Algebraic Spin Liquid in an Exactly Solvable Spin Model

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(Received 29 October 2008; published 29 May 2009)

We have proposed an exactly solvable quantum spin-3/2 model on a square lattice. Its ground state is a quantum spin liquid with a half-integer spin per unit cell. The fermionic excitations, dubbed as "spinons", are gapless with a linear dispersion, while the topological "vison" excitations are gapped. Moreover, these massless fermionic spinon excitations are topologically stable. Thus, this model is, to the best of our knowledge, the first exactly solvable model of half-integer spins whose ground state is an "algebraic spin liquid."

DOI: 10.1103/PhysRevLett.102.217202 PACS numbers: 75.10.Jm, 71.10.−w, 75.50.Mm

The term "spin liquid" is widely used to give sharp meaning to the more general intuitive notion of a Mott insulator; a spin liquid is an insulating state that cannot be adiabatically connected to a band insulator, i.e., to an insulating Slater determinant state. In a system that preserves time reversal symmetry, any insulating state with an odd number of electrons (or a half-integer spin) per unit cell is a spin liquid. Interesting proposals [1,2] have been made concerning the relevance of such states to the theory of high temperature superconductivity in the cuprates and other materials. Indeed, various spin liquid phases have been proposed, which are distinguished by the character of any gapless spinons and the exchange statistics of the topological vison excitations.

Since they are new and "exotic" quantum phases of matter, it is desirable to construct solvable models with short range interactions with stable spin liquid ground-state phases. A breakthrough occurred when Moessner and Sondhi [3] demonstrated the existence of a gapped spin liquid ground state in the quantum dimer model [4], analogous to the short range version of the RVB state [5,6]. An exactly solvable spin-1/2 model in a gapped Z_2 spin-liquid phase was later constructed by Wen [7]. However, much of the recent interest, spurred in part by the possible observation of such a state in $\kappa-(\mathrm{ET})_2\mathrm{Cu}_2(\mathrm{CN})_3$ [8–10] and $\mathrm{Zn}(\mathrm{Cu})_3(\mathrm{OH})_6\mathrm{Cl}_2$ [11], has focused on spin liquids with gapless spinon excitations, so-called "algebraic spin liquids."

The exactly solvable Kitaev model on the honeycomb lattice [12] can exhibit gapless excitations. However, because the honeycomb lattice has two sites per unit cell, this model has an integer spin, hence an even number of electrons per unit cell. In the present paper we construct an exactly solvable model, in much the same spirit as the Kitaev model, whose ground state is a spin liquid with an odd number electrons per unit cell and stable gapless fermionic spinon excitations. To the best of our knowledge, this is the first exactly solvable model with this sort of spin liquid ground state algebraic spin liquid.

The Kitaev model has a spin-1/2 on each site of a trivalent lattice, where the coordination number is dictated by the existence of three Pauli matrices. In order to study a model on a square lattice, we instead consider a model with a spin-3/2 on each lattice site. The resulting larger Hilbert space, with 4 spin polarizations per site, permits us to express the model in terms of the 4×4 anticommuting Gamma matrices, Γ^a ($a=1,\ldots,5$) which form Clifford algebra, $\{\Gamma^a,\Gamma^b\}=2\delta^{ab}$. Specifically, the 5 Gamma matrices can be represented [13] by symmetric bilinear combinations of the components of a spin 3/2 operator, S^α, as

$$\Gamma^1=\frac{1}{\sqrt{3}}\{S^y,S^z\},\qquad \Gamma^2=\frac{1}{\sqrt{3}}\{S^z,S^x\},$$
$$\Gamma^3=\frac{1}{\sqrt{3}}\{S^x,S^y\},\qquad \Gamma^4=\frac{1}{\sqrt{3}}[(S^x)^2-(S^y)^2], \tag{1}$$
$$\Gamma^5=(S^z)^2-\frac{5}{4}.$$

Model Hamiltonian.—We define our model on a square lattice, with a spin-3/2 on each site, and corresponding Γ matrices defined as in Eq. (1). In terms of these,

$$\mathcal{H}=\sum_i[J_x\Gamma_i^1\Gamma_{i+\hat{x}}^2+J_y\Gamma_i^3\Gamma_{i+\hat{y}}^4]+\sum_i[J_x'\Gamma_i^{15}\Gamma_{i+\hat{x}}^{25}+J_y'\Gamma_i^{35}\Gamma_{i+\hat{y}}^{45}]-J_5\sum_i\Gamma_i^5, \tag{2}$$

where $\Gamma^{ab}\equiv[\Gamma^a,\Gamma^b]/(2i)$ and i labels the lattice site at $\mathbf{r}_i=(x_i,y_i)$. We call this model the Gamma matrix model (GMM). Suppose that the square lattice has $N=L_xL_y$ sites, where L_x and L_y are the lattice's linear sizes and are assumed, for simplicity, to be even in this Letter. Moreover, we consider periodic boundary conditions. Obviously, the GMM can be written explicitly as a spin-3/2 model. The GMM model respects translational symmetry and time reversal symmetry (TRS). It does not have global spin SU(2) or even U(1) rotational symmetry, but is invariant under 180° rotations about the z axis in spin space; i.e., it has global Ising symmetry. Note that, due to the lack of SU(2) or U(1) symmetry in the present model, the fermionic spinon excitations discussed here do not have well-defined spin quantum number. A feature of the model which makes it solvable is an infinite set of conserved "fluxes:" $[\hat{W}_i, \mathcal{H}] = 0$ for any i and $[\hat{W}_i, \hat{W}_j] = 0$, where $\hat{W}_i \equiv \Gamma_i^{13}\Gamma_{i+\hat{x}}^{23}\Gamma_{i+\hat{y}}^{14}\Gamma_{i+\hat{x}+\hat{y}}^{24}$ is the plaquette operator on plaquette i.

Note that, in contrast to the spin-1/2 Kitaev model on the honeycomb lattice, the GMM has an odd number (namely, 3) of electrons per unit cell. The present model in the limit $J_x = J_x'$, $J_y = J_y'$, and $J_5 = 0$ is similar to a model proposed by Wen in Ref. [14], where, however, the Gamma matrices were constructed from two spin-1/2 operators on each site, and therefore behave differently under time reversal than in the present realization. Moreover, the present model generically does not possess the special symmetries that are responsible for some of the behaviors of Wen's.

Fermionic representation.—Spin-3/2 operators can be expressed as bilinear forms involving three flavors of fermion operators, $S^z = a^\dagger a + 2b^\dagger b - 3/2$ and $S^+ = \sqrt{3}f^\dagger a + \sqrt{3}af + 2a^\dagger b$, subject to the constraint that the physical states are only those with odd "fermion parity," $(-1)^{\hat{N}} = -1$, where $\hat{N} = f^\dagger f + a^\dagger a + b^\dagger b$. Rather than using this representation in terms of three Dirac fermions, we will directly represent the Gamma matrices in terms of a related set of 6 Majorana fermions:

$$\Gamma_i^\mu = ic_i^\mu d_i, \quad \Gamma_i^{\mu 5} = ic_i^\mu d_i', \quad \mu = 1,2,3,4, \quad \Gamma_i^5 = id_i d_i', \tag{3}$$

where c_i^μ, d_i, and d_i' are Majorana fermions on site i. Six Majorana fermions form an eight dimensional Hilbert space, which is an enlarged one from the physical Hilbert space of an spin-3/2. In terms of spin-3/2 operators in Eq. (1), $\Gamma_i^1\Gamma_i^2\Gamma_i^3\Gamma_i^4\Gamma_i^5 = -1$ for all i. Consequently, all allowed physical states $|\Psi\rangle$ in terms of Majorana fermions must satisfy the following constraint, for all i,

$$D_i|\Psi\rangle \equiv [-ic_i^1c_i^2c_i^3c_i^4d_id_i']|\Psi\rangle = |\Psi\rangle. \tag{4}$$

In terms of Majorana fermions, the Hamiltonian in the enlarged Hilbert space can be written as

$$\mathcal{H} = \sum_i [J_x\hat{u}_{i,x}id_id_{i+\hat{x}} + J_y\hat{u}_{i,y}id_id_{i+\hat{y}} + J_x'\hat{u}_{i,x}id_i'd_{i+\hat{x}}' + J_y'\hat{u}_{i,y}id_i'd_{i+\hat{y}}' - J_5id_id_i'], \tag{5}$$

where $\hat{u}_{i,x} \equiv -ic_i^1c_{i+\hat{x}}^2$ and $\hat{u}_{i,y} \equiv -ic_i^3c_{i+\hat{y}}^4$. It is obvious that $\hat{u}_{i,\lambda}$ are conserved quantities with eigenvalues $u_{i,\lambda} = \pm 1$, $\lambda = x, y$. Consequently, the enlarged Hilbert space can be divided into sectors $\{u\}$. In each sector, the Hamiltonian Eq. (5) describes free Majorana fermions:

$$\mathcal{H}(\{u\}) = \sum_i [J_xu_{i,x}id_id_{i+\hat{x}} + J_yu_{i,y}id_id_{i+\hat{y}} + J_x'u_{i,x}id_i'd_{i+\hat{x}}' + J_y'u_{i,y}id_i'd_{i+\hat{y}}' - J_5id_id_i'], \tag{6}$$

where $u_{i,\lambda}$ are emergent Z_2 gauge fields. The Z_2 gauge transformations are given by $d_i \to \Lambda_i d_i$, $d_i' \to \Lambda_i d_i'$, and $u_{i,\lambda} \to \Lambda_i u_{i,\lambda}\Lambda_{i+\lambda}$, where $\Lambda_i = \pm 1$. In the enlarged Hilbert space, the eigenstates $|\psi\rangle = |\psi\rangle_c \otimes |\psi\rangle_{d,d'}$ can be written as a direct product of a part that involves the c^μ fermions and a part that involves the d and d' fermions, respectively. Here $|\psi\rangle_c$ is defined by $\hat{u}_{i,\lambda}|\psi\rangle_c = u_{i,\lambda}|\psi\rangle_c$ and $|\psi\rangle_{d,d'}$ are eigenstates of Eq. (6) with the corresponding $u_{i,\lambda}$'s.

The spectrum of $\mathcal{H}(\{u\})$ depends only on gauge invariant quantities—the flux on local plaquettes, $\exp(i\phi_i) \equiv u_{i,x}u_{i+\hat{x},y}u_{i,y}u_{i+\hat{y},x}$, and two global fluxes $\exp(i\phi_x) \equiv \prod_{i(y_i=1)}u_{i,x}$ and $\exp(i\phi_y) \equiv \prod_{i(x_i=1)}u_{i,y}$, where ϕ_i and $\phi_{x,y} = 0, \pi$. [Note that the previously defined $W_i = -\exp(i\phi_i)$ and that ϕ_i are good quantum numbers.] It is obvious that $\sum_i\phi_i = 0 \pmod{2\pi}$, so there are $N - 1$ independent local fluxes. Including the two global fluxes, the number of independent fluxes is $N + 1$. Since there are $2N$ Z_2 gauge fields, the number of different gauge field choices corresponding to each flux sector $\{\phi\}$ is 2^{N-1}. In other words, in the enlarged Hilbert space, each state is 2^{N-1}-fold degenerate. Note that in the thermodynamic limit, the energy is independent of the two global fluxes, which gives rise to a fourfold topological degeneracy of the physical ground states.

Projection operators.—Most of the states in the enlarged Hilbert space are not physical states. To obtain a physical eigenstate, we must find a linear combination of the degenerate eigenstates which is simultaneously an eigenstate of every D_i with eigenvalue 1. This is realized through the projection operator P:

$$|\Psi\rangle = P|\psi\rangle \equiv \prod_i[(1 + D_i)/2]|\psi\rangle. \tag{7}$$

Clearly, Eq. (7) implies $D_i|\Psi\rangle = |\Psi\rangle$ for any i. Explicitly, P is given by

$$P = \left[1 + \sum_i D_i + \sum_{i_1<i_2} D_{i_1}D_{i_2} + \cdots + \prod_i D_i\right]\Big/2^N. \tag{8}$$

D_i acting on a direct product state $|\psi\rangle$ is equivalent to a gauge transformation on site i. A subtlety here is that there are 2^N operators in the sum in Eq. (8), but only 2^{N-1} inequivalent gauge transformations. In fact, $D \equiv \prod_i D_i$ implements a gauge transformation on every site, thus leaving all gauge fields unchanged. It follows that $P = P'(1 + D)$, where P' includes all inequivalent transformations. Since $[D, \mathcal{H}] = 0$ and $D^2 = 1$, $D|\psi\rangle = \pm|\psi\rangle$. Moreover, $D = \prod_i[\hat{u}_{i,x}\hat{u}_{i,y}]\prod_i[id_id_i']$. By introducing the following Dirac fermions,

$$f_j \equiv i^j(d_j + id_j')/2, \tag{9}$$

we obtain $D = (-1)^{\hat{N}_\phi + \hat{N}_f}$, where $\hat{N}_f = \sum_i f_i^\dagger f_i$ is the number of Dirac fermions which is conserved modulo 2 by the Hamiltonian, and $\hat{N}_\phi$ is defined by dividing the plaquettes into two sublattices and counting the number of π-fluxes through one or the other sublattice [15].

Thus, depending on the fermion and flux parity, P either annihilates a given direct product state, $|\psi\rangle$, or maps it to the equal weight linear superposition of all gauge transformations acting on $|\psi\rangle$. For instance, when there is a π-flux through each plaquette, which, as discussed below, is the ground state sector, $D = (-1)^{\hat{N}_f}$; i.e., all physical states must have an even number of fermions. Conversely, where 1 π-flux is added to each sublattice, only states with $\hat{N}_f =$ odd survive projection. Conserving the parity of fermion number reflects the fact that physical fermionic excitations are created by nonlocal (string) operators [14,16].

π-flux state and gapped visons.—In each flux sector $\{\phi\}$, the lowest energy of the Hamiltonian is denoted by $E_0(\{\phi\})$. The ground state energy of the model is achieved by minimizing $E_0(\{\phi\})$ with respect to $\{\phi\}$. Formally, by integrating out the fermions, an effective action for the Z_2 gauge fields can be derived. However, in general, it is nontrivial to obtain an explicit form of the effective action.

For $J_5 = 0$, fortunately, there is a theorem due to Lieb [17] which implies that the energy minimizing flux sector of a half-filled band of electrons hopping on a planar, bipartite lattice is π per square plaquette. This uniform π-flux sector, namely the ground state sector, is defined as being vortex-free. Z_2 vortex excitations, "visons", are defined to be plaquettes with $\phi_i = 0$. Because of the constraint $\sum_i \phi_i = 0 \,(\mathrm{mod}\, 2\pi)$, only an even number of visons are allowed. It is special for this model that visons do not have dynamics even though they interact with one another. (Numerical calculations of the two vison energy reveals that the interaction between visons is repulsive.) The minimum energy cost of creating two visons in the ground state is always finite and is defined as the vison gap energy $\Delta_v \sim (\sqrt{|J_x J_y|} + \sqrt{|J'_x J'_y|})$. Since the visons are gapped, the uniform π-flux is still the ground state sector as long as J_5 is much smaller than Δ_v. In the rest of Letter, we restrict our attention to cases (e.g., small J_5) in which the ground states lie in the π-flux sector.

Spin correlation.—The state with uniform π-fluxes preserves the translational symmetry and TRS of the model. To prove the ground state of the model is a spin liquid, we need to show that there is no magnetic order and that the spin correlations are short-ranged. Spin-3/2 operators can be expressed as bilinear forms of Majorana fermions. For instance, $S_i^z = ic_i^3 c_i^4 + \frac{1}{2} ic_i^1 c_i^2$. The action of S_i^z on the ground state $|\Psi\rangle$ creates visons in the four surrounding plaquettes around site i [18]. The effect of S_i^x and S_i^y on the ground state is similar. Because visons are nondynamical in the present model, $\langle\Psi|S_i^\alpha S_j^\beta|\Psi\rangle$ is identically zero unless sites i and j are nearest neighbors; i.e., the spin correlations are unphysically short-ranged. Presumably, if additional small, local terms are added to the Hamiltonian, perturbative corrections to this correlator would lead to a finite correlation length.

Gapless fermions.—To obtain the excitation spectrum in the π-flux (ground state) sector, we fix the gauge by choosing $u_{i,x} = 1$, $u_{i,y} = (-1)^i$. The corresponding Hamiltonian is given by

$$H_0 = \sum_i [t_x f_i^\dagger f_{i+\hat{x}} + t_y (-1)^i f_i^\dagger f_{i+\hat{y}} - J_5 f_i^\dagger f_i - \Delta_x (-1)^i f_i^\dagger f_{i+\hat{x}}^\dagger - \Delta_y f_i^\dagger f_{i+\hat{y}}^\dagger + \mathrm{H.c.}], \quad (10)$$

which describes a p-wave superconductor of spinless fermions [19]. Here $t_\lambda \equiv J_\lambda + J'_\lambda$ and $\Delta_\lambda \equiv J_\lambda - J'_\lambda$. (Note that in Ref. [14], the pairing terms are absent due to "projective symmetries.") In terms of the Bloch states, $f_{\mathbf{k}} = \sum_i \exp(-i\mathbf{k}\cdot\mathbf{r}_i) f_i/\sqrt{N}$, Eq. (10) in momentum space is given by

$$H_0 = \sum_{\mathbf{k}} \Phi_{\mathbf{k}}^\dagger H_{\mathbf{k}} \Phi_{\mathbf{k}}, \quad \Phi_{\mathbf{k}}^\dagger = (f_{\mathbf{k}}^\dagger, f_{\mathbf{k}+\mathbf{Q}}^\dagger, f_{-\mathbf{k}}, f_{-\mathbf{k}-\mathbf{Q}}), \quad (11)$$

where $\mathbf{Q} = (\pi, \pi)$ and the summation is over only half of the Brillouin zone since $\mathbf{k}$ is equivalent to $\mathbf{k} + \mathbf{Q}$. In general, the analytical form of the eigenvalues of the 4×4 matrix $H_{\mathbf{k}}$ are complicated. Because of time reversal symmetry, however, it is straightforward to derive the quasiparticle spinon excitation spectrum as follows:

$$E_{\pm,\mathbf{k}} = 2\sqrt{J_5^2 + 2g_{+,\mathbf{k}} \pm 2\sqrt{g_{-,\mathbf{k}}^2 + J_5^2 g_{\mathbf{k}}}}, \quad (12)$$

where $g_{\pm,\mathbf{k}} = (J_x^2 \pm J_x'^2)\cos^2 k_x + (J_y^2 \pm J_y'^2)\sin^2 k_y$ and $g_{\mathbf{k}} = (J_x + J'_x)^2 \cos^2 k_x + (J_y + J'_y)^2 \sin^2 k_y$.

When $J_5 = 0$, $E_{+,\mathbf{k}} = 4(J_x^2 \cos^2 k_x + J_y^2 \sin^2 k_y)^{1/2}$ and $E_{-,\mathbf{k}} = 4(J_x'^2 \cos^2 k_x + J_y'^2 \sin^2 k_y)^{1/2}$. Both spectra are gapless at nodes $\pm K = \pm(\pi/2, 0)$, around which the spectrum is linear in momentum; the spinon excitations are massless Dirac fermions. However, the massless spinons are unstable in the sense that additional small, local (further neighbor hopping and pairing) terms can gap them [16].

When $0 < J_5 \ll \Delta_v$, (so the ground state lies in the π-flux sector) it is clear that $E_{+,\mathbf{k}}$ is always gapped. The conditions for $E_{-,\mathbf{k}}$ to have gapless excitations are

$$J_x J'_x \cos^2 k_x + J_y J'_y \sin^2 k_y = J_5^2/4, \quad (13)$$

$$(J_x J'_y - J_y J'_x) \cos k_x \sin k_y = 0. \quad (14)$$

For simplicity, we consider the case in which $J_x, J_y \gg J_5 > 0$ so that $J_5 \ll \Delta_v$ is satisfied for arbitrary J'_x and J'_y. From these two conditions, we analyze the number of nodes in the spinon excitation spectrum and obtain the phase diagram accordingly, shown in Fig. 1, as a function of J'_x and J'_y: (i) When $J'_x > J_5^2/(4J_x)$, $J'_y > J_5^2/(4J_y)$, and $J'_x/J'_y \neq J_x/J_y$, the fermion spectrum has eight Dirac nodes at $(\pm\pi/2 \pm \theta_x, 0)$ and $(\pm\pi/2, \pm\theta_y)$ with $\theta_\lambda = \arcsin(J_5/\sqrt{4J_\lambda J'_\lambda})$; (ii) when $J'_x > J_5^2/(4J_x)$ and $J'_y < J_5^2/(4J_x)$, there are four Dirac nodes at $(\pm\pi/2 \pm \theta_x, 0)$; (iii) when $J'_x < J_5^2/(4J_x)$ and $J'_y > J_5^2/(4J_x)$, there are also four Dirac nodes but at $(\pm\pi/2, \pm\theta_y)$; (iv) when $J'_x < J_5^2/(4J_x)$ and $J'_y < J_5^2/(4J_y)$, all spinon excitations are gapped.

For $0 < J_5 \ll \Delta_v$, the gapless spinon excitations, if they exist, are topologically stable in the sense of Wigner–Von Neumann theorem. Any weak translational and time reversal invariant perturbation only shifts the positions of the nodes [16]. Consequently, the present phase with Dirac

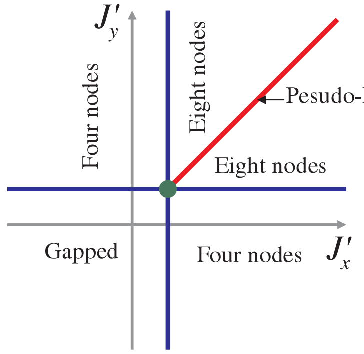

FIG. 1 (color online). The quantum phase diagram of the Gamma matrix model as a function of J'_x and J'_y in the case $J_5 \ll J_x, J_y$, where the ground state lies in the uniform π-flux sector. The Dirac nodes in the phase diagram are topologically stable. At the critical (red) line $J'_x/J'_y = J_x/J_y$, fermionic spinons form a Fermi surface (FS). The other two critical (blue) lines are defined as $J'_x = J_5^2/(4J_x)$ or $J'_y = J_5^2/(4J_y)$.

nodes is characteristic of a stable quantum phase of matter, i.e., an algebraic spin liquid [20–22].

It is worth noting that along the critical line $J'_x/J'_y = J_x/J_y$ and $J'_x > J_5^2/(4J_x)$, corresponding to the red line in Fig. 1, the discrete nodes broaden into a line of nodes. In short, in this special case the present model realizes a Fermi surface. Since spin-3/2 operators can be written as bilinear in term of Schwinger bosons, $S^\alpha = b_s^\dagger \sigma^\alpha_{ss'} b_{s'}/2$, $s = \uparrow, \downarrow$ with the constraint $b_\uparrow^\dagger b_\uparrow + b_\downarrow^\dagger b_\downarrow = 3$, the present model can be written as a bosonic model, albeit one with four-body interactions. It follows as a corollary that by tuning some coupling constants to critical values, a purely bosonic model can exhibit an emergent Fermi surface.

Upon approach to the critical line $J'_x = J_5^2/(4J_x)$ from the eight node phase, both of the two Dirac cones at $\pm(\pi/2 + \theta_x, 0)$ approach $(\pi, 0)$ leading, at criticality, to a single node with the unusual dispersion $E_{\mathbf{k}} \approx \sqrt{Aq_x^4 + Bq_y^2}$ for small $|\mathbf{q} = \mathbf{k} - (\pi, 0)|$, where A and B are constants depending on J'_λ, J_λ, and J_5. Another two nodes at $\pm(\pi/2 - \theta_x, 0)$ approach $(0, 0)$ leading to the same unusual dispersion. Similar physics is obtained at the other critical line $J'_y = J_5^2/(4J_y)$.

Large-J_5 limit.—The ground states at the limit $J_\lambda = J'_\lambda = 0$ form the low energy manifold $S_i^z = \pm 3/2$ for each i. Defining an effective spin-1/2 $\vec{\sigma}$, such that $S_i^z = \pm 3/2$ corresponds to $\sigma_i^z = \pm 1$, and employing degenerate perturbation theory in J_λ and J'_λ, the effective Hamiltonian can be shown be $H_{\text{eff}} = J_{\text{eff}} \sum_i \sigma_i^x \sigma_{i+\hat{x}}^y \sigma_{i+\hat{x}+\hat{y}}^x \sigma_{i+\hat{y}}^y$, $J_{\text{eff}} = (J_x - J'_x)^2 (J_y - J'_y)^2/(16J_5^3)$, which is exactly Wen's plaquette model [7] and is equivalent to the toric code model [23]. The visons have the same nontrivial statistics as in the toric code model and thus may be relevant in the context of topological quantum computation [24]. In terms of fluxes, $H_{\text{eff}} = \sum_i J_{\text{eff}} \exp[i\phi_i]$. Consequently, the ground state is still in the π-flux sector. We expect that the gapped phase in small J_5 limit can be adiabatically connected to the large J_5 gapped phase.

Discussion.—It is straightforward to generalize the GMM to other lattices in 2D and also to higher dimensions. For the 2D triangular lattice, a quantum spin-7/2 (or spin-$\frac{1}{2}$-$\frac{1}{2}$-$\frac{1}{2}$ [14]) model can be defined via the seven Gamma matrices. Because of the non-bi-partiteness of the lattice, the model spontaneously breaks TRS. Thus, it is expected that a chiral spin liquid [25] could be realized in this model on a triangular lattice [16].

We would like to thank E. Fradkin, Z.-C. Gu, T. Hughes, A. Kitaev, R. B. Laughlin, J. Maciejko, J. Moore, C. Nayak, X.-L. Qi, and S. Sondhi for helpful discussions. We are grateful to X.-G. Wen for pointing out a different interpretation of the present Gamma matrix model in Ref. [14]. S. A. K. and H. Y. are supported in part by DOE grant DEFG03-01ER45925. S. C. Z. is supported by NSF DMR-0342832 and DOE grant DE-AC03-76SF00515. H. Y. is supported by SGF at Stanford University.

[1] P. W. Anderson, Science **235**, 1196 (1987).
[2] For a review, see P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
[3] R. Moessner and S. Sondhi, Phys. Rev. Lett. **86**, 1881 (2001).
[4] D. S. Rokhsar and S. A. Kivelson, Phys. Rev. Lett. **61**, 2376 (1988).
[5] P. W. Anderson, Mater. Res. Bull. **8**, 153 (1973).
[6] S. A. Kivelson, D. S. Rokhsar, and J. P. Sethna, Phys. Rev. B **35**, 8865 (1987).
[7] X. G. Wen, Phys. Rev. Lett. **90**, 016803 (2003).
[8] Y. Shimizu *et al.*, Phys. Rev. Lett. **91**, 107001 (2003).
[9] S. Yamashita *et al.*, Nature Phys. **4**, 459 (2008).
[10] Y. Qi, C. Xu, and S. Sachdev, arXiv: 0809.0694.
[11] J. S. Helton *et al.*, Phys. Rev. Lett. **98**, 107204 (2007).
[12] A. Kitaev, Ann. Phys. (N.Y.) **321**, 2 (2006).
[13] S. Murakami, N. Nagaosa, and S. C. Zhang, Phys. Rev. B **69**, 235206 (2004).
[14] X.-G. Wen, Phys. Rev. D **68**, 065003 (2003).
[15] The total number of π-fluxes must be even, so the choice of which sublattice is immaterial.
[16] H. Yao *et al.* (unpublished).
[17] E. H. Lieb, Phys. Rev. Lett. **73**, 2158 (1994).
[18] G. Baskaran, S. Mandal, and R. Shankar, Phys. Rev. Lett. **98**, 247201 (2007).
[19] H.-D.Chen and J.-P.Hu, Phys. Rev. B **76**, 193101 (2007).
[20] W. Rantner and X.-G. Wen, Phys. Rev. Lett. **86**, 3871 (2001).
[21] M. Hermele *et al.*, Phys. Rev. B **70**, 214437 (2004).
[22] M. Hermele, T. Senthil, and M. P. A. Fisher, Phys. Rev. B **72**, 104404 (2005).
[23] A. Kitaev, Ann. Phys. (N.Y.) **303**, 2 (2003).
[24] C. Nayak *et al.*, Rev. Mod. Phys. **80**, 1083 (2008).
[25] H. Yao and S. A. Kivelson, Phys. Rev. Lett. **99**, 247203 (2007).