Destabilizing Effect of Dynamical Friction on Fast-Particle-Driven Waves in a Near-Threshold Nonlinear Regime

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The nonlinear evolution of waves excited by the resonant interaction with energetic particles, just above the instability threshold, is shown to depend on the type of relaxation process that restores the unstable distribution function. When dynamical friction dominates over diffusion in the phase space region surrounding the wave-particle resonance, an explosive evolution of the wave is found to be the only solution. This is in contrast with the case of dominant diffusion when the wave may exhibit steady-state, amplitude modulation, chaotic and explosive regimes near marginal stability. The experimentally observed differences between Alfvénic instabilities driven by neutral beam injection and those driven by ion-cyclotron resonance heating are interpreted.

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The study of driven kinetic systems, and their associated complex nonlinear behavior, is not only of pure academic interest but also has applications to magnetically confined fusion plasmas. Waves excited by energetic particles have the potential to eject energetic particles from the plasma [1], which is undesirable. Such waves can also provide information about the plasma conditions [2] which will be necessary for the burning plasma environment that cannot be accessed directly. The nonlinear temporal behavior of individual waves is an essential element of systematic analysis with predictive capability.

A general theory [3] has been developed for describing the nonlinear evolution of a wave near the threshold satisfying $\gamma \equiv |\gamma_L - \gamma_d| \ll \gamma_d \leq \gamma_L$ where γ_L is the energetic particle contribution to the wave growth rate (which is assumed to be constant but can be extended to slowly time varying without consequence [4]) and γ_d is the wave damping rate due to dissipation in the bulk plasma. It was theoretically shown in [3] that the mode evolution just above the threshold reflects an interplay between the wave electric field, that tends to flatten the distribution function of energetic particles, and the relaxation processes, which tend to restore the unstable distribution function with a characteristic time scale $1/\nu_{\text{eff}}$. The relaxation process restoring the unstable distribution function was modeled in [3] via an ''annihilation'' (Krook [5]) collision operator that treats the effect of collisions as $-\nu_{\text{eff}}(F - F_0)$, with F_0 and F being the equilibrium and perturbed distribution functions, respectively. Within this model it was found that a steady-state solution does not always establish itself near the threshold, and four main regimes of the near-threshold nonlinear amplitude evolution have been predicted in [3] depending on the ratio of ν_{eff}/γ: (i) a steady-state regime; (ii) a regime with periodic amplitude modulation; (iii) a chaotic regime, and (iv) an ''explosive'' regime. The case of velocity-space diffusion was also investigated in [6] and produced very similar nonlinear behavior to the Krook collisions.

The first three regimes have been identified in JET experiments on Toroidal Alfvén Eigenmode (TAE) excitation by ICRH (ion-cyclotron resonance heating) [4,7]. The explosive regime leading to a strongly nonlinear phase was identified in MAST experiments with TAEs driven by NBI (neutral beam injection) [8]. Because of the strong nonlinearity that develops in the explosive scenario, the instability on MAST was observed in the form of TAE ''bursts'', representing a near-threshold type [9] of a general bursting nonlinear scenario described in [10].

A comparison of the nonlinear TAE evolution [8] with that on other machines has shown that there is a tendency for NBI-driven Alfvénic instabilities to exhibit a bursting behavior on NSTX [11], TFTR [12], DIII-D [13], and JT-60U [14]. On the other hand, ICRH-driven modes in the Alfvén frequency range, similar to those observed in [4,7], show predominantly the first three types of mode evolution on TFTR [15], JT-60U [16], DIII-D [17], and C-MOD [18]. Taking into account that the distribution function of NBI-produced ions establishes itself due to electron dynamical friction (also called drag and slowing down) [19], while the distribution function of ICRH-accelerated ions [20] is formed via a quasilinear diffusive process, a comparison between dynamical friction and velocity-space diffusion becomes an important issue for kinetic instabilities. In this Letter we make such a comparison for a bump-on-tail instability and extend the results to toroidal systems with the TAE instability.

In the aforementioned theory [3] the bump-on-tail problem was solved kinetically, with a simplified collision operator, for the beam distribution function $F(x, v, t)$ under the influence of an electrostatic field $E = 1/2[\hat{E}(t)e^{i(kx-\omega t)} + \text{c.c}]$, where $\hat{E}(t)$ is allowed to be complex to permit nonlinear frequency shifting. Despite the specifics of the idealized bump-on-tail model, the nonlinear behavior of this system is in fact universal [6] and can be applied to electromagnetic waves in toroidal geometry, by transforming to appropriate variables. Previously investigated fast particle relaxation mechanisms included velocity-space diffusion [4,6] and annihilation (Krook-type collisions) [3], which were found to display similar properties; however, the effect of a drag collision operator was not considered. The appropriate collision operator for the problem is determined by what collisional process is dominant at the wave-particle resonance in phase space. For fast ions with velocities v_f satisfying $v_i \ll v_f \ll v_e$ where v_i and v_e are the background ion and electron thermal velocities, Coulomb collisions can be described as a combination of pitch angle scattering and electron drag [19,21]. The former can be represented by a diffusive operator as investigated in [6], while the latter introduces a slowing down operator to the kinetic equation. With all three (diffusive, drag, and Krook-type) collision operators included, the relevant kinetic equation for the bump-on-tail problem becomes

$$\frac{\partial F}{\partial t} + \left(\frac{u+\omega}{k}\right)\frac{\partial F}{\partial x} + \frac{ek}{2m}[\hat{E}(t)e^{i(kx-\omega t)} + \text{c.c}]\frac{\partial F}{\partial u} - \nu^3\frac{\partial^2 F}{\partial u^2} - \alpha^2\frac{\partial F}{\partial u} + \beta F = -\nu^3\frac{\partial^2 F_0}{\partial u^2} - \alpha^2\frac{\partial F_0}{\partial u} + \beta F_0, \tag{1}$$

where F_0 is the equilibrium distribution function in the absence of any wave field, $u = kv - \omega$ and ν, α, and β are constants characterizing the velocity-space diffusion, dynamical friction, and Krook operator, respectively, in the narrow vicinity of the resonance. We represent F as a Fourier series $F = F_0 + f_0 + \sum_{n=1}^{\infty}[f_n \exp(in\psi) + \text{c.c}]$ with slowly varying Fourier amplitudes f_n and $\psi = kx - \omega t$. By assuming that any change in $\hat{E}(t)$ occurs on a slower time scale than the wave frequency, the wave equation can be written as

$$\frac{\partial \hat{E}}{\partial t} + 4\pi e\frac{\omega}{k^2}\int f_1 du + \gamma_d \hat{E} = 0. \tag{2}$$

Following Refs. [3,6] we expand the perturbed distribution function in powers of the wave amplitude $\hat{E}$ and we use the ordering $F_0 \gg f_1 \gg f_0, f_2$ to truncate Eq. (1) at the lowest order nonlinearity (cubic) to give

$$\frac{\partial f_0}{\partial t} - \nu^3\frac{\partial^2 f_0}{\partial u^2} - \alpha^2\frac{\partial f_0}{\partial u} + \beta f_0 = -\frac{ek}{2m}\hat{E}\frac{\partial f_1^*}{\partial u} - \frac{ek}{2m}\hat{E}^*\frac{\partial f_1}{\partial u}, \tag{3a}$$

$$\frac{\partial f_1}{\partial t} + iuf_1 - \nu^3\frac{\partial^2 f_1}{\partial u^2} - \alpha^2\frac{\partial f_1}{\partial u} + \beta f_1 = -\frac{ek}{2m}\hat{E}\left[\frac{\partial F_0}{\partial u} + \frac{\partial f_0}{\partial u}\right]. \tag{3b}$$

Here, f_2 has been neglected as it does not contribute to the final expression [Eq. (4)] for the wave amplitude. The actual expansion parameter is $\omega_B\tau$, where $\omega_B \equiv \sqrt{eEk/m}$ is the bounce frequency of the resonant particles, and τ is the time scale of interest. As seen from the resulting Eq. (4), the nonlinear correction to the wave growth rate can be estimated as $\gamma_L(\omega_b\tau)^4$ whereas the linear growth rate itself is $\gamma_L - \gamma_d \ll \gamma_L$. The lowest order nonlinearity becomes important when $(\omega_b\tau)^4 \approx (\gamma_L - \gamma_d)/\gamma_L \ll 1$. At this level, the next-order nonlinear term, $\gamma_L(\omega_b\tau)^8$, is still negligible. Thus, the inequality $(\gamma_L - \gamma_d)/\gamma_L \le (\omega_b\tau)^4 \ll 1$ defines a window in which the dynamics are already nonlinear but the nonlinearity can still be treated perturbatively. By comparing the second term in Eq. (3b) to the collision terms separately, resonance widths can be constructed to characterise the role of various collisions at the wave-particle resonance with $\Delta u_\nu = \nu$, $\Delta u_\alpha = \alpha$ and $\Delta u_\beta = \beta$ for diffusive, drag and Krook collisions, respectively. We solve Eq. (3) iteratively for f_1 as in [3], and the equation for the evolution of the wave amplitude takes the following form

$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \times \int_0^{\tau-2z} dx e^{-\hat{\nu}^3 z^2(2z/3+x) - \hat{\beta}(2z+x) + i\hat{\alpha}^2 z(z+x)} \times A(\tau - z - x)A^*(\tau - 2z - x), \tag{4}$$

where $A = [ek\hat{E}(t)/m/(\gamma_l - \gamma_d)^2][\gamma_l/(\gamma_l - \gamma_d)]^{1/2}$, $\tau = (\gamma_l - \gamma_d)t$, $\hat{\nu}^3 = \nu^3/(\gamma_l - \gamma_d)^3$, $\hat{\alpha}^2 = \alpha^2/(\gamma_l - \gamma_d)^2$, $\hat{\beta} = \beta/(\gamma_l - \gamma_d)$ and $\gamma_l = 2\pi^2(e^2\omega/mk^2)\partial F_0(\omega/k)/\partial v$. The case of no dynamical friction, $\hat{\alpha} = 0$, in Eq. (4) was considered in [6]. The inclusion of a nonzero $\hat{\alpha}$ introduces an oscillatory dependence to the integral, which has a profound effect on the nonlinear behavior of the mode amplitude, as the integral in Eq. (4) can then easily change sign. Previously the nonlinear amplitude evolution was divided into soft and hard nonlinear regimes. The amplitude evolves to a low level in the soft case, reflecting the closeness to the threshold, whereas the hard case leads to a solution which "explodes" in a finite time.

For the case of pure drag ($\hat{\nu} = \hat{\beta} = 0$) the amplitude does not saturate at a low level, since Eq. (4) does not admit "steady-state solutions" in the form $A_0 = |A_0| \exp(ib\tau)$ as $\tau \to \infty$ for any value of $\hat{\alpha}$. Numerical simulations show that the behavior is indeed explosive, similar to Ref. [3]. This is in contrast to the previously studied Krook operator and diffusive cases. By adding diffusion along with drag, the existence of steady-state solutions of Eq. (4) is then only prohibited when the integral in Eq. (4) has a negative real part as $\tau \to \infty$, in which case $\hat{\nu}/\hat{\alpha} < 1.043$ (Fig. 1 dotted line). However the steady solutions that do exist become unstable for smaller values of $\hat{\alpha}$. By perturbing the steady-state solution $A = A_0(1 + \delta A)$ with $\delta A = C\exp(\lambda\tau) + D\exp(\lambda^*\tau)$, a stability boundary in $\hat{\nu}$, $\hat{\alpha}$ space can be calculated by solving the dispersion relation in Eq. (5) for the case when $\mathrm{Re}\{\lambda\} = 0$, with $P(z) = z^2\exp(-2\hat{\nu}^3 z^3/3 + i\hat{\alpha}^2 z^2)$ and $|A_0|^{-2} = \mathrm{Re}\int_0^\infty dz P(z)/(2\hat{\nu}^3 z^2 - 2i\hat{\alpha}^2 z)$. The resultant boundary is displayed in Fig. 1 (solid line)

$$0 = \left\{\lambda + ib - 1 + \frac{|A_0|^2}{2}\int_0^\infty dz P(z)\left[\frac{e^{-\lambda z}}{\hat{\nu}^3 z^2 - i\hat{\alpha}^2 z + \lambda} + \frac{e^{-\lambda z}}{\hat{\nu}^3 z^2 - i\hat{\alpha}^2 z}\right]\right\}\left\{\lambda - ib - 1 + \frac{|A_0|^2}{2}\int_0^\infty dz P^*(z) \times\left[\frac{e^{-\lambda z}}{\hat{\nu}^3 z^2 + i\hat{\alpha}^2 z + \lambda} + \frac{e^{-\lambda z}}{\hat{\nu}^3 z^2 + i\hat{\alpha}^2 z}\right]\right\} - \left\{\frac{|A_0|^2}{2}\int_0^\infty dz P(z)\frac{e^{-2\lambda z}}{\hat{\nu}^3 z^2 - i\hat{\alpha}^2 z + \lambda}\right\}\left\{\frac{|A_0|^2}{2}\int_0^\infty dz P^*(z)\frac{e^{-2\lambda z}}{\hat{\nu}^3 z^2 + i\hat{\alpha}^2 z + \lambda}\right\}. \quad (5)$$

For the bump-on-tail case, the F_0 close to the resonance is a straight line with constant gradient, Fig. 2 (dotted). For the analysis presented here to be valid, the perturbation from equilibrium must not be strong. For the diffusive case, Fig. 2 (dashed) shows that there is a slight modification of the distribution function, which flattens the slope close to the resonance. This is consistent with a system driven to saturation. As $\hat{\nu}$ is increased, the flattened region increases but the slope of F remains positive. However, by adding a significant drag component to the collision term, Fig. 2 (solid) shows that large perturbations in the distribution are formed. As $\hat{\alpha}$ increases, these perturbations in the distribution function increase, which shows that the perturbative approach of this analysis breaks down and indicates that the system is being driven towards a hard nonlinear regime.

In order to generalize the results obtained above to the TAE instability in toroidal geometry, one can follow the procedure in [6] and introduce action-angle variables to represent the wave-particle resonance condition as $\Omega \equiv \omega - n\langle\omega_\phi\rangle - l\langle\omega_\theta\rangle = 0$, where $\omega \equiv \omega_{\mathrm{TAE}}$ is the TAE frequency, $\langle\omega_\phi\rangle \equiv \langle\partial\phi/\partial t\rangle$ and $\langle\omega_\theta\rangle \equiv \langle\partial\theta/\partial t\rangle$ are the orbit frequencies of energetic ions along the toroidal, ϕ, and poloidal, θ coordinates, $\langle\ldots\rangle$ represents the orbit averaging, n is toroidal mode number of the TAE, and l is an integer value. For describing fast particles moving across this resonance, due to the drag and diffusion, the Fokker-Planck operator is represented in the form

$$\left.\frac{dF}{dt}\right|_{\mathrm{coll}} = \frac{\partial}{\partial \mathbf{v}}\cdot\mathbf{D}\cdot\frac{\partial f}{\partial \mathbf{v}} + \frac{\partial}{\partial \mathbf{v}}\cdot\mathbf{b}f = \left\langle\frac{\partial P_\phi}{\partial \mathbf{v}}\cdot\mathbf{D}\cdot\frac{\partial P_\phi}{\partial \mathbf{v}}\right\rangle\left(\frac{\partial\Omega}{\partial P_\phi}\right)^2\frac{\partial^2 f}{\partial\Omega^2} + \left\langle\frac{\partial P_\phi}{\partial \mathbf{v}}\cdot\mathbf{b}\right\rangle\left(\frac{\partial\Omega}{\partial P_\phi}\right)\frac{\partial f}{\partial\Omega}, \quad (6)$$

where P_ϕ is the toroidal canonical momentum and the derivatives $\partial\Omega/\partial P_\phi$ are taken at a constant value of $E - (\omega/n)P_\phi$ (E is energy). Considering TAEs driven by strongly passing beam ions with the resonance condition

$$\Omega \equiv \omega - n\frac{v_\parallel}{R} - l\frac{v_\parallel}{Rq(r)} = 0, \quad (7)$$

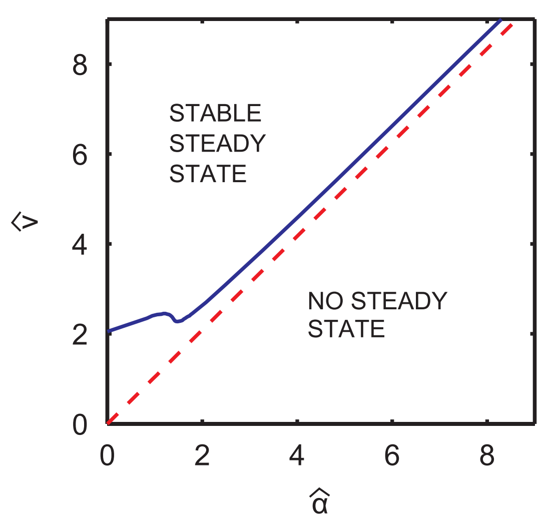

FIG. 1 (color online). Displays the boundaries in parameter space that give stable, unstable and no steady-state solutions to Eq. (4). The unstable solution lies in between the solid and dashed lines.

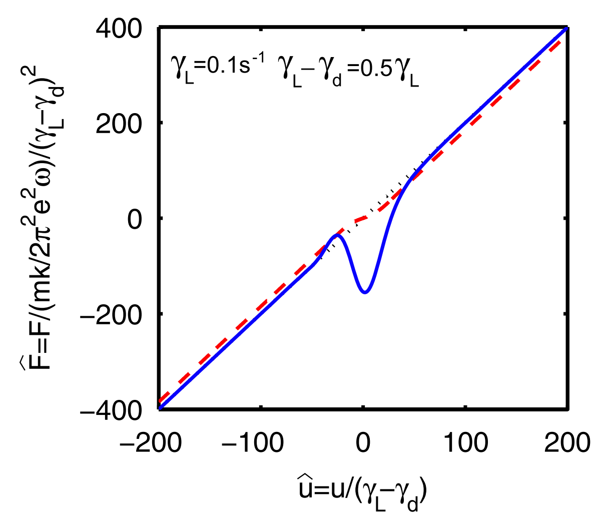

FIG. 2 (color online). Saturated distribution function for the case of no wave (dotted line), pure diffusion with $\hat{\nu} = 10$ (dashed line), and diffusion + drag with $\hat{\nu} = 10$ $\hat{\alpha} = 8.9$ (solid line).

where R is the major radius, r is the minor radius, q is the safety factor and $v_\parallel$ is the velocity parallel to the equilibrium magnetic field, one can estimate, from Eqs. (6) and (3b), the width of the resonance due to diffusion ($\Delta\Omega_{\rm Diff}$) and drag ($\Delta\Omega_{\rm Drag}$) similar to Δu_ν and Δu_α. The ratio of the two gives an estimate for which process dominates at the resonance

$$\frac{(\Delta\Omega_{\rm Diff})^6}{(\Delta\Omega_{\rm Drag})^6} \approx \left\langle \frac{\partial P_\phi}{\partial \mathbf{v}} \cdot \mathbf{D} \cdot \frac{\partial P_\phi}{\partial \mathbf{v}} \right\rangle^2 \left(\frac{\partial \Omega}{\partial P_\phi}\right) \left\langle \frac{\partial P_\phi}{\partial \mathbf{v}} \cdot \mathbf{b} \right\rangle^{-3} \tag{8}$$

and upon substituting the appropriate $\mathbf{D}$ and $\mathbf{b}$ [21], Eq. (8) becomes

$$\frac{(\Delta\Omega_{\rm Diff})^6}{(\Delta\Omega_{\rm Drag})^6} \approx mS\tau \frac{c}{eB_0} \frac{E_{\rm A}}{r^2} \frac{\theta_b^4}{2} \frac{27}{64} \left(\frac{\pi m_b}{m_e}\right)^{3/2} \left(\frac{T_e}{E_{\rm A}}\right)^{9/2}, \tag{9}$$

where m is the poloidal mode number (of order unity), m_b is the mass of the beam species (deuterium in MAST), $r \approx 50$ cm is the minor radius, $B_0 \approx 5$ kG, $T_e \approx 0.1$–1 keV is the electron temperature, $E_{\rm A} \approx 10$ keV is the resonant Alfvénic energy, $\theta_b \approx 0.6$ rad, $S \approx 10^{-2}$–10^{-1} is the magnetic shear, $\tau \equiv E_{\rm A}^{3/2} m_b^{1/2} / \pi Z_b^2 e^4 n_e \ln\Lambda \sqrt{2} \approx 0.025s$. Using these MAST parameters the collisional diffusion vs drag is calculated to be $\Delta\Omega_{\rm Diff}/\Delta\Omega_{\rm Drag} \approx 0.2$–$1.6$, demonstrating that drag can dominate over the collisional diffusion in the vicinity of the TAE resonance.

From this one would expect that steady-state nonlinear TAE behavior is hardly possible for the case of NBI-produced slowing-down distribution functions, as is indeed observed in the bursting TAE experiments with NBI. In contrast, ICRH-accelerated ions have a distribution function for which the dominant relaxation process is a quasilinear diffusion due to the ICRH wave field. Here $\nu_{\rm eff}$ due to the wave exceeds the Coulomb collision frequency by an order of magnitude [4]. With such strong diffusion dominating over the drag, the marginally unstable TAEs are expected to exhibit the set of four regimes [3], as is indeed the case [4,7].

Finally, we note that for ITER relevant parameters ($T_e \sim 10$ keV, $B_0 \sim 50$ kG, $E_{\rm A} \sim 1$ MeV, $n_e \sim 10^{14}$ cm^{-3}, $r \sim 200$ cm, $\theta_b \sim 1$), the alpha particle excited TAEs are expected to be dominated by diffusion with $\Delta\Omega_{\rm Diff}/\Delta\Omega_{\rm Drag} \approx 1.4$. Since drag is not negligibly small its effect should still be included, as described in this paper (Fig. 1), in any predictive ITER calculations. The relevance of drag is further emphasized by noting the sensitivity of this estimate to the electron temperature, more specifically $\Delta\Omega_{\rm Diff}/\Delta\Omega_{\rm Drag}$ scales as $T_e^{3/4}$.

In summary, the destabilizing effect of dynamical friction, leading to an explosive behavior, has been demonstrated in this Letter in the framework of the near-threshold nonlinear theory. A description of the mode evolution beyond the early explosive phase, i.e., in the strongly nonlinear regime, with the addition of dynamical friction will be the subject of further study.

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