

Localization in Correlated Bilayer Structures: From Photonic Crystals to Metamaterials and Semiconductor Superlattices

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In a unified approach, we study the transport properties of periodic-on-average bilayered photonic crystals, metamaterials, and semiconductor superlattices. Our consideration is based on the analytical expression for the localization length derived for the case of weakly fluctuating widths of layers and takes into account possible correlations in disorder. We analyze how the correlations lead to anomalous properties of transport. In particular, we show that for quarter stack layered media specific correlations can result in a ω^2 dependence of the Lyapunov exponent in *all* spectral bands.

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Introduction.—In recent years, much attention was paid to the propagation of waves (electrons) in periodic one-dimensional structures (see, e.g., [1] and references therein). The interest in this subject is due to various applications in which one needs to create materials, metamaterials, or semiconductor superlattices with given transmission properties. One of the important problems that still remains open is the role of a disorder that cannot be avoided in experimental devices due to fluctuations of the width of layers or due to variations of the medium parameters, such as the dielectric constant, magnetic permeability, or barrier height (for electrons) [2,3].

As is well known, the main quantity that absorbs the influence of a disorder is the localization length $l(\omega)$ (LL) entirely determining transport properties in a 1D geometry [4]. In contrast to many studies based on various numerical methods, in this Letter we develop an analytical approach allowing us to derive the unique expression for the LL that is valid for photonic crystals, metamaterials, and semiconductor superlattices. Another key point is that we explicitly take into account possible correlations within a disorder that may be imposed experimentally. As was recently shown, both theoretically [5–8] and experimentally [9,10], specific long-range correlations can significantly enhance or suppress the localization length in desired windows of frequency of incident waves.

Model.—We consider the propagation of an electromagnetic wave of frequency ω through an infinite array of two alternating a and b layers (slabs). The slabs are specified by the dielectric constant $\varepsilon_{a,b}$, magnetic permeability $\mu_{a,b}$, refractive index $n_{a,b} = \sqrt{\varepsilon_{a,b}\mu_{a,b}}$, impedance $Z_{a,b} = \sqrt{\mu_{a,b}/\varepsilon_{a,b}}$, and wave number $k_{a,b} = \omega n_{a,b}/c$. We assume that the z axis is directed along the array of bilayers perpendicular to the stratification. Within the layers, the electric field obeys the wave equation

$$\left(\frac{d^2}{dz^2} + k_{a,b}^2\right)\psi_{a,b}(z) = 0, \quad (1)$$

with two boundary conditions on the interfaces $z = z_i$ between slabs: $\psi_a(z_i) = \psi_b(z_i)$ and $\mu_a^{-1}\psi'_a(z_i) = \mu_b^{-1}\psi'_b(z_i)$.

A disorder is incorporated in the structure via the random widths of the slabs $\tilde{a}(n) = a + \varrho_a(n)$ and $\tilde{b}(n) = b + \varrho_b(n)$. Here n enumerates the elementary ab cells, a and b are the average widths of layers, and $\varrho_a(n)$ and $\varrho_b(n)$ stand for small variations of the widths. In the absence of disorder, the array of slabs is periodic with the period $d = a + b$. The random sequences $\varrho_a(n)$ and $\varrho_b(n)$ are statistically homogeneous with zero average $\langle\varrho_{a,b}(n)\rangle = 0$ and binary correlation functions defined by

$$\begin{aligned} \langle\varrho_j(n)\varrho_j(n')\rangle &= \langle\varrho_j^2(n)\rangle K_j(n-n'), & j &= a, b; \\ \langle\varrho_a(n)\varrho_b(n')\rangle &= \langle\varrho_a(n)\varrho_b(n)\rangle K_{ab}(n-n'). \end{aligned} \quad (2)$$

In what follows, the average $\langle\dots\rangle$ is performed over the whole array or due to the ensemble averaging, that is assumed to be the same. The autocorrelators $K_j(r)$ as well as the intercorrelator $K_{ab}(r)$ are normalized to one: $K_a(0) = K_b(0) = K_{ab}(0) = 1$. The variances $\langle\varrho_j^2(n)\rangle$ are positive, while $\langle\varrho_a(n)\varrho_b(n)\rangle$ can be of arbitrary value. We assume the positional disorder to be weak, $k_j^2\langle\varrho_j^2(n)\rangle \ll 1$, allowing us to use an appropriate perturbation theory. In this case all transport properties are determined by the randomness power spectra $\mathcal{K}_a(k)$, $\mathcal{K}_b(k)$, and $\mathcal{K}_{ab}(k)$, defined by the relation $\mathcal{K}(k) = \sum_{r=-\infty}^{\infty} K(r) \exp(-ikr)$.

Method.—Our aim is to derive the expression for the LL in the general case of either uncorrelated or correlated disorder. To do this, we generalize the method developed in Refs. [5–7] for more simple models. On the scale of individual slabs, we present the solution of Eq. (1) in the form of two maps for the n th a and b layer, respectively,

with corresponding phase shifts $\tilde{\varphi}_{a,b}(n) = \varphi_{a,b} + \xi_{a,b}(n)$, where $\varphi_a = k_a a$, $\varphi_b = k_b b$, and $\xi_{a,b}(n) = k_{a,b} \varrho_{a,b}(n)$. Then, by combining these maps with the boundary conditions, we write the map for the whole n th elementary ab cell:

$$x_{n+1} = \tilde{A}_n x_n + \tilde{B}_n y_n, \quad y_{n+1} = -\tilde{C}_n x_n + \tilde{D}_n y_n. \quad (3)$$

Here $x_n = \psi_a(z_{an})$ and $y_n = k_a^{-1} \psi'_a(z_{an})$, the indices n and $n+1$ stand for left and right edges of the n th cell, and

$$\begin{aligned} \tilde{A}_n &= \cos \tilde{\varphi}_a(n) \cos \tilde{\varphi}_b(n) - Z_a^{-1} Z_b \sin \tilde{\varphi}_a(n) \sin \tilde{\varphi}_b(n), \\ \tilde{B}_n &= \sin \tilde{\varphi}_a(n) \cos \tilde{\varphi}_b(n) + Z_a^{-1} Z_b \cos \tilde{\varphi}_a(n) \sin \tilde{\varphi}_b(n), \\ \tilde{C}_n &= \sin \tilde{\varphi}_a(n) \cos \tilde{\varphi}_b(n) + Z_a Z_b^{-1} \cos \tilde{\varphi}_a(n) \sin \tilde{\varphi}_b(n), \\ \tilde{D}_n &= \cos \tilde{\varphi}_a(n) \cos \tilde{\varphi}_b(n) - Z_a Z_b^{-1} \sin \tilde{\varphi}_a(n) \sin \tilde{\varphi}_b(n). \end{aligned}$$

Equation (3) can be treated as the map of a linear oscillator with time-dependent parametric force [11]. Without disorder $\xi_{a,b}(n) = 0$, the trajectory x_n, y_n creates an ellipse in the phase space (x, y) that is an image of the unperturbed motion. It is convenient to make the transformation $x_n = v^{-1} Q_n \cos \tau - v P_n \sin \tau$, $y_n = v^{-1} Q_n \sin \tau + v P_n \cos \tau$ to new coordinates Q_n and P_n , in which the unperturbed trajectory occupies the circle $Q_{n+1} = Q_n \cos \gamma + P_n \sin \gamma$, $P_{n+1} = -Q_n \sin \gamma + P_n \cos \gamma$ in the phase space (Q, P) . Here τ and v can be found from Eq. (3), and γ determines the Bloch wave number $\kappa = \gamma/d$ arising in the relation $\psi(z+d) = \exp(i\kappa d) \psi(z)$ for the periodic array

$$\cos \gamma = \cos \varphi_a \cos \varphi_b - \frac{1}{2} \left(\frac{Z_b}{Z_a} + \frac{Z_a}{Z_b} \right) \sin \varphi_a \sin \varphi_b. \quad (4)$$

The perturbation in the normal coordinates Q_n and P_n results in a weak distortion of a circle and can be evaluated in a relatively simple way. Specifically, we expand the constants $\tilde{A}_n, \tilde{B}_n, \tilde{C}_n$, and \tilde{D}_n up to the second order in the perturbation parameters $\xi_{a,b}(n) \ll 1$. Then we transform the coordinates x_n and y_n into Q_n and P_n and pass to action-angle variables R_n and θ_n via the standard transformations, $Q_n = R_n \cos \theta_n$ and $P_n = R_n \sin \theta_n$. This allows us to derive the relation between R_{n+1} and R_n :

$$\begin{aligned} \frac{R_{n+1}^2}{R_n^2} &= 1 + \xi_a(n) V_a(n) + \xi_b(n) V_b(n) + \xi_a^2(n) W_a(n) \\ &+ \xi_b^2(n) W_b(n) + \xi_a(n) \xi_b(n) W_{ab}(n), \end{aligned} \quad (5)$$

where $V_a(n), V_b(n), W_a(n), W_b(n)$, and $W_{ab}(n)$ are some functions of θ_n and the model parameters (details will be presented elsewhere).

Localization length.—The LL can be expressed via the Lyapunov exponent (LE) $\lambda = d/l(\omega)$ defined by [11]

$$\lambda = \frac{1}{2} \left\langle \ln \left(\frac{R_{n+1}}{R_n} \right)^2 \right\rangle. \quad (6)$$

Now we substitute Eq. (5) into Eq. (6) and perform the averaging over both the disorder and phase θ_n . We assume that the distribution of θ_n is homogenous within the first

order of approximation. This assumption is correct apart from the band edges $\gamma = 0; \pi$ and the vicinity of the center $\gamma = \pi/2$ [7]. After quite cumbersome calculations we arrive at the final result for the LE:

$$\begin{aligned} \lambda &= \frac{\varpi^2}{8 \sin^2 \gamma} \left[\sigma_a^2 \mathcal{K}_a(2\gamma) \sin^2 \varphi_b + \sigma_b^2 \mathcal{K}_b(2\gamma) \sin^2 \varphi_a \right. \\ &\quad \left. - 2 \sigma_{ab}^2 \mathcal{K}_{ab}(2\gamma) \sin \varphi_a \sin \varphi_b \cos \gamma \right], \end{aligned} \quad (7)$$

where $\sigma_a^2 = k_a^2 \langle \varrho_a^2(n) \rangle$, $\sigma_b^2 = k_b^2 \langle \varrho_b^2(n) \rangle$, $\sigma_{ab}^2 = k_a k_b \langle \varrho_a(n) \varrho_b(n) \rangle$, and $\varpi = Z_a/Z_b - Z_b/Z_a$ is the *mismatching factor*. Expression (7) generalizes the results obtained in Refs. [6,7,12] for particular cases and is in a complete correspondence with them. Let us now discuss the derived expression in some applications.

Conventional photonic layered media.—In this case all parameters $\varepsilon_{a,b}, \mu_{a,b}, n_{a,b}$, and $Z_{a,b}$ are *positive constants*. If the impedances of a and b slabs are equal $Z_a = Z_b$, the mismatching factor ϖ in Eq. (7) vanishes and the perfect transparency emerges ($\lambda = 0$) even in the presence of a disorder. This conclusion is general [1] and does not depend on the strength of disorder. In this case the stack structure is effectively equivalent to the homogeneous medium with the linear spectrum $\kappa \sim \omega$.

For the Fabry-Perot resonances appearing when $\omega/c = s_a \pi/n_a a$ or $\omega/c = s_b \pi/n_b b$, with $s_{a,b} = 1, 2, 3, \dots$, the factor $\sin \varphi_a$ or $\sin \varphi_b$ in Eq. (7) vanishes, thus giving rise to the *resonance increase* of the LL. In a special case when $n_a a/n_b b = s_a/s_b$, which can arise only at the edges of spectral bands ($\gamma = 0, \pi$), the LL gets a finite value. In the vicinity of the bottom of the spectrum $\omega = 0$ and for a white noise $\mathcal{K}_a = \mathcal{K}_b = 1, \mathcal{K}_{ab} = 0, \pm 1$, the LE obeys the conventional dependence $\lambda \propto \omega^2$ for $\omega \rightarrow 0$.

Of special interest are long-range correlations leading to the divergence of the LL in the controlled frequency window. This effect is similar to that found in more simple 1D models with correlated disorder [5–7,10]. For example, one can have $\mathcal{K}_a = \mathcal{K}_b = \mathcal{K}_{ab} = 0$ in some range of frequency ω . Thus, one can artificially construct an array of random bilayers with such power spectra that abruptly vanish within prescribed intervals of ω , resulting in the divergence of the LL. Also, specific correlations [13] in a disorder can be used to cancel a sharp frequency dependence that is due to the term $\sin^2 \gamma$ in the denominator of Eq. (7). Note that in the middle of spectral Bloch bands, $\gamma = \pi/2$, the third term vanishes, and the intercorrelations do not influence the LL. The typical dependence $\lambda(\omega)$ for the conventional photonic bilayer stack is shown in Fig. 1.

Quarter stack layered medium.—This term is typically used when two basic layers a and b have the same optical width $n_a a = n_b b$ (see, e.g., [1]). Since $\varphi_a = \varphi_b$, in this case the dispersion relation (4) takes the form

$$\cos \gamma = 1 - \frac{1}{2} \frac{(Z_a + Z_b)^2}{Z_a Z_b} \sin^2(k_a a). \quad (8)$$

One can see that, starting from the second band, the top of

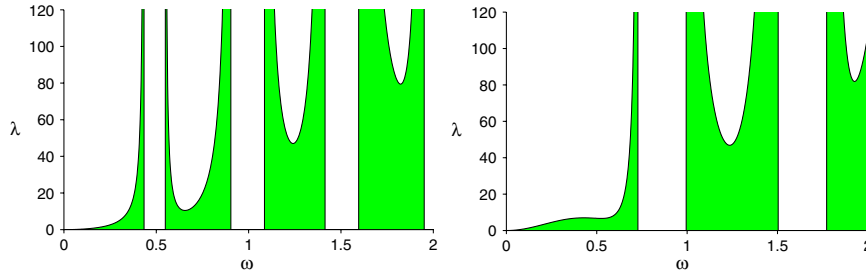


FIG. 1 (color online). Lyapunov exponent versus frequency in arbitrary units for $\varpi^2 n_a^2 \langle \varrho_a^2(n) \rangle / (2c^2) \approx 12.28$, $\varpi^2 n_b^2 \langle \varrho_b^2(n) \rangle / (2c^2) \approx 0.27$ and $n_a a / c = 1.6$, $n_b b / c = 0.4$. Left: Photonic layered medium, Eqs. (4) and (7). Right: The RH-LH bilayers with plus in Eq. (4).

every even band coincides with the bottom of the next odd band at $\gamma = 0$. The gaps arise only at $\gamma = \pi$.

With the use of Eq. (8) the LE can be written as

$$\lambda = \frac{Zf(\gamma)}{8\cos^2(\gamma/2)}, \quad Z = \frac{(Z_a - Z_b)^2}{Z_a Z_b},$$

$$f(\gamma) = \sigma_a^2 \mathcal{K}_a(2\gamma) + \sigma_b^2 \mathcal{K}_b(2\gamma) - 2\sigma_{ab}^2 \mathcal{K}_{ab}(2\gamma) \cos\gamma. \quad (9)$$

Thus, the LE is finite or vanishes at $\gamma = 0$ and diverges at $\gamma = \pi$.

It is instructive to analyze the simplest case of correlations when either $\xi_a(n) = \xi_b(n)$ (*plus correlations*) or $\xi_a(n) = -\xi_b(n)$ (*minus correlations*). In this case one gets $f(\gamma) = 2\sigma_a^2 \mathcal{K}_a(2\gamma)(1 \mp \cos\gamma)$, respectively. As a result, for the LE one can obtain

$$\lambda_+ = \frac{Z\sigma_a^2 \mathcal{K}_a(2\gamma)}{2} \tan^2 \frac{\gamma}{2}; \quad \lambda_- = \frac{Z\sigma_a^2 \mathcal{K}_a(2\gamma)}{2}. \quad (10)$$

As one can see, $\lambda_+ \propto \omega^4$ at the bottom of the spectrum ($\omega \rightarrow 0$), in contrast to the conventional dependence $\lambda \propto \omega^2$. Another nonconventional dependence $\lambda \propto \omega^6$ was recently found [3] in a different layered model with left-handed material. It is interesting that, for the minus correlations and $\mathcal{K}_a(2\gamma) = 1$, the LE is quadratic in ω inside any spectral band. In this case the total optical length is constant within any pair of a and b layers, although the width of both a and b layers fluctuates randomly. For such correlations the quadratic ω dependence seems to survive in the nonperturbative regime.

In Fig. 2, we have used the parameters for a silicon-air stack with plus and minus correlations between two disorders. In order to check our analytical predictions, in the

inset we present a numerical solution of Eq. (1) for the LE in the third frequency band for $N = 4 \times 10^4$ ab cells.

Metamaterials.—Let us now consider mixed systems in which the a layer is a conventional right-handed (RH) material and the b layer is a left-handed (LH) material. Therefore, $\varepsilon_a, \mu_a, n_a > 0$, whereas $\varepsilon_b, \mu_b, n_b < 0$; however, the impedances are positive: $Z_a, Z_b > 0$. Remarkably, in comparison with the conventional stack structure, Eq. (7) for the LE stays the same. The only difference is that now the sign “plus” stands at the second term in Eq. (4) (note that $\varphi_b \equiv k_b b = -\omega |n_b| b / c$). Such a “minor” correction drastically changes the ω dependence of the LE; see Fig. 1. Nevertheless, the LE obeys the conventional dependence $\lambda \propto \omega^2$ when $\omega \rightarrow 0$. Note also that the ideal mixed stack ($\varepsilon_a = \mu_a = n_a = 1$, $\varepsilon_b = \mu_b = n_b = -1$, $Z_a = Z_b = 1$) has perfect transmission $\lambda = 0$ even in the presence of a positional disorder.

It is important that in reality the permittivity $\varepsilon_b(\omega)$ and permeability $\mu_b(\omega)$ are ω -dependent [1]. This fact is crucial in applications. In particular, the mismatching factor ϖ in Eq. (7) can vanish for specific values of ω only, thus resulting in a resonancelike dependence for the transmission. Also, for a typical frequency dependence, the refractive index of b slabs can be imaginary giving rise to an emergence of a new gap at the origin of spectrum $\omega = 0$, in contrast to conventional photonic crystals. It can be seen that in many aspects the wave transport through the bilayered metamaterials resembles that of the electrons through double-barrier structures.

Electrons.—Our approach can be also applied to the electron transport through the structures with alternating potential barriers of amplitudes U_a and U_b and slightly perturbed widths. One possible application is the fabrication of nanostructured electronic systems (see, for ex-

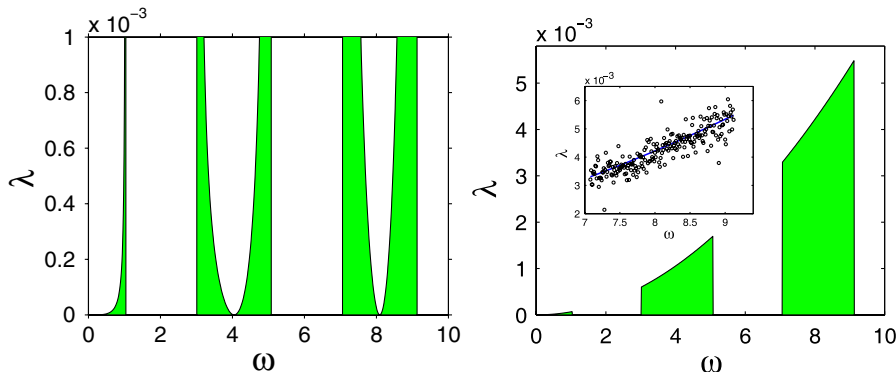


FIG. 2 (color online). Lyapunov exponent versus frequency for the quarter stack layered medium, for the plus correlations (left) and minus correlations (right); see Eqs. (8) and (10). Here $n_a = 3.47$ (silicon), $d = 1 \mu\text{m}$, $a = 0.224 \mu\text{m}$, $\langle \varrho_a^2(n) \rangle = 3.75 \times 10^{-5} \mu\text{m}^2$, $c = 1.0$, and $\mathcal{K}_a = 1$.

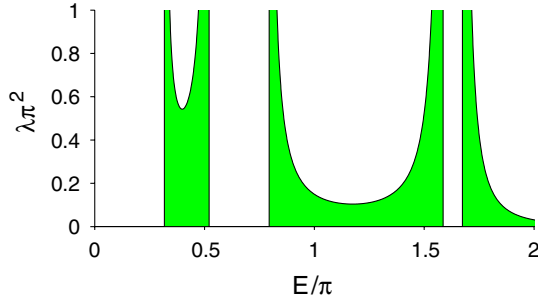


FIG. 3 (color online). Lyapunov exponent versus energy for electrons in a bilayered structure. Here $2m_a/\hbar^2 = 2m_b/\hbar^2 = 1$, $U_a = 0.15$, $U_b = 1.6$, $a = 0.35\pi$, $b = 0.65\pi$, $\langle \varrho_a^2(n) \rangle = (0.1a)^2$, and $\langle \varrho_b^2(n) \rangle = (0.2b)^2$. The transition between tunneling and overbarrier scattering occurs at $E/\pi = 0.5$.

ample, Ref. [14] and references therein). The stationary 1D Schrödinger equation for the wave functions $\psi_{a,b}(z)$ of an electron with effective masses m_a and m_b inside the barriers and total energy E can be written in the form of Eq. (1), where $k_a = \sqrt{2m_a(E - U_a)}/\hbar$ and $k_b = \sqrt{2m_b(E - U_b)}/\hbar$. Another change, $\mu_{a,b} \rightarrow m_{a,b}$, should be done in the boundary condition on hetero-interfaces: $m_a^{-1}\psi'_a(z_i) = m_b^{-1}\psi'_b(z_i)$. Correspondingly, $Z_b/Z_a = k_a m_b/k_b m_a$ in the dispersion relation (4) and in Eq. (7) for the LE.

If the energy E is smaller than the heights of both barriers $E < U_a, U_b$, the electron wave numbers are imaginary. As a consequence, the electron states are strongly localized and the structure is not transparent. When $U_a < E < U_b$, the *tunneling propagation* of electrons emerges. In this case k_a is real, while k_b is imaginary. Therefore, the electron moves freely within any a barrier and tunnels through the b barriers. Thus, the expressions (7) for λ and (4) for γ have to be modified according to the change: $k_b \rightarrow i|k_b|$ and $\sin(k_b b) \rightarrow i \sinh(|k_b|b)$. As a result, the increase of the LL due to Fabry-Perot resonances arises only due to the second and third terms of Eq. (7). Note that from our general expression (7) one can easily get the LE for a particular case of an array with delta-like potential barriers, analyzed in Ref. [6].

For the *overbarrier scattering*, when $U_a < U_b < E$, both wave numbers k_a and k_b are positive, and the electron transport is similar to that for the conventional photonic stack, however, with dispersive parameters. The example of energy dependence of the LE is given in Fig. 3. It is interesting that when $E = (U_b m_a - U_a m_b)/(m_a - m_b)$, an electron does not change its velocity in the barriers $\hbar k_a/m_a = \hbar k_b/m_b$, although its momentum changes: $\hbar k_a > \hbar k_b$. The LE vanishes in this case. This effect is equivalent to that in a homogeneous medium with perfect transmission.

Conclusion.—We have developed the method allowing us to derive the expression for the localization length l for quasiperiodic bilayer structures whose widths are weakly perturbed. The knowledge of l is very important in prac-

tice, since it is directly related to the transmission coefficient T for finite samples of size L , according to the famous relation $\langle \ln T \rangle = -2L/l$. Our results can be applied to both conventional photonic crystals and metamaterials, as well as to semiconductor superlattices. The distinctive peculiarity of the approach is that it takes into account possible correlations in the disorder that can lead to anomalous frequency (energy) dependence of transport properties. Because of the correlations, one can significantly enhance or suppress the transmission/reflection through the bilayered devices within the prescribed windows of frequency (energy) of electromagnetic (electron) waves. The results may have a strong impact for the fabrication of a new class of disordered optic crystals, left- and right-handed metamaterials, and electron nanodevices with *selective* transmission and/or reflection.

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