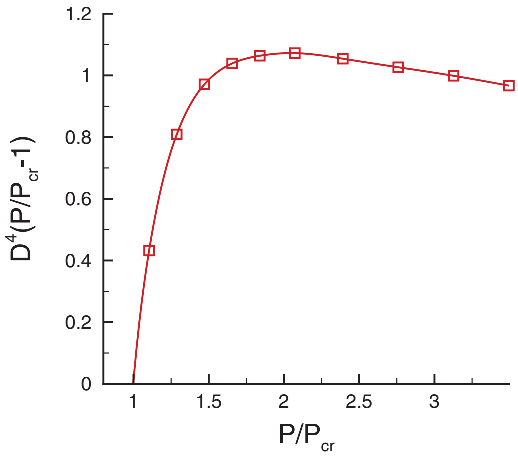

FIG. 5 (color). The dependence $D^4(\frac{P}{P_{cr}} - 1)$ on power P/P_{cr} required to start the filamentation.

tion $L \cong h$ leads to the following relation:

$$\frac{P}{P_{cr}} - 1 \sim \left(\frac{\lambda F^2}{hD^2}\right)^2. \tag{5}$$

Thus, the power is related to the mirror diameter as,

$$D^4\left(\frac{P}{P_{cr}} - 1\right) = \text{const.} \tag{6}$$

Equation (6) presents an effective design rule for maximal compression without filamentation of the transported beam. The results of the numerical modeling presented in Fig. 5 confirm that apart from the small region corresponding to powers close to P_{cr} Eq. (6) holds and can be used in an estimate of the beam or system parameters that allow for laser power transportation without beam splitting and filamentation. Two effects limit the applicability of the scaling relation (5). For large mirror radii, the Raleigh length becomes comparable with atmosphere height (for $R >$ 10 cm), so the estimates for beam parameters entering the atmosphere must be modified. For small mirror sizes, the footprint on the ground can be comparable with the mirror radius and beam parameters must be modified accordingly (3) and (4).

Let us discuss the applicability of our modeling. To practically achieve power above the critical one, we have to use the pulsed lasers. The Eq. (1) is valid when the pulse duration is not too short and one can neglect the group velocity dispersion term. For this the propagation length (atmospheric height) must be shorter then the dispersive length. For propagation in air dispersion the coefficient is about 0.02 ps^2/km and for atmosphere height 6 km the stationary model (1) is applicable for a laser pulse much longer then 0.4 ps. The laser design becomes much more simple, pulse energy higher and laser much lighter, if the pulse duration is longer then 10 ps. This justifies the choice of our model.

For such high power regimes, scattering by atmospheric turbulence, beam aberrations and air breakdown are likely to be much more important than in low power situations. In this Letter we present a concept rather than a thorough examination of all details describing powerful laser beam propagation from orbit to ground through the inhomogeneous terrestrial atmosphere. Detailed analysis will be presented elsewhere. Our preliminary studies demonstrate that all of these effects produce only minimal restriction on the parameters of the laser system. This is, mainly, due to geometry of the considered problem. All atmospheric effects take place near the surface when the scattered light has no time to deviate significantly from the undisturbed trajectory. This situation is quite different from the commonly considered case of when the laser beam propagates from the ground to space, in which case, even a small angular scattering in the beginning of the pass produces a large distortion at the focal spot in space.

The key result presented in this Letter is that the beam footprints on the ground can be made small without running into filamentation and nonlinear beam breakup. This expands the possibilities for power collection on the ground. As an example, one can consider the concept of a power sphere [9]; an integrating sphere which efficiently captures and converts the laser beam to electricity.

We have demonstrated that self-focusing in atmosphere can substantially compress the laser beam without beam quality degradation. The atmosphere density variation suppresses the filamentation process and makes possible the nonlinear lens-assisted delivery of the high quality laser beam to a small spot size on the ground.

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