Quantum Noise Interference and Backaction Cooling in Cavity Nanomechanics

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We present a theoretical analysis of a novel cavity electromechanical system where a mechanical resonator directly modulates the damping rate κ of a driven electromagnetic cavity. We show that via a destructive interference of quantum noise, the driven cavity can effectively act like a zero-temperature bath irrespective of the ratio κ/ω_M, where ω_M is the mechanical frequency. This scheme thus allows one to cool the mechanical resonator to its ground state without requiring the cavity to be in the so-called good cavity limit $\kappa \ll \omega_M$. The system described here could be implemented directly using setups similar to those used in recent experiments in cavity electromechanics.

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Introduction.—Much of the rapid progress in fabricating and controlling nanomechanical devices has been fueled by numerous promising technological applications. However, progress has also been motivated by the realization that such systems are ideally poised to allow the exploration of several fundamental quantum mechanical effects. Various studies have addressed such issues as entanglement, quantum-limited measurement, and Fock state detection in systems containing nanoscale mechanical resonators [1–4]. The issue of quantum backaction has also received considerable attention in these systems [5,6]. Understanding the backaction properties of a detector in a quantum electromechanical or optomechanical system is necessary if one wishes to do quantum-limited position detection. Further, this backaction can in some cases be exploited to achieve significant nonequilibrium cooling of the mechanical resonator. This is of particular importance, since a prerequisite to seeing truly quantum behavior in these systems is the ability to cool the mechanical resonator to near its ground state.

A particularly effective backaction cooling scheme is ‘‘cavity cooling’’ [7–10]. Here, a mechanical resonator is dispersively coupled to a driven electromagnetic cavity (i.e., the cavity's frequency depends on the mechanical displacement x). By monitoring the frequency of the cavity, one can make a sensitive measurement of x. In addition, the cavity photon number necessarily acts as a noisy force on the mechanics; for a suitably chosen cavity drive, this force can be used to effectively cool the mechanical resonator. This approach has been used in recent experiments, both with optical cavities coupled to mechanical resonators [11,12], as well as in systems using microwave cavities [13,14]. Theoretically, it has been shown that such schemes can allow ground-state cooling of the mechanical resonator [9,10]. Similar physics even occurs in seemingly very different systems, e.g., a superconducting single-electron transistor coupled to a mechanical resonator [15,16].

In this Letter, we present a theoretical analysis of a generic electromechanical (or optomechanical) system which is the dual of the dispersive coupling discussed above. We again consider a driven cavity coupled to a mechanical resonator: now, however, the mechanical displacement x does not change the cavity frequency, but rather changes its damping rate κ. In the ideal case, this damping will be dominated by the coupling of the cavity to the port used to drive it. As we will discuss, such a dissipative coupling arises naturally in systems where a microwave cavity is coupled to a nanomechanical resonator [17]; it could also be realized in an optical cavity containing a moveable ‘‘mebrane in the middle’’ [12,18]. We show that for such a dissipative cavity-mechanical resonator coupling, interference effects are important in determining the quantum backaction effects on the mechanical system; this is not the case for a purely dispersive coupling. In particular, we show that one can use destructive interference to allow the cavity to act as an effective zero-temperature bath, irrespective of the ratio of the mechanical frequency ω_M to the cavity linewidth κ; as such, ground-state cooling is possible without requiring the ‘‘good cavity’’ limit $\omega_M \gg \kappa$. This is in sharp contrast to the case of a dispersive coupling, where ground-state cooling is only possible if $\omega_M \gg \kappa$. From a practical perspective, being able to deviate from the good cavity limit is advantageous, as it allows one to use small drive detunings and hence achieve much larger effective cavity-mechanical resonator couplings. We show that this destructive interference effect persists in the case where one has both a dissipative and dispersive coupling; we also show that a dissipative coupling can allow for a quantum-limited position measurement.

Model.—We consider a mechanical oscillator (frequency ω_M, displacement x) whose motion weakly modulates the damping rate κ and resonant frequency ω_R of a driven electromagnetic cavity. For small displacements, both ω_R and κ will have a linear dependence on x, and we can describe the system via the Hamiltonian ($\hbar = 1$): $\hat{H} = \omega_R \hat{a}^\dagger \hat{a} + \omega_M \hat{c}^\dagger \hat{c} + \sum_q \omega_q \hat{b}_q^\dagger \hat{b}_q + \hat{H}_{\text{damp}} + \hat{H}_{\text{int}} + \hat{H}_\gamma$. The first two terms describe the cavity and mechanical

Hamiltonians, while H_γ describes the intrinsic mechanical damping by an equilibrium bath at temperature $T_{\rm eq}$. The third term in $\hat{H}$ describes the bosonic bath responsible for the dissipation and driving of the cavity. Working in the usual Markovian limit where $\kappa \ll \omega_R$, and where the bath density of states ρ can be treated as energy independent over relevant frequencies, the cavity-bath interaction takes the form [19,20]

$$\hat{H}_{\rm damp} = -i\sqrt{\frac{\kappa}{2\pi\rho}}\sum_q (\hat{a}^\dagger \hat{b}_q - \hat{b}_q^\dagger \hat{a}). \tag{1}$$

The cavity-mechanical coupling is then described by

$$\hat{H}_{\rm int} = (\hat{x}/x_{\rm zpt})(\tfrac{1}{2}\tilde{B}\hat{H}_{\rm damp} + \tilde{A}\kappa\hat{a}^\dagger\hat{a}). \tag{2}$$

The dimensionless coupling strengths $\tilde{A}$, $\tilde{B}$ above are given by $\tilde{B}\kappa = (d\kappa/dx)x_{\rm zpt}$, $\tilde{A}\kappa = (d\omega_R/dx)x_{\rm zpt}$, where $x_{\rm zpt}$ is the zero-point motion amplitude of the mechanical oscillator.

To proceed, we linearize the full quantum dynamics about the solutions of the classical equations of motion for the uncoupled cavity, making use of the input-output formalism of quantum optics [19,20]. Working in frame rotating at the drive frequency, and writing $\hat{a} = \bar{a} + \hat{d}$ (where $\bar{a} = \langle\hat{a}\rangle$), the linearized Heisenberg equations of motion take the form

$$\dot{\hat{d}} = \left(i\Delta - \frac{\kappa}{2}\right)\hat{d} - \sqrt{\kappa}\hat{\xi} - \kappa\left[\frac{\tilde{B}}{2}\left(\bar{a} + \frac{\bar{b}_{\rm in}}{\sqrt{\kappa}}\right) + i\tilde{A}\bar{a}\right](\hat{c} + \hat{c}^\dagger), \tag{3}$$

$$\dot{\hat{c}} = -\left(i\omega_M + \frac{\gamma}{2}\right)\hat{c} - \sqrt{\gamma}\hat{\eta} + ix_{\rm zpt}\hat{F}. \tag{4}$$

Here $\bar{b}_{\rm in}$ is the amplitude of the coherent cavity drive, $\Delta = \omega_{\rm drive} - \omega_R$ is the detuning of the cavity drive, $\hat{\xi}$ describes vacuum noise entering the cavity from the drive port, and $\hat{\eta}$ describes thermal noise associated with the mechanical damping γ [19,20]. The backaction force operator $\hat{F} = -(d/dx)\hat{H}_{\rm int}$ in Eq. (4) takes the form

$$\hat{F} = -\left(\frac{\tilde{A}\kappa}{x_{\rm zpt}}\bar{a}^*\hat{d} - \frac{i\tilde{B}\sqrt{\kappa}}{2x_{\rm zpt}}(\bar{a}^*\hat{\xi} - \bar{b}_{\rm in}^*\hat{d})\right) + \text{H.c.} \tag{5}$$

Quantum noise.—For a sufficiently weak optomechanical coupling, linear-response theory applies, and the unperturbed quantum noise spectrum of $\hat{F}$ (i.e., calculated at $\tilde{A} = \tilde{B} = 0$) determines both the backaction damping and heating of the mechanical resonator by the driven cavity [15,20]. The relevant spectrum is $S_{FF}(\omega) = \int d\tau e^{i\omega\tau}\langle\hat{F}(\tau)\hat{F}(0)\rangle_0$. Recall that $S_{FF}(\omega_M)$ is proportional to the Fermi's golden rule rate for the absorption of a mechanical quantum by the driven cavity, while $S_{FF}(-\omega_M)$ is proportional to the corresponding emission rate. The effective temperature associated with the backaction noise as seen by the mechanical oscillator is then given by $k_B T_{\rm eff} \equiv \omega_M(\log[S_{FF}(\omega_M)/S_{FF}(-\omega_M)])^{-1}$, while the backaction damping is given by $\gamma_{\rm BA} = x_{\rm zpt}^2[S_{FF}(\omega_M) - S_{FF}(-\omega_M)]$. These linear-response results presume the total mechanical damping rate to be small enough that the resonator is only sensitive to noise at $\omega = \pm\omega_M$; as we will see, deviations will occur when the effective optomechanical coupling becomes large enough to make $\gamma_{\rm BA}$ comparable to ω_M.

One finds from Eqs. (3) and (4) that the uncoupled cavity's backaction force noise spectrum $S_{FF}(\omega)$ is given by

$$S_{FF}(\omega) = \kappa\left(\frac{\tilde{B}|\bar{a}|}{2x_{\rm zpt}}\right)^2 \frac{[\omega + 2\Delta - \frac{2\tilde{A}}{\tilde{B}}\kappa]^2}{(\omega + \Delta)^2 + \kappa^2/4}. \tag{6}$$

In the limit $\tilde{B} \to 0$ of a purely dispersive coupling, Eq. (6) reduces to a simple Lorentzian [9,10]. This form has a simple interpretation: it corresponds to the Lorentzian density of final states relevant to a Raman process where an incident drive photon gains an energy $\hbar\omega$ while attempting to enter the cavity. The optimal backaction cooling discussed in Refs. [9,10] requires $\Delta = -\omega_M$ and $\kappa \ll \omega_M$. For these parameters, the drive photons are initially far from being on resonance with the cavity. The absorption of energy from the oscillator is resonantly enhanced, as it moves an incident drive photon onto the cavity resonance. In contrast, emission of energy to the oscillator is greatly suppressed, as the drive photon is moved even further from resonance. Thus, the $\tilde{B} = 0$ form of $S_{FF}(\omega)$ and the resulting cooling are simply explained as a density of states effect.

In the more general case where the dissipative optomechanical coupling $\tilde{B}$ is also nonzero, $S_{FF}(\omega)$ is not a simple Lorentzian; as such, the backaction physics cannot be interpreted solely as a density of states effect. In general, $S_{FF}(\omega)$ has an asymmetric Fano line shape (see Fig. 1), with the cavity emission noise $S_{FF}(-\omega_M)$ vanishing whenever the detuning $\Delta = \omega_M/2 + (\tilde{A}/\tilde{B})\kappa \equiv \Delta_{\rm opt}$. Thus, for this optimal detuning, one finds that the cavity acts as an effective zero-temperature bath, irrespective of the ratio κ/ω_M. For $\Delta = \Delta_{\rm opt}$, one has

$$\gamma_{\rm BA,opt} = \tilde{B}^2|\bar{a}|^2\kappa\frac{\omega_M^2}{[3\omega_M/2 + (\tilde{A}/\tilde{B})\kappa]^2 + \kappa^2/4}. \tag{7}$$

For $\Delta = \Delta_{\rm opt}$, the weak-coupling, quantum noise approach yields the equilibrium number of quanta in the oscillator to be $n_{\rm osc} = \gamma n_{\rm eq}/(\gamma_{\rm BA,opt} + \gamma)$, where γ is the intrinsic damping rate of the oscillator, and $n_{\rm eq}$ is the Bose-Einstein factor associated with the bath temperature $T_{\rm eq}$. One can thus cool to the ground state for a sufficiently large coupling and/or cavity drive. This possibility of ground-state cooling for an arbitrary κ/ω_M ratio is a main result of this Letter. Note that the optimal detuning $\Delta_{\rm opt}$ is greater than 0: ground-state cooling is possible even though the drive photons would seemingly need to "burn off" energy to enter the cavity. This is in stark contrast to the purely dispersive case where a positive detuning leads to heating and a negative-damping instability. Note finally that there is also an "optical spring" effect associated with the backaction; it can be obtained directly from $S_{FF}(\omega)$ [9].

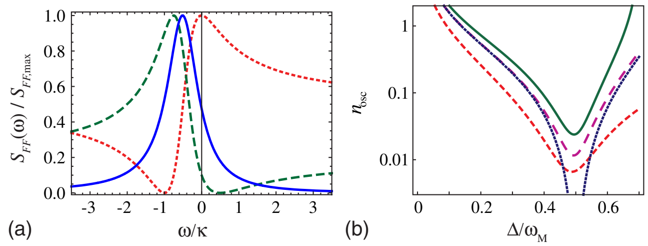

FIG. 1 (color online). (a) Backaction force noise spectral density for $\Delta = \kappa/2$, for different ratios of the dispersive to dissipative optomechanical couplings. The solid blue curve corresponds to $\tilde{A}/\tilde{B} = 10$, the long-dashed green curve to $\tilde{A}/\tilde{B} = 0.75$, and the short-dashed red curve to $\tilde{A}/\tilde{B} = 0$. Each curve has been scaled by its maximum value. (b) Number of oscillator quanta $n_{\rm osc}$ versus drive detuning Δ, obtained by solving the full equations of motion for a purely dissipative optomechanical coupling (i.e., $\tilde{A} = 0$). The dotted blue curve is the Bose-Einstein factor $n_{\rm eff}$ corresponding to the backaction effective temperature $T_{\rm eff}$ when $\kappa = \omega_M$ [i.e., $n_{\rm eff} = S_{FF}(-\omega_M)/[S_{FF}(\omega_M) - S_{FF}(-\omega_M)]$]. The remaining curves correspond to $\tilde{B}\,\bar{a} = 0.2$, $n_{\rm eq} = 50$, $\gamma = 10^{-6}\omega_M$, and $\kappa/\omega_M = 0.2$ (green solid curve), $\kappa/\omega_M = 1.0$ (purple long-dashed curve), $\kappa/\omega_M = 5.0$ (red short-dashed curve).

Fano line shapes arise generically as a result of interference between resonant and nonresonant processes; the situation is no different here. In the usual $\tilde{B} = 0$ case, the only source of backaction force noise is the number fluctuations of the cavity field $\hat{a}$. However, when $\tilde{B} \neq 0$, the mechanical oscillator mediates the coupling between the cavity and the cavity's dissipative bath. As a result, it is subject to two sources of noise, corresponding to the two terms in Eq. (5): the shot noise associated with the driving of the cavity, as well as the fluctuations of $\hat{a}$. The first of these noise processes is white, whereas the second is not: it is simply the shot noise incident on the cavity filtered by the ω-dependent cavity susceptibility. The interference between these two noises yields a Fano line shape for $S_{FF}(\omega)$, and the destructive interference at $\Delta = \Delta_{\rm opt}$ which causes $T_{\rm eff} = 0$. Note that Fano interference in electromechanical systems has recently been discussed in Ref. [21], albeit in a very different context.

Full solution.—To address whether the destructive noise interference effect persists beyond the simplest weak-coupling regime, we now examine the full solution of Eqs. (3) and (4); for simplicity, we focus on $\tilde{A} = 0$. One finds for the oscillator spectrum $S_{cc} = \int dt \langle \hat{c}^\dagger(t)\hat{c}(0)\rangle e^{-i\omega t}$,

$$S_{cc}(\omega) = |\tilde{\chi}_M(-\omega)|^2[\sigma_{\rm eq}(-\omega) + S_{FF}(\omega)], \tag{8}$$

where

$$\tilde{\chi}_M(\omega) = \chi_M(\omega)/[1 + i\chi_M(\omega)\Sigma(\omega)], \tag{9}$$

$$\Sigma(\omega) = -i\tilde{B}^2|\bar{a}|^2\chi_M^*(-\omega)\chi_R(\omega)\chi_R^*(-\omega) \times \omega_M\Delta[\Delta^2 - \tfrac{3}{4}\kappa^2 + i\omega\kappa], \tag{10}$$

$$\sigma_{\rm eq}(\omega) = \gamma\left[n_{\rm eq}\left(1 + \frac{{\rm Re}\Sigma(\omega)}{\omega_M}\right) + \left|\frac{\Sigma(\omega)}{2\omega_M}\right|^2(1 + 2n_{\rm eq})\right]. \tag{11}$$

Here, we denote the bare mechanical and cavity susceptibilities by $\chi_M(\omega) = [-i(\omega - \omega_M) + \gamma/2]^{-1}$ and $\chi_R(\omega) = [-i(\omega + \Delta) + \kappa/2]^{-1}$. Equation (8) has a simple interpretation: the oscillator responds with a modified susceptibility $\tilde{\chi}_M$ to two independent fluctuating forces, corresponding to the two terms in the equation. The first, described by $\sigma_{\rm eq}$, is the fluctuating thermal force associated with the intrinsic oscillator damping γ. The second is the backaction from the driven cavity. We see that its form is not affected by the coupling strength: the same spectral density $S_{FF}(\omega)$ (evaluated at $\tilde{A} = 0$) found earlier in the weak-coupling regime [c.f. Eq. (6)] appears here. Thus, the quantum noise interference discussed above continues to play a role even for moderate coupling strengths. A strong cavity-mechanical resonator coupling will nonetheless modify the physics, as the destructive interference occurring when $\Delta = \Delta_{\rm opt}$ only occurs at the single frequency ω_M. For a sufficiently strong coupling, the oscillator's total damping will become large enough that the oscillator will be sensitive to noise at frequencies away from ω_M, frequencies where the destructive interference is not complete. As a result, the cavity will no longer appear to the oscillator as an effective zero-temperature bath. Note that when $\tilde{A} \neq 0$, the spectrum $S_{cc}(\omega)$ still has the general form given by Eqs. (8), (9), and (11): one now simply uses the full form of $S_{FF}(\omega)$, as well as a self-energy that contains extra terms $\propto \tilde{A}$.

To see whether this resonance-broadening effect as well as other strong-coupling effects preclude ground-state cooling at the optimal detuning $\Delta = \Delta_{\rm opt}$, we calculate the average number of oscillator quanta $n_{\rm osc}$ directly by integrating $S_{cc}(\omega)$ in Eq. (8). In Fig. 2 we show the expected cooling for realistic parameter values, as a function of the cavity drive strength. Strong-coupling effects lead to an

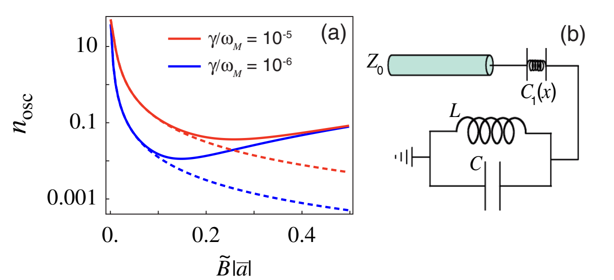

FIG. 2 (color online). (a) Number of mechanical quanta $n_{\rm osc}$ versus coupling strength $\tilde{B}|\bar{a}|$, for $\tilde{A} = 0$, $n_{\rm eq} = 50$, $\kappa/\omega_M = 1$ and for an optimal detuning $\Delta = \omega_M/2$; the mechanical damping γ is as marked. Dashed curves are results from the linear-response, quantum noise approach, whereas solid curves are obtained from the full solutions of the equations of motion. (b) Schematic of a cavity (modeled as an LC resonator) coupled to a transmission line (impedance Z_0) via an x-dependent capacitance $C_1(x)$.

optimal drive strength, beyond which $n_{\rm osc}$ starts to increase; this additional heating effect scales as $(\gamma_{\rm BA}/\omega_M)^2$. Nonetheless, the minimum value of $n_{\rm osc}$ can still be significantly less than 1.

Physical realization.—The dissipative optomechanical coupling analyzed here could be realized in a microwave electromechanical system similar to those studied in Refs. [13,14]. In such systems, a capacitor C_1 couples the cavity to a transmission line (impedance Z_0) which both drives and damps the cavity. One would now need to make C_1 mechanically compliant [see Fig. 2(b)]. In general, such a setup will have both dissipative and dispersive optomechanical couplings (i.e., $\tilde{A} \neq 0$, $\tilde{B} \neq 0$). A careful analysis shows that in the physically relevant regime $C_1 \ll C$, one can have $\tilde{A} \simeq \tilde{B}$ if the cavity impedance $Z_R = \sqrt{L/C}$ is made slightly smaller than $Z_0 \simeq 50\Omega$. For example, taking experimentally achievable parameters $\omega_R = 2\pi \times 10\,\text{GHz}$, $C = 3.2\,\text{pF}$ and $C_1 = 0.01C$ results in $\kappa/\omega_R \simeq 10^{-3}$ and $|\tilde{A}/\tilde{B}| \simeq 2.5$. Equation (6) then implies that ground-state cooling via destructive noise interference is possible if one uses a drive detuning $\Delta = \Delta_{\rm opt} = \omega_M/2 + 2.5\kappa$. We stress this conclusion is independent of the magnitude of ω_M/κ; in contrast, if one had a purely dispersive cavity-mechanical resonator coupling, ground-state cooling is only possible if $\omega_M \gg \kappa$. A second setup where the dissipative optomechanical coupling could be realized is an optical Fabry-Pérot cavity containing a thin, moveable dielectric slab. If the cavity is constructed with mirrors having different reflectivities κ_L and κ_R, the motion of the membrane will naturally modulate the damping rate of the optical modes. The resulting value of $\tilde{B}$ can be easily derived using the approach of Ref. [18]. Note that in such a system, one can position the membrane such that the optical mode frequency is insensitive to linear changes in x [12]: one thus naturally achieves $\tilde{A} = 0$ and $\tilde{B} \propto (\kappa_R - \kappa_L) \neq 0$.

Position detection.—A straightforward calculation shows that the power spectrum of $\hat{b}_{\rm out}$, the output field from the cavity, is modified in a simple way by the coupling to the oscillator: the oscillator adds a term $S_{xx}(-\omega)S_{FF}(\omega)$ (where ω is measured from the drive frequency); this holds for both $\tilde{A}$, $\tilde{B} \neq 0$. Here, $S_{xx}(\omega) = \int d\tau e^{i\omega\tau}\langle\hat{x}(\tau)\hat{x}(0)\rangle$. One can thus use the peaks at $\omega = \pm\omega_M$ in the output spectrum to infer the temperature of the oscillator. Note that for an optimal detuning $\Delta = \Delta_{\rm opt}$, the destructive interference effect means that the peak at $\omega = \omega_M$ in the output spectrum will vanish. This could serve as an interesting test of our predictions.

Finally, specializing to the case where $\omega_M \ll \kappa$, one can also use the dissipative optomechanical coupling to make a quantum-limited, weak continuous position measurement of the oscillator. One drives the cavity off resonance, and performs a homodyne detection of the appropriate quadrature of the output field from the cavity. One finds from Eqs. (3) and (4) that for any nonzero detuning Δ, the corresponding backaction noise is as small as is allowed by the uncertainty principle, meaning that one can reach the quantum limit on the total added noise [20].

Conclusions.—We have considered a generic system where a mechanical resonator modulates the damping of a driven quantum cavity, and demonstrated how Fano interference is important to the backaction physics. In particular, one can have a perfect destructive interference which allows the backaction to mimic a zero-temperature environment; as such, such a system should allow near ground-state cooling of the mechanics.

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