Chiral Primordial Gravitational Waves from a Lifshitz Point

Tomohiro Takahashi and Jiro Soda
Department of Physics, Kyoto University, Kyoto, 606-8501, Japan
(Received 11 April 2009; published 11 June 2009)

We study primordial gravitational waves produced during inflation in quantum gravity at a Lifshitz point proposed by Hořava. Assuming power-counting renormalizability, foliation-preserving diffeomorphism invariance, and the condition of detailed balance, we show that primordial gravitational waves are circularly polarized due to parity violation. The chirality of primordial gravitational waves is a quite robust prediction of quantum gravity at a Lifshitz point which can be tested through observations of cosmic microwave background radiation and stochastic gravitational waves.

DOI: 10.1103/PhysRevLett.102.231301 PACS numbers: 98.80.Cq, 04.30.Db, 04.60.Nc

Introduction.—For a long time, the inflationary scenario has been regarded as an academic theory which provides an elegant solution to conceptual problems in cosmology such as the flatness problem, the horizon problem, and the origin of structure of the Universe. However, the fact that all recent cosmological observations strongly support the inflationary scenario with high precision encourages us to take inflation more seriously. Then, by taking into account that inflation magnifies microscopic scales to macroscopic ones, it is reasonable to regard inflation as a probe to investigate physics at the Planck scale, namely, quantum gravity.

It is widely believed that string theory is a promising candidate for quantum gravity. However, it is premature to discuss a Planckian regime of the Universe using string theory. Therefore, so far, study of the trans-Planckian effect on inflationary predictions has been phenomenological [1,2]. In fact, there are various phenomenological models which can mimic trans-Planckian physics and lead to a modification of the power spectrum of curvature perturbations [3]. It is known that these quantitative trans-Planckian corrections suffer from severe constraints due to the back-reaction problem [4,5]. However, there may be more qualitative effects due to trans-Planckian physics. For example, polarization of primordial gravitational waves could be an important smoking gun of trans-Planckian physics [6,7]. In fact, a parity-violating gravitational Chern-Simons term which is ubiquitous in string theory can generate circular polarization in primordial gravitational waves [8–10]. However, it has been shown that the effect of parity violation is negligibly small for slow roll inflation [11]. Recently, it is argued that sizable circular polarization could be generated [12,13] by resorting to a peculiar feature due to the Gauss-Bonnet term [14]. One defect in these models is the appearance of divergence in one of the circular polarization modes. This divergence suggests the necessity of a consistent quantum theory of gravity.

Recently, quantum gravity at a Lifshitz point which is power-counting renormalizable was proposed by Hořava [15,16]. In contrast to string theory, the theory is not intended to be a unified theory but just quantum gravity in 4 dimensions. In this "small" framework, one can discuss the trans-Planckian effect on cosmology in a self-consistent manner. In Hořava's formulation of quantum gravity, the action necessarily contains a Cotton tensor, which violates parity invariance. Hence, we can expect circular polarization of primordial gravitational waves. Moreover, there exists no divergence in this model. Then the purpose of this Letter is to calculate the degree of circular polarization during inflation and show observability of chiral primordial gravitational waves which is a robust prediction of quantum gravity at a Lifshitz point.

Quantum gravity at a Lifshitz point.—The quantum gravity proposed by Hořava can be characterized by anisotropic scaling at an ultraviolet fixed point $\mathbf{x} \to b\mathbf{x}$, $t \to b^3 t$, where b, $\mathbf{x}$, and t are a scaling factor, spatial coordinates, and a time coordinate, respectively. This scaling guarantees the renormalizability of the theory [16]. Because of the anisotropic scaling, the time direction plays a privileged role. In other words, the spacetime has a codimension-one foliation structure in which leaves of the foliation are hypersurfaces of constant time. Since the spacetime has the anisotropic scaling and the foliation structure, the theory is not diffeomorphism invariant but invariant under the foliation-preserving diffeomorphism defined by $\tilde{x}^i = \tilde{x}^i(x^j, t)$, $\tilde{t} = \tilde{t}(t)$. Here, indices $i, j, k, \ldots$ represent spatial coordinates.

To describe the foliation, it is convenient to use Arnowitt-Deser-Misner decomposition of the metric $ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$, where N, N_i, and g_{ij} are the lapse function, the shift function, and the three-dimensional induced metric, respectively. In order for the theory to be unitary, the number of time derivatives should be at most two in the action. The renormalizable kinetic part is then given by

$$S_K = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N (K^{ij} K_{ij} - \lambda K^2), \qquad (1)$$

where K_{ij} is the extrinsic curvature of the constant time hypersurface defined by $K_{ij} = (\dot{g}_{ij} - N_{i|j} - N_{j|i})/2N$ and

K is the trace part of K_{ij}. Note that κ and λ are dimensionless coupling constants which run according to the renormalization group flow. Hereafter, we assume λ has already settled down to the infrared fixed point $\lambda = 1$ at the beginning of inflation although we keep λ in subsequent formulas.

The most crucial assumption of Hořava's theory is the detailed balance condition

$$S_V = \frac{\kappa^2}{8} \int dt d^3\mathbf{x}\sqrt{g} N E^{ij} \mathcal{G}_{ijkl} E^{kl}, \tag{2}$$

where we have defined $\sqrt{g} E^{ij} = \delta W[g_{ij}]/\delta g_{ij}$ with some functional W. Here we introduced the inverse of De Witt metric $\mathcal{G}^{ijkl} = (g^{ik}g^{jl} + g^{il}g^{jk})/2 - \lambda g^{ij}g^{kl}$. The renormalizability of the theory requires E^{ij} must be third order in spatial derivatives. The requirement uniquely selects the Cotton tensor

$$C^{ij} = \varepsilon^{ikl}\nabla_k (R^j_l - \tfrac{1}{4} R \delta^j_l), \tag{3}$$

where ϵ^{ijk} denotes the totally antisymmetric tensor and R_{ij} and R are the three-dimensional Ricci tensor and Ricci scalar, respectively. Including relevant deformations, we have

$$W = \frac{1}{w^2}\int d^3\mathbf{x}\sqrt{g}\epsilon^{ijk}\left(\Gamma^m_{il}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}\right) + \mu\int d^3\mathbf{x}\sqrt{g}(R - 2\Lambda_w), \tag{4}$$

where Γ^i_{jk} and Λ_w are Christoffel symbols and the three-dimensional "cosmological constant," respectively. Here we have introduced new coupling constants w and μ. Note that the first two terms lead to the Cotton tensor. Thus, we obtain the potential part of the four-dimensional action [16]

$$S_V = \int dt d^3x\sqrt{g}N\left[-\frac{\kappa^2}{2w^4}C^{ij}C_{ij} + \frac{\kappa^2\mu}{2w^2}\varepsilon^{ijk}R_{il}R^l_{k|j} - \frac{\kappa^2\mu^2}{8}R_{ij}R^{ij} + \frac{\kappa^2\mu^2}{8(1-3\lambda)}\left(\frac{1-4\lambda}{4}R^2 + \Lambda_w R - 3\Lambda_w^2\right)\right], \tag{5}$$

where a stroke | denotes a covariant derivative with respect to spatial coordinates. In the above action (5), the coefficient of scalar curvature R is $\kappa^2\mu^2\Lambda_w/8(1-3\lambda)$, and then the gravitational constant becomes negative in the low energy limit unless $\Lambda_w/(1-3\lambda) > 0$.

In addition to this gravity sector, we consider the action for an inflaton ϕ

$$S_M = \int dt d^3\mathbf{x}\sqrt{g}N\left[-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]. \tag{6}$$

Let us assume slow roll inflation and take the slow roll limit. Then we can replace the action (6) with the effective cosmological constant $\bar{\Lambda}$; namely, we have $S_M = -\int dt d^3\mathbf{x}\sqrt{g}N\bar{\Lambda}$.

Thus, the total action is given by $S = S_K + S_V + S_M$. The total action S breaks the detailed balance condition softly [16]. It should be emphasized that the total action S reduces to the conventional Einstein theory at low energy.

Primordial gravitational waves.—Let us consider the background spacetime with spatial isotropy and homogeneity $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$, where a is the scale factor. Using this metric ansatz, we can get the Friedmann equation with $\lambda = 1$:

$$\frac{\dot{a}^2}{a^2} = \frac{\kappa^2}{12}\left(\bar{\Lambda} - \frac{3\kappa^2\mu^2\Lambda_w^2}{16}\right) \equiv H^2. \tag{7}$$

The above equation leads to de Sitter spacetime $a(t) \propto e^{Ht}$. Here we assumed $\bar{\Lambda} > 3\kappa^2\mu^2\Lambda_w^2/16$. Note that if we chose $\bar{\Lambda} = 0$, there is no Minkowski solution. Hence, there must exist residual vacuum energy in the matter sector at the end of the day. This is related to the issue of the cosmological constant, which is beyond the scope of this Letter.

Now we consider tensor perturbations $ds^2 = -dt^2 + a(t)^2[\delta_{ij} + h_{ij}(t,\mathbf{x})]dx^i dx^j$, where h_{ij} satisfies the transverse-traceless conditions. Substituting this metric into the total action, we obtain the quadratic action

$$\delta^2 S = \int dt d^3x a^3\left[\frac{1}{2\kappa^2}\dot{h}^i_j\dot{h}^j_i + \frac{\kappa^2}{8w^4a^6}\Delta^2 h^i_j\Delta h^j_i + \frac{\kappa^2\mu}{8w^2a^5}\epsilon^{ijk}\Delta h_{il}\Delta h^l_{k|j} - \frac{\kappa^2\mu^2}{32a^4}\Delta h^i_j\Delta h^j_i + \frac{\kappa^2\mu^2\Lambda_w}{32(1-3\lambda)a^2}h^i_j\Delta h^j_i\right], \tag{8}$$

where Δ represents the Laplace operator. The transverse-traceless tensor h_{ij} can be expanded in terms of plane waves with wave number $\mathbf{k}$ as

$$h_{ij}(t,\mathbf{x}) = \sum_{A=R,L}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\psi^A_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}p^A_{ij}, \tag{9}$$

where p^A_{ij} are circular polarization tensors which are defined by $ik_s\epsilon^{rsj}p^A_{ij} = k\rho^A p^{r}{}_i{}^A$ [12]. Here $\rho^R = 1$ and $\rho^L = -1$ modes are called the right-handed mode and the left-handed mode, respectively. We also impose normalization conditions $p^{*i}{}_j{}^A p^j{}_i{}^B = \delta^{AB}$, where $p^{*i}{}_j{}^A$ is the complex conjugate of $p^i{}_j{}^A$. Substituting the expansion (9) into the gravitational action (8), we obtain

$$\delta^2 S = \sum_{A=R,L}\int dt\frac{d^3\mathbf{k}}{(2\pi)^3}a^3\left[\frac{1}{2\kappa^2}|\dot{\psi}^A_{\mathbf{k}}|^2 - \left\{\frac{\kappa^2k^6}{8w^4a^6} - \rho^A\frac{\kappa^2\mu k^5}{8w^2a^5} + \frac{\kappa^2\mu^2k^4}{32a^4} + \frac{\kappa^2\mu^2\Lambda_w k^2}{32(1-3\lambda)a^2}\right\}|\psi^A_{\mathbf{k}}|^2\right]. \tag{10}$$

Using the variable $v^A_{\mathbf{k}} \equiv a\psi^A_{\mathbf{k}}$ and conformal time η defined by $d\eta/dt = 1/a$, we obtain the equations of motion

$$\frac{\partial^2}{\partial \eta^2} v_{\mathbf{k}}^A + \left(k_{\mathrm{eff}}^{A\,2} - \frac{2}{\eta^2} \right) v_{\mathbf{k}}^A = 0, \tag{11}$$

where we used $a = -1/H\eta$ and defined $k_{\mathrm{eff}}^{A\,2} = \alpha^2 k^2 \{1 + \beta(\alpha k \eta)^2 (1 + \rho^A \gamma \alpha k \eta)^2\}$. We have also defined

$$\alpha^2 = \frac{\kappa^4 \mu^2 \Lambda_w}{16(1-3\lambda)}, \qquad \beta = H^2 \frac{1-3\lambda}{\Lambda_w \alpha^2}, \qquad \gamma = H \frac{2}{w^2 \mu \alpha}. \tag{12}$$

Here α is "the emergent speed of light" [16], and β and γ are dimensionless parameters.

Since there appears ρ^A in Eq. (11), the evolution of the right-handed mode is different from that of the left-handed mode. Hence, the dimensionless parameter γ characterizes "the parity violation." If $\beta = 0$ and α is exactly the speed of light, Eq. (11) becomes the equation for gravitational waves in a pure de Sitter background in Einstein theory. Then β measures the "deviation from Einstein theory."

Circular polarization.—Now we calculate the power spectrum $|\psi_{\mathbf{k}}^A|^2$ numerically and evaluate the degree of circular polarization of primordial gravitational waves.

For the numerical analysis, it is convenient to introduce dimensionless variables $k' \equiv \alpha k/H$ and $y \equiv k' H \eta$. Using these variables and the transformation $\zeta^A \equiv \sqrt{k' H} v_{\mathbf{k}}^A/\kappa$, we can write down the basic equation

$$\frac{d^2}{dy^2}\zeta^A + \omega^2(y)\zeta^A = 0, \tag{13}$$

where $\omega^2(y) = 1 + \beta y^2 (1 + \rho^A \gamma y)^2 - 2/y^2$. Since WKB approximation is pretty good in the asymptotic past $y \to -\infty$, we can choose the adiabatic vacuum as the initial condition. More precisely, we set the positive frequency modes as

$$\zeta^A = \frac{1}{\sqrt{2\omega(y)}} \exp\left\{ -i \int_{y_i}^{y} \omega(y') dy' \right\}. \tag{14}$$

On superhorizon scales $y \to 0$, Eq. (13) has asymptotic solution $\zeta^A = C^A/y + D^A y^2$ with constants of integration C^A and D^A. Hence, the power spectrum defined by $k^3 |\psi_{\mathbf{k}}^A|^2 = k^3 |v_{\mathbf{k}}^A/a|^2 = \kappa^2 H^2 |y\zeta^A|^2/\alpha^3$ reduces to

$$k^3 |\psi_{\mathbf{k}}^A|^2 = \frac{\kappa^2 H^2}{\alpha^3} |C^A|^2 \tag{15}$$

on superhorizon scales $y \to 0$. So we need only to calculate C^A using Eq. (13) with the initial condition (14). Notice that the power spectrum $P(k) = k^3 |\psi_{\mathbf{k}}^A|^2$ is scale free. From the mode function (14), we see that the vacuum depends on the chirality. In the WKB regime, the amplitude of the right-handed mode grows, while that of the left-handed mode decays. These two effects make the difference. In Fig. 1, we plotted the time evolution of the power for a right-handed mode and a left-handed mode and also displayed the case of Einstein theory for comparison.

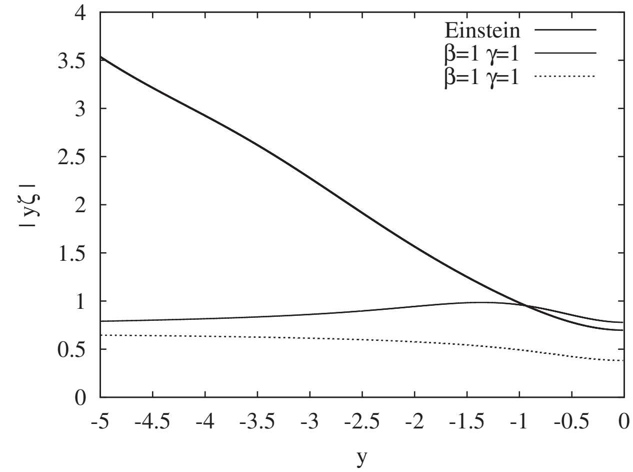

FIG. 1. The time evolution of the power is depicted. The thick solid line represents the evolution for conventional Einstein gravity. The thin solid line and the dotted line show the time evolution of the right-handed mode and left-handed mode, respectively.

Clearly, one can see that the differences of initial amplitude and the growth rate during the WKB regime lead to circular polarization.

Now we are in a position to discuss observability of circular polarization. For this aim, we need to quantify polarization by defining the degree of circular polarization

$$\Pi = \frac{|\psi_{\mathbf{k}}^R|^2 - |\psi_{\mathbf{k}}^L|^2}{|\psi_{\mathbf{k}}^R|^2 + |\psi_{\mathbf{k}}^L|^2} = \frac{|C^R|^2 - |C^L|^2}{|C^R|^2 + |C^L|^2}. \tag{16}$$

Numerical results are plotted in Fig. 2. There are two possible channels to observe circular polarization of pri-

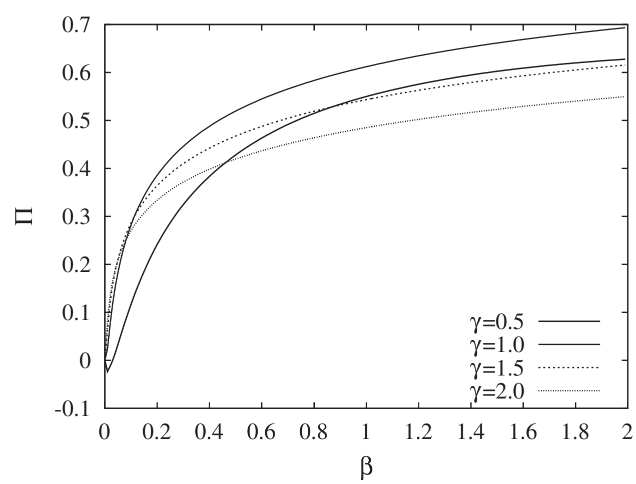

FIG. 2. The degree of circular polarization Π for various γ as a function of β is shown. As can be seen from the figure, Π grows as the β becomes large. The dependence on γ is not monotonic; rather, there is a value which gives the maximum polarization for fixed β.

mordial gravitational waves. One is the indirect detection of circular polarization through the cosmic microwave background radiation; the required degree of circular polarization has been obtained as $|\Pi| \gtrsim 0.35(r/0.05)^{-0.6}$ in [17], where r is the tensor-to-scalar ratio. The relevant frequency of gravitational waves in this case is around $f \sim 10^{-17}$ Hz. From Fig. 2, supposing $r = 0.05$ and $\gamma = 1$, we see that we can detect the circular polarization through the temperature and B-mode polarization correlation if $\beta > 0.2$. The other is the direct detection of circular polarization; the required degree of circular polarization has been estimated as $\Pi \sim 0.08(\Omega_{\rm GW}/10^{-15})^{-1}(\mathrm{SNR}/5)$ around the frequency $f \sim 1$ Hz [18], where $\Omega_{\rm GW}$ is the density parameter of the stochastic gravitational waves and SNR is the signal to the noise ratio [18–20]. Here, 10 years of observational time is assumed. Taking a look at Fig. 2, one can see that it is easy to get the circular polarization of the order of 0.08 in the present model. Hence, we can prove or disprove quantum gravity at a Lifshitz point by these observations.

Conclusion.—We have considered the inflationary scenario in the context of quantum gravity at a Lifshitz point which is supposed to be a power-counting renormalizable theory. Because of the detailed balance condition, the action necessarily contains a Cotton tensor which violates the parity invariance. We have calculated the degree of circular polarization of primordial gravitational waves. As a consequence, we find that chiral primordial gravitational waves exist for generic parameters. It should be emphasized that the existence of circular polarization is a robust prediction of the theory.

In the usual discussions on the trans-Planckian effects, phenomenological approaches have been adopted, and mostly a modification of the spectrum has been discussed, while we have used a candidate of quantum gravity and discussed chirality of primordial gravitational waves. The point is that we have found a modification of the nature of gravitational waves rather than a modification of the shape of the spectrum for gravitational waves. It is also apparent that the power spectrum for curvature perturbations is scale free. Thus, quantum gravity at a Lifshitz point is consistent with all current observations. Moreover, we have a testable smoking gun of quantum gravity.

There are many issues to be pursued. One of those is to find exact solutions which represent black holes. When black hole solutions are found, it will be very interesting to examine their spacetime structures, thermodynamics, and Hawking radiation. It would also be intriguing to apply the idea of the anisotropic scaling in gravity to braneworld cosmology [21]. Furthermore, the possibility to generalize the anisotropic scaling in gravity to cases where spatial isotropy is broken would be interesting from the point of anisotropic inflationary scenarios [22,23].

J. S. is supported by the Japan-U.K. Research Cooperative Program and Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Science and Culture of Japan No. 18540262.

Note added.—After submitting our Letter, we found a related work in the archive where inflation caused by a Lifshitz scalar is considered [24].

[1] J. Martin and R. H. Brandenberger, Phys. Rev. D **63**, 123501 (2001).
[2] R. H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001).
[3] R. H. Brandenberger, arXiv:hep-th/0210186.
[4] T. Tanaka, arXiv:astro-ph/0012431.
[5] A. A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **73**, 415 (2001) [JETP Lett. **73**, 371 (2001)].
[6] A. Lue, L. M. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
[7] C. R. Contaldi, J. Magueijo, and L. Smolin, Phys. Rev. Lett. **101**, 141101 (2008).
[8] K. Choi, J. C. Hwang, and K. W. Hwang, Phys. Rev. D **61**, 084026 (2000).
[9] S. H. S. Alexander, M. E. Peskin, and M. M. Sheikh-Jabbari, Phys. Rev. Lett. **96**, 081301 (2006).
[10] D. H. Lyth, C. Quimbay, and Y. Rodriguez, J. High Energy Phys. 03 (2005) 016.
[11] S. Alexander and J. Martin, Phys. Rev. D **71**, 063526 (2005).
[12] M. Satoh, S. Kanno, and J. Soda, Phys. Rev. D **77**, 023526 (2008).
[13] M. Satoh and J. Soda, J. Cosmol. Astropart. Phys. 09 (2008) 019.
[14] S. Kawai, M. a. Sakagami, and J. Soda, Phys. Lett. B **437**, 284 (1998).
[15] P. Hořava, J. High Energy Phys. 03 (2009) 020.
[16] P. Hořava, Phys. Rev. D **79**, 084008 (2009).
[17] S. Saito, K. Ichiki, and A. Taruya, J. Cosmol. Astropart. Phys. 09 (2007) 002.
[18] N. Seto, Phys. Rev. Lett. **97**, 151101 (2006).
[19] N. Seto, Phys. Rev. D **75**, 061302(R) (2007).
[20] N. Seto and A. Taruya, Phys. Rev. Lett. **99**, 121101 (2007).
[21] J. Soda and S. Kanno, arXiv:gr-qc/0612069.
[22] S. Kanno, M. Kimura, J. Soda, and S. Yokoyama, J. Cosmol. Astropart. Phys. 08 (2008) 034.
[23] M. a. Watanabe, S. Kanno, and J. Soda, Phys. Rev. Lett. **102**, 191302 (2009).
[24] G. Calcagni, arXiv:0904.0829.