Precise Predictions for $W + 3$ Jet Production at Hadron Colliders

C. F. Berger,[1] Z. Bern,[2] L. J. Dixon,[3] F. Febres Cordero,[2] D. Forde,[3] T. Gleisberg,[3] H. Ita,[2] D. A. Kosower,[4] and D. Maître[5]

[1]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA*
[3]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*
[4]*Institut de Physique Théorique, CEA-Saclay, F-91191 Gif-sur-Yvette cedex, France*
[5]*Department of Physics, University of Durham, DH1 3LE, United Kingdom*
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We report on the first next-to-leading order QCD computation of $W + 3$-jet production in hadronic collisions including all partonic subprocesses. We compare the results with data from the Tevatron and find excellent agreement. The required one-loop matrix elements are computed using on-shell methods, implemented in a numerical program, BLACKHAT. We use the SHERPA package to generate the real-emission contributions and to integrate the various contributions over phase space. We use a leading-color (large-N_c) approximation for the virtual part, which we confirm in $W + 1$, 2-jet production to be valid to within three percent.

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Particle physicists have long anticipated the discovery of new physics beyond the Standard Model at the Large Hadron Collider (LHC) at CERN. In many channels, discovering, understanding, and measuring new physics signals will require quantitatively reliable predictions for Standard Model background processes. Next-to-leading order (NLO) calculations in perturbative QCD are crucial to providing such predictions. Leading-order (LO) cross sections suffer from large normalization uncertainties, up to a factor of two in complex processes. NLO corrections typically reduce the uncertainties to 10–20% [1].

The production of a vector boson in association with multiple jets of hadrons is an important process. It forms a background to Standard Model processes such as top quark production, as well as to searches for supersymmetry. Here, we present the first NLO computation of $W + 3$-jet production that can be compared directly to data, namely, CDF results [2] from the Tevatron.

The development of methods for computing high-multiplicity processes at NLO has involved a dedicated effort over many years, summarized in Ref. [1]. The long-standing bottleneck to NLO computations with four or more final-state objects—including jets—has been in evaluating one-loop (virtual) corrections. Feynman-diagram techniques suffer from a rapid growth in complexity as the number of legs increases; in QCD, NLO corrections to processes with four final-state objects have been limited to the case of all external quarks [3]. On-shell methods [4–12], in contrast, do not use Feynman diagrams, but rely on the analyticity and unitarity of scattering amplitudes to generate new amplitudes from previously computed ones. Such methods scale extremely well as the number of external legs increases [9,13,14], offering a solution to these difficulties.

In an on-shell approach, terms in a one-loop amplitude containing branch cuts are computed by matching the unitarity cuts (products of tree amplitudes) with an expansion of the amplitude in terms of a basis of scalar integrals [4]. Recent refinements [6,10,11,15], exploiting complexified loop momenta, greatly enhance the effectiveness of generalized (multiple) cuts [5]. Evaluating the cuts in four dimensions allows the use of compact forms for the tree amplitudes which enter as ingredients. This procedure drops rational terms, which could be computed by evaluating the cuts in D dimensions [16]. One may also obtain the rational terms using on-shell recursion, developed by Britto, Cachazo, Feng, and Witten at tree level [7], and extended to loop level in Refs. [8,9].

Within the BLACKHAT program [13], we determine coefficients of scalar integrals using Forde's analytic approach [11], also incorporating elements from the approach of Ossola, Papadopoulos, and Pittau (OPP) [10]. For the rational terms, we have implemented both loop-level on-shell recursion and a massive continuation approach (related to D-dimensional unitarity) along the lines of Badger's method [15]. The on-shell recursion code is faster at present, so we use it here. The requisite speed and numerical stability of BLACKHAT have been validated for one-loop six-, seven- and eight-gluon amplitudes [13], and for leading-color amplitudes for a vector boson with up to five partons [17], required for the present study. (A subsequent computation of one-loop matrix elements needed for $W + 3$-jet production using D-dimensional generalized unitarity within the OPP formalism was described in Ref. [18].) Other numerical programs along similar lines are presented in Refs. [14,19].

To speed up the evaluation of the virtual cross section, we make use of a leading-color (large-N_c) approximation for the finite parts of the one-loop amplitudes, keeping the exact color dependence in all other parts of the calculation. Such approximations have long been known to be excellent for the four-jet rate in e^+e^- annihilation [20]. A similar approximation was used recently for an investigation of $W + 3$-jet production [21], which, however, also omitted many partonic subprocesses. Our study retains all subprocesses. In addition, we keep all subleading-color terms in the real-emission contributions. In the finite virtual terms of each subprocess, we drop certain subleading-color contributions. "Finite" refers to the ϵ^0 term in the Laurent expansion of the infrared-divergent one-loop amplitudes in $\epsilon = (4 - D)/2$, after extracting a multiplicative factor of $c_\Gamma(\epsilon) \equiv \Gamma(1+\epsilon)\Gamma^2(1-\epsilon)/\Gamma(1-2\epsilon)/(4\pi)^{2-\epsilon}$. "Subleading-color" refers to the part of the ratio of the virtual terms to tree cross section that is suppressed by at least one power of either $1/N_c^2$ or n_f/N_c (virtual quark loops). We multiply the surviving, leading-color terms in this ratio back by the tree cross section, with its full-color dependence.

For this approximation, we need only the color-ordered (primitive) amplitudes in which the W boson is adjacent to the two external quarks forming the quark line to which it attaches. Representative Feynman diagrams for these primitive amplitudes are shown in Fig. 1. Other primitive amplitudes have external gluons (or a gluon splitting to a $\bar{Q}Q$ pair) attached between the W boson and the two above-mentioned external quarks; they only contribute [22] to the subleading-color terms that we drop. As discussed below, we have confirmed that for $W + 1$, 2-jet production, this leading-color approximation is valid to within three percent, so we expect corrections to the $W +$ 3-jet cross sections from subleading-color terms also to be small.

In addition to the virtual corrections to the cross section provided by BLACKHAT, the NLO result also requires the real-emission corrections to the LO process. The latter arises from tree-level amplitudes with one additional parton, either an additional gluon, or a quark-antiquark pair replacing a gluon. Infrared singularities develop when the extra parton momentum is integrated over unresolved phase-space regions. They cancel against singular terms in the virtual corrections, and against counterterms associated with the evolution of parton distributions. We use the program AMEGIC++ [23] to implement these cancellations via the Catani-Seymour dipole subtraction method [24]. The SHERPA framework [25] incorporates AMEGIC++, making it easy to analyze the results and construct a wide variety of distributions. For other automated implementations of the dipole subtraction method, see Refs. [26].

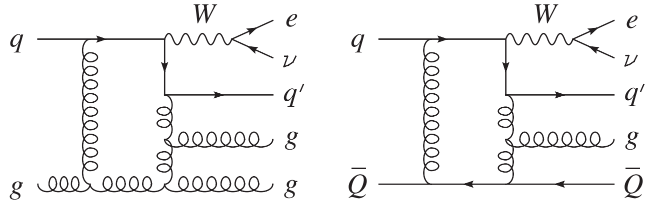

FIG. 1. Sample diagrams for the seven-point amplitudes $qg \to Wq'gg$ and $q\bar{Q} \to Wq'g\bar{Q}$, followed by W decay to $e\nu$.

The CDF analysis [2] employs the JETCLU cone algorithm [27] with a cone radius $R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2} = 0.4$. However, this algorithm is not generally infrared safe at NLO, so we instead use the seedless cone algorithm SISCONE [28]. In general, at the partonic level, we expect similar results from any infrared-safe cone algorithm. For $W + 1$, 2 jets we have confirmed that distributions using SISCONE are within a few percent of those obtained with the midpoint cone algorithm [29].

Both electron and positron final states are counted, and the following cuts are imposed: $E_T^e > 20$ GeV, $|\eta^e| < 1.1$, $\not{E}_T > 30$ GeV, $M_T^W > 20$ GeV, and $E_T^{\rm jet} > 20$ GeV. Here, E_T is the transverse energy, $\not{E}_T$ is the missing transverse energy, M_T^W the transverse mass of the $e\nu$ pair, and η the pseudorapidity. Jets are ordered by E_T, and are required to have $|\eta| < 2$. Total cross sections are quoted with a tighter jet cut, $E_T^{\rm jet} > 25$ GeV. CDF also imposes a minimum ΔR between the charged decay lepton and any jet; the effect of this cut, however, is removed by the acceptance corrections.

CDF compared [2] their measured $W + n$-jet cross sections to LO (matched to parton showers [30]) and the then-available NLO theoretical predictions. The LO calculations differ substantially from the data, especially at lower E_T, and have large scale dependence bands. In contrast, the NLO calculations for $n \leq 2$ jets (using the MCFM code [31], with the $W + 4$-parton one-loop matrix elements of Ref. [5]) show much better agreement, and narrow scale-dependence bands. See Ref. [2] for details.

Our aim in this Letter is to extend this comparison to $n = 3$ jets. We apply the same lepton and jet cuts as CDF, replacing the $\not{E}_T$ cut by one on the neutrino E_T, and ignoring the lepton-jet ΔR cut removed by acceptance. We approximate the Cabibbo-Kobayashi-Maskawa matrix by the unit matrix, express the W coupling to fermions using the Standard Model parameters $\alpha_{\rm QED} = 1/128.802$ and $\sin^2\theta_W = 0.230$, and use $m_W = 80.419$ GeV and $\Gamma_W = 2.06$ GeV. We use the CTEQ6M [32] parton distribution functions (PDFs) and an event-by-event common renormalization and factorization scale, $\mu = \sqrt{m_W^2 + p_T^2(W)}$. To estimate the scale dependence, we choose five values in the range $(\frac{1}{2}, 2) \times \mu$. The numerical integration errors are on the order of a half percent. We do not include PDF uncertainties. For $W + 1$, 2-jet production, these uncertainties have been estimated in Ref. [2]. In general, they are smaller than the scale uncertainties at low E_T but larger at high E_T. The LO calculation uses the CTEQ6L1 PDF set. For $n = 1, 2$ jets, NLO total cross sections agree with those from MCFM [31], for various cuts. We do not apply correc-

TABLE I. Total cross sections in pb for $W + n$ jets with $E_T^{n\text{th-jet}} > 25$ GeV as measured by CDF [2]. The results are compared to NLO QCD. For $W + 1$ and $W + 2$ jets, the difference between the leading-color approximation and the complete NLO result is under three percent. For $W + 3$ jets, only the LC NLO result is currently available, but we expect a similarly small deviation for the full NLO result. Experimental statistical, systematic, and luminosity uncertainties have been combined for the CDF results.

number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.908^{+0.044}_{-0.142}$	$\cdots$

tions for the underlying event or hadronization. Such corrections are expected to be under ten percent [2].

In Table I, we collect the results for the total cross section, comparing CDF data to the NLO theoretical predictions computed using BLACKHAT and SHERPA. The columns labeled "LC NLO" and "NLO" show, respectively, the results for our leading-color approximation to NLO, and for the full NLO calculation. The leading-color NLO and full NLO cross sections for $W + 1$- and $W + 2$-jet production agree to within three percent. We thus expect only a small change in the results for $W + 3$-jet production once the missing subleading-color contributions are incorporated.

We have also compared the E_T distribution of the nth jet in CDF data to the NLO predictions for $W + 1$, 2, 3-jet production. For $W + 2$, 3-jets, these comparisons are shown in Fig. 2, including scale-dependence bands obtained as described above. For reference, we also show the LO distributions and corresponding scale-dependence band. (The calculations matching to parton showers [30] used in Ref. [2] make different choices for the scale variation and are not directly comparable to the parton-level predictions shown here.) The NLO predictions match the data very well, and uniformly in all but the highest E_T bin. The central values of the LO predictions, in contrast, have different shapes from the data. The scale dependence of the NLO predictions is substantially smaller than that of the LO ones. In the $W + 2$-jet case, we also show the ratio of the leading-color approximation to the full-color result within the NLO calculation: the two results differ by less than three percent over the entire transverse energy range, considerably smaller than the scale dependence (and experimental uncertainties).

In Fig. 3, we show the distribution for the total transverse energy H_T, given by the scalar sum of the jet and lepton transverse energies, $H_T = \sum_j E_{T,j}^{\text{jet}} + E_T^e + \not{E}_T$. We show the NLO and LO predictions, along with their scale-uncertainty bands. As in the E_T distributions, the NLO band is much narrower, and the shape of the distribution is altered at NLO from the LO prediction.

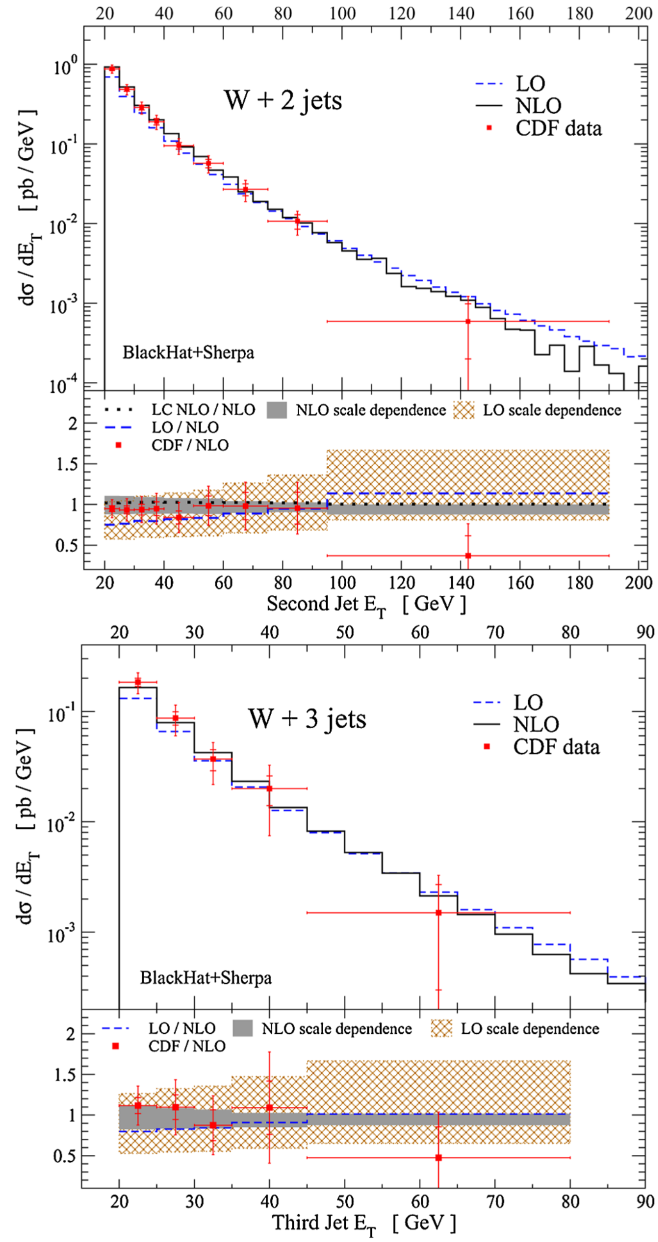

FIG. 2 (color online). The measured cross section $d\sigma(W \to e\nu + \geq n\text{-jets})/dE_T^{n\text{th-jet}}$ compared to NLO predictions for $n = 2$, 3. In the upper panels, the NLO distribution is the solid (black) histogram, and CDF data points are the (red) points, whose inner and outer error bars denote the statistical and total uncertainties on the measurements. The LO predictions are shown as dashed (blue) lines. The lower panels show the distribution normalized to an NLO prediction, the full one for $n = 2$ and the leading-color one for $n = 3$, in the experimental bins (that is, averaging over several bins in the upper panel). The scale-uncertainty bands are shaded (gray) for NLO and cross-hatched (brown) for LO. In the $n = 2$ case, the dotted (black) line shows the ratio of the leading-color approximation to the full-color calculation.

In summary, we have presented the first phenomenologically useful NLO study of $W + 3$-jet production, and compared the total cross section and the jet E_T distribution to Tevatron data [2]. The results demonstrate the utility of

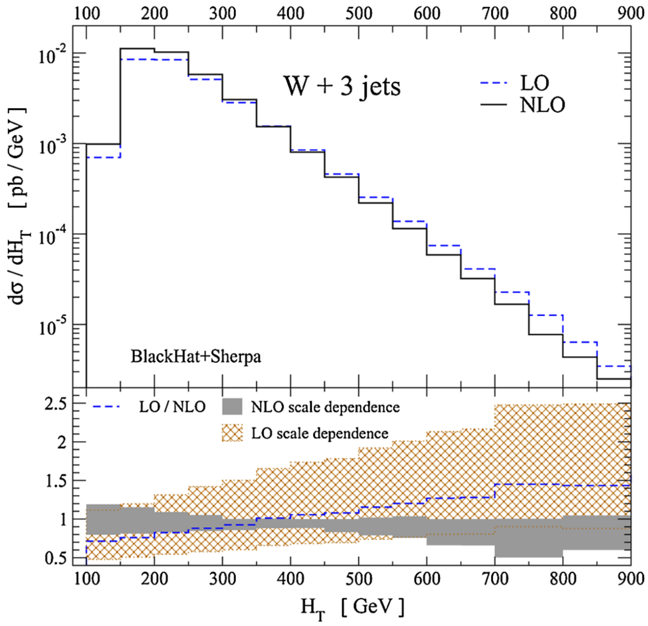

FIG. 3 (color online). The theoretical prediction for the H_T distribution in $W + 3$-jet production.

the on-shell method and its numerical implementation in the BLACKHAT code for NLO computations of phenomenologically important processes at the LHC.

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