Proposal for a Search for Cosmic Axions Using an Optical Cavity

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A high finesse optical cavity can be used to search for cosmic axions in the mass range $10^{-6} < m_a < 10^{-4}$ eV. Either a two-arm or a single-arm cavity is suitable, and in either case the signal appears as resonant sidebands imposed on the carrier. Assuming for the local axion density the usual figure of $\rho_a = 500\ \text{Mev/cm}^3$, the KSVZ axion line $g_{a\gamma\gamma}/m_a = 0.4\ \text{GeV}^{-2}$, can be reached over the full mass range in a 1 yr search.

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The existence of axions, light pseudoscalars particles, was postulated 30 years ago [1–3] to explain why the strong interactions conserve P and CP in spite of the fact that the QCD Lagrangian does not [4,5]. Axions remain an attractive candidate for the cold dark matter of the universe [6,7]. A detailed review of axion theory can be found in [6]. As a result of their gravitational attraction and very weak interaction with ordinary matter, axions are expected to condense into galactic halos. The local axion density is estimated to exceed their average density by a factor of $\sim 10^5$ [8]. We will use units of $\hbar = c = 1$ throughout unless otherwise indicated.

Axions couple to two photons through the triangle anomaly and the effective action density can be written [7,9]

$$\mathcal{L} = \frac{1}{2}[E^2 - B^2] + \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - \frac{1}{2}m_a^2\phi^2 - g\vec{E}\cdot\vec{B}\phi; \tag{1}$$

$\vec{E}$ and $\vec{B}$ are the electric and magnetic field, and ϕ, m_a the axion field and axion mass. The coupling of the axion to two photons is designated by g and is proportional to the axion mass. In order of magnitude

$$g \equiv g_{a\gamma\gamma} = \frac{1}{\Lambda} \simeq \frac{\alpha}{\pi}\frac{m_a}{m_\pi f_\pi} \tag{2}$$

with m_π, f_π the pion mass and decay constant, $m_\pi f_\pi \sim 10^{-2}\ \text{GeV}^2$. In all axion models the product of the inverse coupling constant, Λ(GeV) and axion mass is constant with $\Lambda m_a \sim 1\ \text{GeV}^2$. The classical equations of motion for the fields derived from Eq. (1) are

$$\vec{\nabla}\cdot\vec{E} = g\vec{B}\cdot\vec{\nabla}\phi, \tag{3}$$

$$\vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = g\left[\vec{E}\times\vec{\nabla}\phi - \vec{B}\frac{\partial\phi}{\partial t}\right], \tag{4}$$

$$\left[\frac{\partial^2}{\partial t^2} - \nabla^2\right]\phi + m_a^2\phi = -g\vec{E}\cdot\vec{B}. \tag{5}$$

Over the past two decades several experiments have searched for cosmic axions [6,10–12], and for axions produced in the sun [13,14]. There have also been efforts to observe axion production using laser beams [15–18]. Dark matter candidate axions are expected in the mass range $10^{-3} < m_a < 10^{-6}$ eV [6,19], with, correspondingly, weak couplings to the electromagnetic field. The most sensitive searches for μeV axions in the galactic halo are based on the conversion of axions to microwave photons in a static magnetic field. The converted photons are detected in a cavity which is resonant at the frequency corresponding to the axion mass [6].

Here we propose an analogous process where the axions are absorbed (but also emitted) by (from) an optical field of frequency ω_0, typically in the visible. Therefore sidebands $\omega_\pm = \omega_0 \pm \omega_a$ appear on the carrier, displaced by the axion frequency $\omega_a = E_a \simeq m_a$. For this process to be efficient, the sidebands must resonate in the optical cavity. We discuss later how this is achieved in practice.

We start from Eqs. (3)–(5) and designate the carrier fields by $\vec{E}_0$, $\vec{B}_0$, and the sideband fields by $\vec{E}_\pm$, $\vec{B}_\pm$ for the upper and lower sideband respectively;

$$\vec{E} = \vec{E}_0 + \vec{E}_+ + \vec{E}_- \quad \text{and} \quad \vec{B} = \vec{B}_0 + \vec{B}_+ + \vec{B}_-. \tag{6}$$

The fields $\vec{E}_\pm$, $\vec{B}_\pm$ are of order $g\phi_a \sim 10^{-21}$ as compared to the fields $\vec{E}_0$, $\vec{B}_0$. The carrier is a standing wave in a cavity of length L along the x axis

$$\vec{E}_0 = A_0(t)\sin(k_0x)e^{-i\omega_0 t}\hat{u}_z, \quad \vec{B}_0 = A_0(t)\cos(k_0x)e^{-i(\omega_0 t-\pi/2)}\hat{u}_y, \tag{7}$$

where $\omega_0 = k_0 = n_0(\pi/L)$. We seek solutions where the sideband fields are orthogonal to the carrier and are standing waves in an overlapping cavity of length L_s,

$$\vec{E}_\pm = \pm A_\pm(t)\sin(k_\pm x)e^{-i\omega_\pm t}\hat{u}_y, \quad \vec{B}_\pm = \pm A_\pm(t)\cos(k_\pm x)e^{-i(\omega_\pm t-\pi/2)}\hat{u}_z, \tag{8}$$

where $\omega_\pm = k_\pm = \omega_0 \pm \omega_a$. The length L_s is adjusted to make one of the sidebands resonant, i.e., $\omega_+ = n_+(\pi/L_s)$. When the axion frequency coincides with multiples of the free spectral range of the overlapping cavity, $\nu_a = \omega_a/2\pi = q/2L_s$ both the upper and lower sidebands are simultaneously resonant. In Eqs. (8) we have explicitly indicated the slow variation in time of the amplitudes $A_\pm(t)$. We have also written $A_0(t)$ in Eqs. (7) even though, in practice, the carrier amplitude remains constant.

The axion field is assumed spatially homogeneous over the dimensions of the detector

$$\phi(x,t) = \phi_a(e^{-i\omega_a t} + e^{i\omega_a t})/2. \qquad (9)$$

This assumption is justified because the DeBroglie wavelength of the axions $\lambda_{\rm DB} = 2\pi/(\beta_a m_a)$ is much larger than the dimensions of the detector for $m_a < 10^{-3}$ eV. β_a is the velocity of the axions which is that of the virial velocity of the galaxy $\beta_a \simeq 10^{-3}$. The first term in Eq. (9) contributes the upper sideband and the second term the lower sideband.

Introducing Eq. (6) in Eq. (4) and keeping only terms of order $g\phi_a$ we obtain the wave equation for the sideband fields. We made use of Eq. (3) and also of $\omega_a \ll \omega_0$ to neglect terms in ω_a/ω_0. We find

$$(\nabla^2 - \partial^2/\partial t^2)\vec{E}_\pm = \pm g\omega_0\omega_a\vec{B}_0\phi_a/2. \qquad (10)$$

As expected the sideband fields $\vec{E}_\pm$ are directed perpendicular to $\vec{E}_0$ and have time dependence $e^{-i\omega_\pm t}$. The remaining terms give the wave equation for the evolution of the carrier field which, to order $g\phi_a$ is free, $(\nabla^2 - \partial^2/\partial t^2)\vec{E}_0 = 0$.

We can solve Eq. (10) imposing the boundary conditions for a standing wave (vanishing electric field at the cavity boundaries). The same result is obtained by using Eq. (4) directly

$$\vec{\nabla} \times \vec{B}_\pm - \frac{\partial \vec{E}_\pm}{\partial t} = \pm(i/2)g\omega_a\vec{B}_0\phi_a e^{\mp i\omega_a t}. \qquad (11)$$

Using Eqs. (7) and (8) the rapid time dependence cancels leading to

$$\frac{dA_\pm}{dt}\sin(k_\pm x) = \pm g\omega_a A_0\cos(k_0 x)\phi_a/2. \qquad (12)$$

We expand $\cos(k_0x)$ in the modes of the overlapping cavity, $\cos(k_0x) = \sum C_l \sin(k_l x)$ with $k_l = l\pi/L_s$. Hence,

$$\frac{dA_\pm}{dt} = C_\pm g\omega_a A_0\phi_a/2, \qquad (13)$$

$l_\pm = (\omega_0 \pm \omega_a)(L_s/\pi)$ and $C_\pm = [1 - \cos(k_aL_s)]/\omega_a L_s$.

The growth of the amplitude $A_\pm(t)$ is restricted by the losses in the cavity, expressed by the "quality factor" Q

$$\frac{dA_\pm}{dt} = -\frac{\omega_\pm}{2Q}A_\pm. \qquad (14)$$

It follows that in the steady state

$$A_\pm = \pm g\phi_a QC_\pm\frac{\omega_a}{\omega_0}A_0. \qquad (15)$$

The configuration of the coupled cavities is shown in Fig. 1. The carrier resonates in L, between M1 and M2. The sidebands have orthogonal polarization to the carrier and are directed by the (polarizing) beam splitter to mirror 3. The spacing, L_s, between M1 and M3, is tuned to the sideband frequency.

We define the cavity "finesse" in the usual way

$$\mathcal{F} = \pi\frac{\sqrt{r_1r_2}}{1 - r_1r_2}, \qquad (16)$$

where r_1, r_2 are the amplitude reflectivities of the input and output mirrors and other losses are assumed absent. The quality factor of the cavity is

$$Q = \mathcal{F}(2L/\lambda_0) \qquad (17)$$

with L the length of the cavity and λ_0 the wavelength of the carrier. The free spectral range (fsr) of the cavity is $\nu_{\rm fsr} = 1/(2L)$ and the FWHM of the cavity resonance is $\Delta\nu_c = \nu_{\rm fsr}/\mathcal{F} = \nu_0/Q$. We use t_1, t_2 for the amplitude transmissivities of the cavity mirrors, which satisfy $r^2 + t^2 + A = 1$, with A the absorption coefficient (for simplicity we set $A = 0$). The carrier field circulating in the cavity is $E^{\rm circ} = E^{\rm in}t_1/(1 - r_1r_2)$ and the transmitted fields $E^{\rm out} = t_2E^{\rm circ}$; $E^{\rm in}$ is the incident field.

With the above definition of the Q of the optical cavities Eq. (15) can be written in compact form

$$\frac{A_\pm}{A_0} = \pm g\phi_a(\mathcal{F}/\pi)[1 - \cos(k_aL_s)] \equiv R_\pm. \qquad (18)$$

It is interesting, but not surprising, that the experimental signal depends only on the product of the axion coupling and field, which is constant for fixed axion density. This is also true for the microwave cavity axion searches.

The axion amplitude ϕ_a is related to the axion density ρ_a through

$$\langle\phi^2\rangle = \rho_a/m_a^2 \qquad \text{or} \qquad \phi_a = \sqrt{2\rho_a}/m_a. \qquad (19)$$

Using $\rho_a = 0.5$ GeV/cm^3 [8] and $g/m_a = 0.4$ GeV^{-2} appropriate for the KSVZ model [4] we obtain for the dimensionless quantity $g_a\phi_a = 10^{-21}$. We can expect a finesse $\mathcal{F} \sim 10^6$ [20], so that when the spatial form factor is optimized, we must be able to detect sidebands of relative amplitude $A_\pm/A_0 \sim 10^{-15}$, in order to reach the axion limit.

It is not possible to measure directly the power in the sidebands for such a weak signal but one must instead measure the amplitude of the sideband field. This is done by exploiting the coherence between the carrier and the

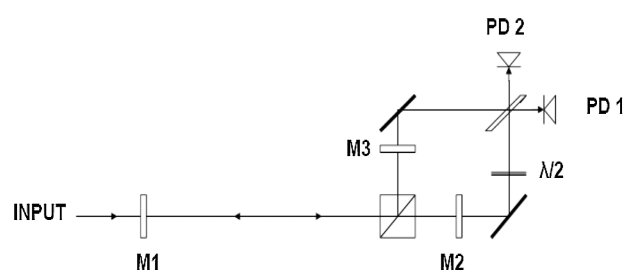

FIG. 1. Proposed layout of the coupled cavities.

sidebands and measuring the interference term; what is referred to as "homodyne" detection.

The carrier reaching the detector is suppressed by the transmission σ through the polarizer. Thus the photodiode current is

$$I_P = \eta|\sigma A_0 + A_\pm|^2 \tag{20}$$

and we can set the quantum efficiency $\eta \sim 1$. The irreducible background is determined by the shot noise fluctuations in the carrier

$$I_N = \sigma^2 \delta|A_0|^2 = \sigma^2|A_0|^2[(4\pi^2\hbar c/\lambda_0)/P_{\rm in}\mathcal{F}]^{1/2}/\sqrt{\rm Hz} \tag{21}$$

with $P_{\rm in} \propto |A_0|^2$ the input laser power. The signal current is $I_S = 2R_\pm\sigma|A_0|^2$, and the signal-to-noise (S/N) ratio

$$\frac{S}{N} = 2R_\pm \frac{1}{\sigma}[(4\pi^2\hbar c/\lambda_0)/P_{\rm in}\mathcal{F}]^{-1/2}. \tag{22}$$

For $P_{\rm in} = 1{\rm W}$, $\lambda_0 \sim 10^{-6}{\rm m}$, $\mathcal{F} = 10^6$, $R_\pm = 10^{-15}$ and $\sigma = 4 \times 10^{-4}$, the shot noise limited S/N is of order 5 for 1 Hz signal bandwidth. The signal power is significantly above amplifier noise. For an amplifier with a 10° K temperature at 24 GHz, the noise power is $P_N = 1.4 \times 10^{-22}$ W/Hz, whereas the (homodyned) signal power, for $\sigma = 4 \times 10^{-4}$ is $\sim 2.5 \times 10^{-18}$ W. The extinction of the carrier is controlled by σ which is eventually chosen so as to optimize the overall S/N.

A critical issue is the detection of the high frequency modulation imposed on the optical carrier. Fast photodiodes with bandwidth of 30 GHz are available. Thus an interesting range of axion masses for this search would be $10^{-6} < m_a < 10^{-4}$ eV, namely, from 240 MHz to 24 GHz. In this frequency range microwave techniques will have to be used to amplify the weak signal in the photocurrent.

The optical cavity needs to be scanned only over one free spectral range in the region of the axion frequency ν_a, namely, at the Nth free spectral range where $N = \nu_a/\nu_{\rm fsr} = 2L\nu_a$. To cover the desired range from N to $N + 1$ the cavity length needs to be changed by

$$\delta L/L = 1/N \qquad \text{or} \qquad \delta L \sim 1/2\nu_a.$$

Assuming a 1 m long cavity, and the range of frequencies of interest, $\delta L = \pm 25$ cm covers the low mass range, $m_a = 10^{-6}$ eV, whereas for $m_a = 10^{-4}$ eV, $\delta L = \pm 2.5$ mm. Scanning over the entire 25 cm range will result in repeated resonances if the axion mass is in the upper part of the search.

The width of the cavity resonance is fixed by the finesse and the fsr frequency, $\Delta\nu = \nu_{\rm fsr}/\mathcal{F} = 150$ Hz for $L = 1$ m. The width of the axion line is determined by the random motion of the axions $\Delta\nu_a = (1/2)m_a\beta^2 = 5 \times 10^{-7}m_a$. This ranges from 120 Hz to 12 kHz. One would scan in steps of the cavity width at the Nth fsr. Namely a total of $\mathcal{F}$ steps with progressively smaller increments, depending on the axion mass that is searched for, i.e. $\delta x_{\rm step} = \delta L(\nu_a)/\mathcal{F}$. Devoting an integration time of 30 s to each step would require about a year for the entire search, at a shot noise limited $S/N = 2.5$. In this search the carrier frequency is locked onto the primary cavity and the length of the sideband cavity is continuously scanned.

The insertion of the polarizing beam splitter in the carrier cavity will introduce losses and prevent the finesse from reaching the design figure of $\mathcal{F} = 10^6$. Instead one can use a single cavity and consider length settings where both the carrier and the sidebands are resonant. All previous calculations remain valid, but now both the upper and lower sidebands are present with equal amplitudes. The resonance conditions are $k_0L/\pi = n_0$ and $k_\pm L/\pi = n_\pm$, and therefore

$$\omega_a L/\pi = (n_\pm - n_0) = 2p + 1, \tag{23}$$

where p is an integer. We impose the odd integer condition so that the coefficient $C_\pm = 2/\omega_a L$, while it vanishes when $(n_\pm - n_0)$ is even.

The drawback of using a single cavity is that the carrier frequency must be continuously adjusted as the cavity length is scanned. In practice this is quite feasible except when using a short cavity to search in the lowest range of axion masses. Since both sidebands resonate and have opposite (real) amplitudes, mixing with the carrier gives a null result. In order to detect the signal one must impose FM sidebands on the carrier and mix the modulated fields with the signal field.

So far it has been implicitly assumed that the axion field is coherent. While this is true when the axion field was first created, during the evolution to the present time the field has retained only partial coherence. The present coherence can be estimated from the time it takes an axion to traverse its de Broglie wavelength [21].

$$\tau_{\rm coherence} \sim 2\pi/m_a\beta_a^2. \tag{24}$$

Numerically, $\tau_c = 4 \times 10^{-3}/[m_a/1\ \mu{\rm eV}]$ s. Such partial coherence times are longer than the "filling" time required for the optical cavity to reach the signal level, $\tau_{\rm acq} = 2Q/\omega_c = 10^{-3}$ s; see Eq. (14). Therefore the data can be acquired at intervals shorter than the coherence time of the axion field. Sampling at a rate of $f_s = 2.5$ kHz ($\tau_s = 4 \times 10^{-4}$ s) will reduce the S/N ratio of each individual measurement to 0.1. However, averaging the $f_s\Delta T = 7.5 \times 10^4$ individual measurements restores the previously quoted sensitivity, in this case the ratio of the signal to the fluctuations of the noise.

As long as the acquisition time is shorter than the coherence time of the axion field, no information is lost. Apart from the complication of faster sampling, and correspondingly larger data sets, the statistical accuracy can be recovered by off-line data processing. Recent studies of axion flows in the vicinity of the earth [22] suggest that the axions form a Bose-Einstein condensate, and predict coherence times in the order of seconds, even for axion masses as high as $m_a = 10^{-4}$.

A search for cosmic axions using an optical cavity can reach the axion limit and cover the entire cosmologically interesting range 10^{-6} to 10^{-4} eV in 1 yr. As compared to the microwave cavity searches the optical technique has the advantage of measuring the amplitude (rather than the power) of the induced em field. Of course, the ''external'' magnetic field is significantly weaker. Another advantage is that with the optical cavity the search is broadband and several axion frequencies are queried at each setting of the cavity length. Finally, there are no special conditions imposed on the geometric dimensions of the cavity and the tuning is straightforward. Exploiting the (partially) coherent nature of the axion field using a microwave cavity has been proposed previously [21]. In that case the loaded microwave cavity is tuned to the axion frequency and the axion signal is extracted from the slow modulation of the cavity power, rather than by the direct detection of the sideband power as proposed here.

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