Order Parameter and Vortices in the Superconducting Q Phase of $CeCoIn_5$

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Recently, it has been reported that the low-temperature high-magnetic field superconducting phase in $CeCoIn_5$ (Q phase), has spin-density wave (SDW) order that only exists within this phase. This indicates that the SDW order is the result of the development of pair density wave (PDW) order in the superconducting phase that coexists with d-wave superconductivity. Here we develop a phenomenological theory for these coexisting orders. This provides selection rules for the PDW order and further shows that the detailed structure of this order is highly constrained. We then apply our theory to the vortex phase. This reveals vortex phases in which the d-wave vortex cores exhibit charge density wave order and further reveals that the SDW order provides detailed information about the vortex phase.

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The low-temperature high-magnetic field phase in $CeCoIn_5$ (Q phase) has been thought to be the best example of a Fulde, Ferrell, Larkin, and Ovchinnikov (FFLO) superconductor [1–4] and has thus generated a tremendous interest [5,6]. However, the recent measurements of Kenzelmann *et al.* [7], suggest that this point of view should be altered. This important experimental discovery shows that the Q phase reveals itself through the appearance of an incommensurate spin-density wave (SDW) order. What is striking about this SDW order is that it vanishes when superconductivity vanishes at high-magnetic fields. This implies that superconducting order is the primary order parameter with the SDW order induced as a secondary order parameter. A possibility for such superconducting order, as pointed out by Kenzelmann *et al.* [7], is pair density wave (PDW) superconductivity. PDW order is defined microscopically through the expectation value $\Delta_{\sigma,\sigma'}(\mathbf{r},\mathbf{r}') = \langle \psi^\dagger_\sigma(\mathbf{r})\psi^\dagger_{\sigma'}(\mathbf{r}')\rangle$ with periodicity in the center of mass coordinate $(\mathbf{r}+\mathbf{r}')/2$, so that the Fourier transform with respect to this coordinate is peaked at a nonzero $\boldsymbol{Q}$ value. Psaltakis and Fenton have shown that PDW order coexisting with usual superconducting order implies the existence of SDW order [8]. If the SDW order is associated with a wave vector $\boldsymbol{Q}$, then the PDW order must have the wave vector $-\boldsymbol{Q}$ to be able to induce the SDW order. The SDW order has $\boldsymbol{Q} = (q, q, 0.5)$, which is too large to be a consequence of the long-wavelength modulation of a FFLO phase [3,4,9]. The PDW order is more akin to the π-triplet staggered pairing suggested by Aperis [10] or to the PDW order suggested in $La_{2-x}Ba_xCuO_4$ at $x = 1/8$ [11]. The ensuing physical picture is then a d-wave superconductor at low fields with PDW order appearing through a second order phase transition at high fields. These two types of superconducting order will coexist in the Q phase (together with the SDW order).

The observation of this PDW order raises a series of deep questions about the origin of this phase. To help address these, we have developed a phenomenological theory for this PDW order. Our approach is based on irreducible representations of the full space group and complements that given by Kenzelmann *et al.* We apply this theory to the vortex lattice phase.

Prior to presenting our detailed results, it is worthwhile highlighting our most important findings. These are presented in Table I, Fig. 2, and Eq. (13). Table I provides a succinct classification of the possible types of PDW order. The PDW order parameter has an independent complex degree of freedom for each $\mathbf{Q}_i$ shown in Fig. 1. Figure 2 reveals the existence of a class of vortex lattice solutions in which three independent superconducting degrees of freedom (two PDW degrees of freedom and the d-wave degree of freedom) *all have vortex cores at different positions*. Finally, Eq. (13) reveals that an experimental investigation of the position of the SDW order Bragg peaks will reveal, not only the vortex lattice, but also the relative displacements between the vortex cores of the two PDW and d-wave order parameters. Now we turn to a detailed derivation of these results.

PDW superconducting order parameter.—Our approach is to classify the PDW order in terms of irreducible representations of the full space group [12]. For $CeCoIn_5$ this is $P4/mmm$. For order appearing at a wave vector $\boldsymbol{Q}$, the order parameter is defined by the irreducible representations of G_Q (set of elements conserving $\boldsymbol{Q}$) and the star of the wave vector $\boldsymbol{Q}$ (set of wave vectors symmetrically equivalent to $\boldsymbol{Q}$). For $\boldsymbol{Q} = (q, q, 0.5) = (q, q, -0.5)$ $G_Q = \{E, C_{2\eta}, \sigma_z, \sigma_\zeta\}$ with $C_{2\eta}$ the 180°-rotation around the axis (1, 1, 0), σ_z and σ_ζ the mirror operations at the basal plane and the plane perpendicular to (1, −1, 0), respectively. Note (0, 0, 1) is a reciprocal lattice vector. In Table I, we give the irreducible representations of G_Q together with representative basis functions for spin-singlet pairing (scalar functions $\psi(\boldsymbol{k})$ [13]), spin-triplet pairing (vector functions $\boldsymbol{d}(\boldsymbol{k})$ [13]), and spin-density order (S_i). To define the

TABLE I. Representative spin-singlet, spin-triplet, and spin-density basis functions for the different irreducible representations that have momentum $\boldsymbol{Q}_1 = (q, q, 0.5)$.

Irrep (Γ_i)	$D_{\Gamma_i}(E)$	$D_{\Gamma_i}(\sigma_z)$	$D_{\Gamma_i}(C_{2\eta})$	$D_{\Gamma_i}(\sigma_\zeta)$	Representative $\psi(\boldsymbol{k})$)	Representative $\boldsymbol{d}(\boldsymbol{k})$	Representative S_i
Γ_1	1	1	1	1	$s,\ k_xk_y$	$\hat{z}(k_x - k_y),\quad k_z(\hat{x} - \hat{y}),$	
Γ_2	1	1	-1	-1	$k_x^2 - k_y^2$	$\hat{z}(k_x + k_y),\quad k_z(\hat{x} + \hat{y})$	S_z
Γ_3	1	-1	-1	1	$k_z(k_x + k_y)$	$\hat{x}k_x - \hat{y}k_y,\quad \hat{x}k_y - \hat{y}k_x$	$S_x - S_y$
Γ_4	1	-1	1	-1	$k_z(k_x - k_y)$	$\hat{x}k_x + \hat{y}k_y,\quad \hat{x}k_y + \hat{y}k_x$	$S_x + S_y$

additional order parameter components at the wave vectors in the star of $\boldsymbol{Q}$ we use the elements $\{E, C_4, C_4^2, C_4^3\}$ (these give the star of $\boldsymbol{Q}$, $\{\boldsymbol{Q}_1, \boldsymbol{Q}_2, \boldsymbol{Q}_3, \boldsymbol{Q}_4\}$ respectively, as shown in Fig. 1). This then defines a superconducting order parameter with four components which we define as $\Delta_{\Gamma_i} = (\Delta_{\Gamma_i,Q_1}, \Delta_{\Gamma_i,Q_2}, \Delta_{\Gamma_i,Q_3}, \Delta_{\Gamma_i,Q_4})$. With these definitions, the symmetry properties of the order parameter are given as follows [$D_{\Gamma_i}(g)$ defined in Table I]: translation $\boldsymbol{T}$, $\Delta_{\Gamma_i,Q_j} \rightarrow e^{i\boldsymbol{Q}_j\cdot\boldsymbol{T}}\Delta_{\Gamma_i,Q_j}$ ($\Delta^*_{\Gamma_i,Q_j} \rightarrow e^{-i\boldsymbol{Q}\cdot\boldsymbol{T}j}\Delta^*_{\Gamma_i,Q_j}$); time-reversal operation $\Delta_{\Gamma_i,Q_j} \rightarrow \Delta^*_{\Gamma_i,-Q_j}$. Moreover, the transformations G_Q lead to

$$\begin{aligned} C_4&: D_{\Gamma_i}(C_4)(\Delta_{\Gamma_i,Q_2}, \Delta_{\Gamma_i,Q_3}, \Delta_{\Gamma_i,Q_4}, \Delta_{\Gamma_i,Q_1}),\\ \sigma_z&: D_{\Gamma_i}(\sigma_z)(\Delta_{\Gamma_i,Q_1}, \Delta_{\Gamma_i,Q_2}, \Delta_{\Gamma_i,Q_3}, \Delta_{\Gamma_i,Q_4}),\\ C_{2\eta}&: D_{\Gamma_i}(C_{2\eta})(\Delta_{\Gamma_i,Q_1}, \Delta_{\Gamma_i,Q_4}, \Delta_{\Gamma_i,Q_3}, \Delta_{\Gamma_i,Q_2}),\\ \sigma_\zeta&: D_{\Gamma_i}(\sigma_\zeta)(\Delta_{\Gamma_i,Q_1}, \Delta_{\Gamma_i,Q_4}, \Delta_{\Gamma_i,Q_3}, \Delta_{\Gamma_i,Q_2}). \end{aligned} \tag{1}$$

Table I reveals that both singlet and triplet order parameters belong to the same representation which implies that singlet and triplet superconductivity are mixed. The phenomenological theory below automatically incorporates this mixing.

Free energy and PDW solutions.—We use a Ginzburg Landau (GL) theory to describe the PDW and d-wave order parameters. This will allow us to correctly identify the properties of the PDW order and make robust experimental predictions. The PDW free energy density is constructed by imposing invariance under the above symmetries (this is the same for all Γ_l),

$$\begin{aligned} f =& \alpha\sum_i|\Delta_{\Gamma_l,Q_i}|^2 + \beta_1\Big(\sum_i|\Delta_{\Gamma_l,Q_i}|^2\Big)^2\\ &+ \beta_2\sum_{i<j}|\Delta_{\Gamma_l,Q_i}|^2|\Delta_{\Gamma_l,Q_j}|^2 + \beta_3(|\Delta_{\Gamma_l,Q_1}|^2|\Delta_{\Gamma_l,Q_3}|^2\\ &+ |\Delta_{\Gamma_l,Q_2}|^2|\Delta_{\Gamma_l,Q_4}|^2) + \beta_4[\Delta_{\Gamma_l,Q_1}\Delta_{\Gamma_l,Q_3}(\Delta_{\Gamma_l,Q_2}\Delta_{\Gamma_l,Q_4})^*\\ &+ (\Delta_{\Gamma_l,Q_1}\Delta_{\Gamma_l,Q_3})^*\Delta_{\Gamma_l,Q_2}\Delta_{\Gamma_l,Q_4}] + \kappa_1\sum_i|\boldsymbol{D}\Delta_{\Gamma_l,Q_i}|^2\\ &+ \kappa_2\sum_i(-1)^i(|D_1\Delta_{\Gamma_l,Q_i}|^2 - |D_2\Delta_{\Gamma_l,Q_i}|^2)\\ &+ \kappa_3\sum_i|D_z\Delta_{\Gamma_l,Q_i}|^2 + \frac{1}{2}(\boldsymbol{\nabla}\times\boldsymbol{A})^2, \end{aligned} \tag{2}$$

where $\boldsymbol{D} = -i\boldsymbol{\nabla} - 2e\boldsymbol{A}$, $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, D_1 corresponds to the (1, 1, 0), and D_2 to the (1, -1, 0) direction. The free energy density for the d-wave order parameter is

$$f_d = \alpha_d|\Delta_d|^2 + \beta_d|\Delta_d|^4 + \kappa|\boldsymbol{D}\Delta_d|^2 + \kappa_c|D_z\Delta_d|^2. \tag{3}$$

The coupling between these order parameters is given by (this is the same for all Γ_l):

$$\begin{aligned} f_c =& \beta_{c1}\sum_i|\Delta_d|^2|\Delta_{\Gamma_l,Q_i}|^2 + \beta_{c2}[\Delta_d^2(\Delta_{\Gamma_l,Q_1}\Delta_{\Gamma_l,Q_3}\\ &+ \Delta_{\Gamma_l,Q_2}\Delta_{\Gamma_l,Q_4})^* + \text{c.c.}]. \end{aligned} \tag{4}$$

This free energy is similar to one studied earlier in the context of PDW order in Ref. [14]. The ''homogeneous'' phase in the absence of a magnetic field has five PDW states distinct by symmetry, if we ignore the d-wave phase. The presence of a d-wave order parameter selects two of these phases [14] (the phase factors ϕ_1, ϕ_2, and ϕ_3 are not determined by the free energy):

$$\begin{aligned} \Delta^{(1)}_{\Gamma_l} &= (e^{i\phi_1}, 0, e^{i\phi_3}, 0),\\ \Delta^{(2)}_{\Gamma_l} &= (e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, e^{i(\phi_1+\phi_3-\phi_2)}). \end{aligned} \tag{5}$$

Finally, a magnetic field along the (1, -1, 0) direction favors the state $\Delta^{(1)}_{\Gamma_l}$ as it removes the degeneracy between the $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ wave vectors (the pairs $\boldsymbol{Q}_1$, $\boldsymbol{Q}_3$and $\boldsymbol{Q}_2$, $\boldsymbol{Q}_4$ remain degenerate). The PDW order then has the spatial dependence $\Delta_{\Gamma_l,Q_1}\cos(\boldsymbol{Q}_1\cdot\boldsymbol{R})$. In view of the coupling to the d-wave order parameter the relative phase between Δ_d and Δ_{Γ_l,Q_1} can be either 0 (π) or $\pm\pi/2$ [14] which are both permitted by the free energy. Therefore, the combined PDW and d-wave superconductivity must then take one

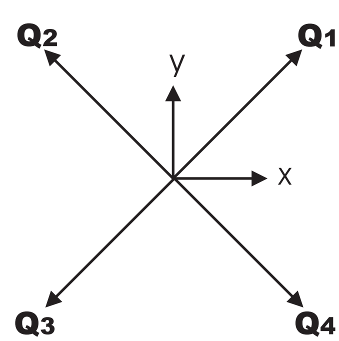

FIG. 1. Directions of $\boldsymbol{Q}_i$ used in the text. The field is applied along the direction $\boldsymbol{Q}_4$.

of two forms when vortices are ignored: $|\Delta_d| + i|\Delta_{\Gamma_i,1}|\cos(\boldsymbol{Q}_1 \cdot \boldsymbol{R})$ (time-reversal violating phase) or $|\Delta_d| + |\Delta_{\Gamma_i,1}|\cos(\boldsymbol{Q}_1 \cdot \boldsymbol{R})$ (time-reversal-invariant phase).

Coupling to spin-density wave.—We assume here that the SDW is sufficiently weak so as to not alter the free energy significantly. This is justified by noting that the observed moment in $CeCoIn_5$ is small ($0.15\mu_B$) [7]. The free energy density we use is $f_{\rm SDW} = \alpha_s S^z_{Q_1} S^z_{-Q_1} + f_{\rm coupling}$ with $\alpha_s > 0$. To determine $f_{\rm coupling}$, it is important to note that the SDW order breaks time reversal and can be generated by the PDW and d-wave order in two ways. The first is by coupling directly to the time-reversal symmetry violating phase and the second is by coupling to the applied magnetic field and the time-reversal-invariant phase. This leads to two possible coupling terms, the first exists without the magnetic field,

$$\gamma_1 i S^z_{Q_1}\{\Delta^*_d \Delta_{\Gamma_1,Q_3} - \Delta_d \Delta^*_{\Gamma_1,Q_1}\} + \text{c.c.}, \tag{6}$$

and the second exists only in a finite magnetic field

$$\gamma_2 H S^z_{Q_1}\{\Delta^*_d \Delta_{\Gamma_4,Q_3} + \Delta_d \Delta^*_{\Gamma_4,Q_1}\} + \text{c.c.}, \tag{7}$$

where we have included H, the magnetic field along the (1, −1, 0) direction. The experimental observation of a nonzero $S^z_{Q_1}$ therefore leads to two possible types of PDW order. In the time-reversal broken phase, the PDW order must belong to the Γ_1 representation. In the time-reversal-invariant phase, the PDW order must belong to the Γ_4 representation. This second possibility is most closely related to the π-triplet staggered phase that has been found within a simple microscopic description of $CeCoIn_5$ [10]. Note that, in principle, both the representations Γ_1 and Γ_4 will appear simultaneously. However, it is reasonable to expect that one of the two representations will give rise to the dominant order parameter.

Role of vortices.—Prior to turning to the detailed analysis, we present the two main results here: (i) The vortex cores of the two PDW degrees of freedom Δ_{Γ_i,Q_1} and Δ_{Γ_i,Q_3} can lie at different positions and also need not coincide with the d-wave vortex cores. We find that there exist stable phases where this happens. These phases are defined by the relative displacements $\boldsymbol{\tau}_i$ of the PDW vortex cores from the d-wave vortex cores. In such phases, the d-wave vortex cores exhibit CDW order. (ii) The SDW order leads to Bragg peaks that are determined by the reciprocal lattice vectors of the vortex lattice *and the displacements* $\boldsymbol{\tau}_i$ [see Eq. (13)].

For a detailed derivation of the above results, we analyze the simplest realistic situation. We assume that the correlation length of the spin-density order is much smaller than the coherence length of the superconducting order [this simplification does not change the main result found in Eq. (13)]. We take Eq. (7) as the term driving the SDW order [the same arguments can be applied if Eq. (6) is used]. From this we obtain

$$S^z_{Q_1}(\boldsymbol{R}) = \frac{\gamma_2 H}{\alpha_s}[\Delta_d(\boldsymbol{R})^* \Delta_{\Gamma_4,Q_1}(\boldsymbol{R}) + \Delta_d(\boldsymbol{R})\Delta^*_{\Gamma_4,Q_3}(\boldsymbol{R})]. \tag{8}$$

The spatial dependence of the PDW and d-wave order parameter can now be determined in the high-field limit for which the field H may be considered uniform. From Eq. (3), one finds that the d-wave component yields an Abrikosov vortex lattice. Using z to represent the (0, 0, 1) and x the (1, 1, 0) direction, the vortex lattice solution can be given by

$$\Delta_d(\tilde{x}, \tilde{z}) = \Delta_{d0} \sum_n c_n e^{iq(n-1/2)\tilde{x}} e^{-(\tilde{z}-z_n)^2/2}, \tag{9}$$

where $\tilde{x} = x/\epsilon$, $\tilde{z} = \epsilon z$, the vortex lattice in the coordinates $\tilde{x}$, $\tilde{z}$ has the basis vectors $\boldsymbol{a} = (a, 0)$ and $\boldsymbol{b} = b(\cos\alpha, \sin\alpha)$ [15], $c_n = e^{i\pi\rho n^2} e^{-i\pi\rho(n+1)}$, $q = 2\pi/a$, $z_n = b\sin\alpha(n + 1/2)$, $\rho = (b/a)\cos\alpha$, and $\epsilon = [(\kappa - \kappa_c)/\kappa]^{1/4}$. The parameter ϵ scales lengths in the x and z directions to take the anisotropy into account. This solution is an $n = 0$ eigenstate of the operator $\tilde{\boldsymbol{D}}^2 = \tilde{D}^2_x + \tilde{D}^2_z = (-i\tilde{\boldsymbol{\nabla}} - 2e\tilde{\boldsymbol{A}})^2$ with eigenvalues $(2n + 1)/l^2$ and $l^2 = \Phi_0/(2\pi H)$ ($n = 0, 1, 2, \ldots$ is the Landau level (LL) index). The macroscopic degeneracy of the eigenstates of $\tilde{\boldsymbol{D}}^2$ is exploited to create the Abrikosov vortex lattice solutions and, at the same time, plays a central part in constructing degenerate solutions for the displaced vortex lattice ($\tilde{\phi}_n$) characterized by a vector $\boldsymbol{\tau}$: $\tilde{\phi}_n(\boldsymbol{r} + \boldsymbol{\tau}) = e^{-i\tau_y x}\phi_n(\boldsymbol{r} + \boldsymbol{\tau})$ with $\phi_n(\boldsymbol{r})$ being a vortex lattice solution in LL n. The states $\tilde{\phi}_n$ and ϕ_n are degenerate eigenstates of the operator $\tilde{\boldsymbol{D}}^2$.

In order to determine the PDW vortex structure it suffices to consider the linear equation for the PDW order parameter, which is found by keeping both Eqs. (3) and (4), and by setting $\beta_i = 0$ in Eq. (3). As a technical simplification, we set $(\kappa_1 - |\kappa_2|)/\kappa_3 = \kappa/(\kappa + \kappa_c)$ to ensure that the d-wave order and the PDW order share the same $\tilde{\boldsymbol{D}}^2$ operator and hence have the same eigenstates. Minimization of the free energy yields the following for the 2 degrees of freedom in the PDW order:

$$\begin{aligned} \tilde{\Pi}\Delta_{\Gamma_4,Q_1} &= -\beta_{c1}|\Delta_d|^2\Delta_{\Gamma_4,Q_1} - \beta_{c2}\Delta_d^2\Delta^*_{\Gamma_4,Q_3}, \\ \tilde{\Pi}\Delta_{\Gamma_4,Q_3} &= -\beta_{c1}|\Delta_d|^2\Delta_{\Gamma_4,Q_3} - \beta_{c2}\Delta_d^2\Delta^*_{\Gamma_4,Q_1}, \end{aligned} \tag{10}$$

with $\tilde{\Pi} = (\alpha + \sqrt{(\kappa_1 - \kappa_2)(\kappa_1 + \kappa_3)}\tilde{\boldsymbol{D}}^2)$. To solve these equations, we expand the PDW order in eigenstates of the $\tilde{\boldsymbol{D}}^2$ operator. At sufficiently high fields, the PDW order will lie predominantly in the $n = 0$ eigenstate for both Δ_{Γ_4,Q_1} and Δ_{Γ_4,Q_3}, and we ignore the smaller higher n contributions here. As mentioned above, these solutions are degenerate, implying the use of two displacement vectors $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_3$. At the second order transition where the PDW order appears, the vortex lattice structure is determined entirely by the d-wave order parameter, so the only undetermined

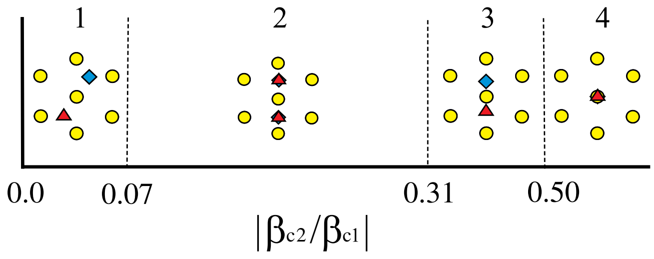

FIG. 2 (color online). Possible vortex configurations for the PDW order. The yellow circles give the zeroes of the d-wave order parameter, the blue diamonds give the positions of the zeroes of Δ_{Γ_4,Q_3}, and the red triangles give the positions of the zeroes of Δ_{Γ_4,Q_1}.

parameters are $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_3$. Solving the resulting linear equation yields the result that the optimal PDW state is found by minimizing $\beta_{c1}\beta_A(\boldsymbol{\tau}_1) - |\beta_{c2}\tilde{\beta}(\boldsymbol{\tau}_1, \boldsymbol{\tau}_3)|$ with respect to $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_3$, where

$$\beta_A(\boldsymbol{\tau}) = \sum_{\mathbf{G}} e^{-(l^2G^2/2)} e^{i\mathbf{G}\cdot\boldsymbol{\tau}}, \tag{11}$$

$$\tilde{\beta}(\boldsymbol{\tau}_1, \boldsymbol{\tau}_3) = \sum_{\mathbf{G}} e^{-(l^2\tilde{G}^2/2)} e^{i\mathbf{G}\cdot\boldsymbol{\tau}_3}, \tag{12}$$

where $\mathbf{G}$ are the reciprocal lattice vectors of the vortex lattice, $\tilde{\mathbf{G}} = \mathbf{G} + \frac{2\pi \boldsymbol{B}}{\Phi_0} \times \boldsymbol{\tau}_1$ and $\tilde{\beta} = 0$ unless $\boldsymbol{\tau}_1 + \boldsymbol{\tau}_3 = \boldsymbol{T}$, where $\boldsymbol{T}$ is a vortex lattice translation vector. For $\beta_{c1} < 0$ it follows immediately that $\boldsymbol{\tau}_1 = \boldsymbol{\tau}_3 = 0$ while the solution for $\beta_{c1} > 0$ requires a numerical minimization to determine $\boldsymbol{\tau}_1$. The resulting configurations are shown in Fig. 2, assuming that the d-wave order forms a hexagonal vortex lattice.

The phase diagram in Fig. 2 depends upon $r = |\beta_{c2}/\beta_{c1}|$ and in all the phases we can choose $\boldsymbol{\tau}_3 = -\boldsymbol{\tau}_1$. We find four phases: in phase 1 ($0 \leq r < 0.07$), $\boldsymbol{\tau}_3 = \gamma(\boldsymbol{a} + \boldsymbol{b})$ and γ evolves continuously from 1/3 to 1/2; in phase 2 ($0.07 \leq r < 0.31$) $\gamma = 1/2$ (Fig. 2 shows $\boldsymbol{\tau}_3 = \boldsymbol{a}/2$ which is equivalent to $\boldsymbol{\tau}_3 = (\boldsymbol{a} + \boldsymbol{b})/2$); in phase 3 ($0.31 \leq r < 0.5$), $\boldsymbol{\tau}_3 = \gamma_2 \boldsymbol{a}$ where γ_2 changes continuously from 1/2 to 0; finally in phase 4 ($r > 0.5$), $\boldsymbol{\tau}_3 = 0$. Arguments similar to Ref. [14] imply that in phases 1 through 3, the d-wave vortex cores have charge density wave order at twice the PDW wave vectors.

From the solution of the vortex lattice, the SDW order can be determined. This is particularly important experimentally, since neutron scattering measures the Fourier transform of $S^z(\boldsymbol{R})$. Equation (8) yields the intriguing result that the SDW order will exhibit Bragg peaks at $\boldsymbol{k}$ positions that depend upon $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_3$:

$$\boldsymbol{k} = \boldsymbol{Q}_1 + \boldsymbol{G} + \frac{2\pi \boldsymbol{B}}{\Phi_0} \times \boldsymbol{\tau}, \tag{13}$$

where $\boldsymbol{G}$ is a reciprocal lattice vector of the vortex lattice and $\boldsymbol{\tau}$ is either $\boldsymbol{\tau}_1$ or $\boldsymbol{\tau}_3$. Consequently, the relative position of the vortex cores of the PDW and d-wave order can be retrieved from the position of the Bragg peaks in the SDW order.

Conclusions.—We have developed a phenomenological theory for the Q phase of $CeCoIn_5$ to identify the possible symmetries for the PDW order. This theory is used to determine phases in which the PDW and d-wave vortex lattice are relatively displaced, leading to CDW order in the d-wave vortex cores. Interestingly, these structures can be probed by the position of the SDW Bragg peaks.

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