

Higher Derivative Corrections to Black Hole Thermodynamics from Supersymmetric Matrix Quantum Mechanics

Masanori Hanada,^{1,*} Yoshifumi Hyakutake,^{2,†} Jun Nishimura,^{3,4,‡} and Shingo Takeuchi^{4,5,§}

¹*Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

²*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

³*High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan*

⁴*Department of Particle and Nuclear Physics, School of High Energy Accelerator Science, Graduate University for Advanced Studies (SOKENDAI), Tsukuba 305-0801, Japan*

⁵*Asia Pacific Center for Theoretical Physics (APCTP), Pohang, Gyeongbuk 790-784, Korea*

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We perform a direct test of the gauge-gravity duality associated with the system of N $D0$ -branes in type IIA superstring theory at finite temperature. Based on the fact that higher derivative corrections to the type IIA supergravity action start at the order of α'^3 , we derive the internal energy in expansion around infinite 't Hooft coupling up to the subleading term with one unknown coefficient. The power of the subleading term is shown to be nicely reproduced by the Monte Carlo data obtained nonperturbatively on the gauge theory side at finite but large effective (dimensionless) 't Hooft coupling constant. This suggests, in particular, that the open strings attached to the $D0$ -branes provide the microscopic origin of the black hole thermodynamics of the dual geometry including α' corrections. The coefficient of the subleading term extracted from the fit to the Monte Carlo data provides a prediction for the gravity side.

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Introduction.—It is widely believed that large- N gauge theory provides a nonperturbative description of superstrings [1,2] and hence of quantum space-time. In pursuing such a direction, it is useful to consider a particular setup with a stack of N D -branes in the so-called decoupling limit. After taking this limit there exists a parameter region, in which the superstring theory in the bulk ten dimensions reduces to a classical supergravity theory so that one only has to consider a particular classical solution that describes the N D -branes. On the other hand, the worldvolume theory of the N D -branes is given by a supersymmetric $U(N)$ gauge theory. In the above parameter region, which corresponds to taking the planar large- N limit with infinite 't Hooft coupling, the gauge theory is conjectured to have a dual description in terms of the supergravity solution [3]. Including α' corrections on the gravity side corresponds to including subleading terms with respect to the inverse 't Hooft coupling constant on the gauge theory side. Similarly, including string loop corrections corresponds to including $1/N$ corrections. In fact the gauge theory is well-defined for arbitrary coupling constant and N , and thus it is expected to be a nonperturbative description of superstrings in a certain curved background.

The system of $D0$ -branes in type IIA superstring theory provides a particularly simple example of the gauge-gravity duality [4] since the gauge theory in this case lives in one dimension, and hence it is nothing but matrix quantum mechanics (MQM). It is also important due to its connection [1] to M theory [5], which is expected to emerge in the strong coupling limit of type IIA superstring theory. Recently Monte Carlo studies of the supersymmetric MQM have been performed [6] by using a nonlattice

regularization [7], which respects supersymmetry maximally. The internal energy calculated for a wide range of the effective 't Hooft coupling at finite temperature interpolates nicely the weak coupling behavior [8], and the leading asymptotic behavior in the strong coupling limit predicted from the black hole thermodynamics of the dual geometry. Consistent results are obtained also from the lattice approach [9]. See Refs. [10] for earlier works on the same system based on the Gaussian expansion method.

In this Letter we perform a precision test of the above gauge-gravity duality by considering α' corrections to the black hole thermodynamics. One of our main results is that the internal energy E at temperature T is given as

$$\frac{1}{N^2} \frac{E}{\lambda^{1/3}} = c_1 \left(\frac{T}{\lambda^{1/3}} \right)^{14/5} - C \left(\frac{T}{\lambda^{1/3}} \right)^{23/5}, \quad (1)$$

$$\lambda = (2\pi)^{-2} \alpha'^{-(3/2)} g_s N \quad (2)$$

in the large- N limit with fixed $\lambda \gg T^3$, where $c_1 \simeq 7.41$. The first term is known [4] from the supergravity analysis [11]. The second term is the one we get from α' corrections, where C is calculable once the $O(\alpha'^3)$ correction to the supergravity action is obtained completely.

On the gauge theory side, λ corresponds to the 't Hooft coupling constant. By comparing Monte Carlo data at large but finite λ with Eq. (1), we can test the gauge-gravity duality including α' corrections.

The dual black hole geometry.—The low-energy effective theory of type IIA superstring theory can be obtained at the tree level as $\mathcal{S} = \mathcal{S}_{(0)} + \mathcal{S}_{(1)} + \dots$ in the α' expansion. The type IIA supergravity action corresponds to the leading term $\mathcal{S}_{(0)}$, which is given by

$$\mathcal{S}_{(0)} = \kappa \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} (R + 4\partial_\mu \phi \partial^\mu \phi) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \right\} \quad (3)$$

in the string frame, where we show the terms which depend only on the metric $g_{\mu\nu}$, the dilaton ϕ and the Ramond-Ramond (R-R) 1-form potential $A = A_\mu dx^\mu$ with the field strength $G = dA$. The coefficient κ is given by $\kappa^{-1} = 16\pi G_N = (2\pi)^7 \alpha'^4 g_s^2$ in terms of the ten-dimensional Newton constant G_N . In the last term of (3), we have absorbed the tree-level dilaton factor into the normalization of the R-R 1-form potential.

In type IIA supergravity, N D0-branes at finite temperature can be described by the nonextremal black 0-brane solution. In the decoupling limit, we are interested in the excitations of the D0-branes with fixed energy in the $\alpha' \rightarrow 0$ limit. Correspondingly, we need to introduce a new radial coordinate $U = r/\alpha'$ and take the near-horizon limit of the above solution, which reads [4]

$$ds^2 = \alpha' \left(-\frac{f}{H^{1/2}} dt^2 + \frac{H^{1/2}}{f} dU^2 + H^{1/2} U^2 d\Omega_8^2 \right), \quad (4)$$

$$e^\phi = \alpha'^{-(3/2)} H^{3/4}, \quad G = \alpha'^2 H^{-2} H' dt \wedge dU,$$

where $d\Omega_8^2$ represents the line element of S^8 . The functions $H(U)$ and $f(U)$ are given as

$$H = \frac{2^4 15 \pi^5 \lambda}{U^7}, \quad f = 1 - \frac{U_0^7}{U^7}, \quad (5)$$

where λ is given by (2). The metric (4) represents a black hole geometry with an event horizon located at $U = U_0$.

Black hole thermodynamics.—Given the black hole geometry, we can obtain thermodynamical quantities associated with it from the geometry at the horizon.

The Hawking temperature T is obtained by requiring that a conical singularity does not appear at $U = U_0$ when one makes the Wick rotation and compactifies the Euclidean time $\tau = it$ to $\beta = T^{-1}$. This gives

$$T = \frac{1}{4\pi} H^{-(1/2)} f'|_{U=U_0} = c_2 \lambda^{1/3} \left(\frac{U_0}{\lambda^{1/3}} \right)^{5/2}, \quad (6)$$

where $c_2 = 7/(2^4 15^{1/2} \pi^{7/2})$. Note that the extremal case ($T = 0$) corresponds to choosing $U_0 = 0$.

The Bekenstein-Hawking entropy S is evaluated by the area \mathcal{A} of the horizon in the Einstein frame as [11]

$$\frac{S}{N^2} = \frac{1}{N^2} \frac{\mathcal{A}}{4G_N} = c_3 \left(\frac{T}{\lambda^{1/3}} \right)^{9/5}, \quad (7)$$

where $c_3 = 4^{13/5} 15^{2/5} (\pi/7)^{14/5}$ and we have used Eq. (6).

The internal energy E is determined by the first law of thermodynamics $dE = TdS$, and it gives the first term in (1) with the coefficient $c_1 = \frac{9}{14} c_3 = 7.407 \dots$

Let us recall the region of validity for this leading behavior [4]. In order for the contributions from string oscillations to be neglected, the curvature radius ρ of the geometry (4) at $U = U_0$ should be much larger than the

string scale $\sqrt{\alpha'}$, i.e.,

$$\frac{\rho^2}{\alpha'} \equiv \frac{4\pi^{5/2} 15^{1/2}}{147} \left(\frac{\lambda}{U_0^3} \right)^{1/2} \gg 1, \quad (8)$$

which corresponds to $T/\lambda^{1/3} \ll 1$ due to (6). In order for the string loop effects to be neglected, the effective string coupling at $U = U_0$ should be small enough, i.e.,

$$g_s e^\phi \equiv \frac{2^5 15^{3/4} \pi^{23/4}}{N} \left(\frac{\lambda}{U_0^3} \right)^{7/4} \ll 1, \quad (9)$$

which corresponds to $T/\lambda^{1/3} \gg N^{-(10/21)}$.

Higher derivative corrections.—When (8) is not met, we need to consider α' corrections to the type IIA supergravity action $\mathcal{S}_{(0)}$. They can be obtained by calculating tree-level scattering amplitudes of the massless modes in type IIA superstring theory.

Explicit calculations show that the two-point and three-point amplitudes contribute only to $\mathcal{S}_{(0)}$, and hence $\mathcal{S}_{(1)} = \mathcal{S}_{(2)} = 0$. On the other hand, the four-point amplitudes are known to give nontrivial contributions to the effective action at the order α'^3 [12].

From this fact alone, we can deduce the power of the subleading term in Eq. (1). On dimensional grounds, the actual expansion parameter in the α' expansion is α'/ρ^2 , which is the inverse of (8). Using (6), this translates to $(T/\lambda^{1/3})^{3/5}$. Since the black hole thermodynamics is expected to receive corrections of the order $(\alpha'/\rho^2)^3 \sim (T/\lambda^{1/3})^{9/5}$, we obtain $\frac{14}{5} + \frac{9}{5} = \frac{23}{5}$ as the power of the subleading term.

More on α' corrections.—Here we present a more detailed analysis of the α' corrections, which yields the power of the second term in Eq. (1). We hope that our analysis will be useful also in calculating the coefficient C once the complete form of $\mathcal{S}_{(3)}$ is obtained.

A typical term in $\mathcal{S}_{(3)}$ is given by [13]

$$\mathcal{S}_{(3)} = \kappa \int d^{10}x \sqrt{-g} \{ \alpha'^3 e^{-2\phi} \mathcal{R}^4 + \dots \}, \quad (10)$$

where \mathcal{R}^4 stands for a scalar quantity obtained by contracting indices of four Riemann tensors and multiplying by some numerical factor. (Its explicit form can be found in Ref. [14], for example.) Dilaton-dependent terms can be obtained by replacing the Riemann tensor by the second covariant derivative $D^2\phi$ of the dilaton field [15]. Other possible terms can be written symbolically as $\alpha'^3 R^3 G^2$, $\alpha'^3 R^2 (DG)^2$ and so on [16,17].

The equations of motion are derived by taking the variation of the effective action $\mathcal{S} = \mathcal{S}_{(0)} + \mathcal{S}_{(3)}$ with respect to ϕ , for instance, as

$$0 = R + 4\partial^\mu \phi \partial_\mu \phi - 2e^{2\phi} D_\mu \partial^\mu e^{-2\phi} + \alpha'^3 \mathcal{R}^4 + \dots, \quad (11)$$

and similarly for $g_{\mu\nu}$ and A_μ . Here we assume that the solution to these equations is given by the same form (4)

with the functions $H(U)$ and $f(U)$ replaced by

$$H = \frac{2^4 15 \pi^5 \lambda}{U^7} (1 + H_{(3)}), \quad f = 1 - \frac{U_0^7}{U^7} + f_{(3)}. \quad (12)$$

Using this ansatz, Eq. (11) becomes

$$0 = -\frac{1}{\alpha'} \left(\frac{U^3}{\lambda} \right)^{1/2} \left[\left(1 - \frac{U_0^7}{U^7} \right) \left(-3UH'_{(3)} + \frac{U^2}{2} H''_{(3)} \right) \right. \\ \left. + (56f_{(3)} + 16Uf'_{(3)} + U^2f''_{(3)}) \right] + \frac{1}{\alpha'} \left(\frac{U^3}{\lambda} \right)^2 h \left(\frac{U_0}{U} \right). \quad (13)$$

The last term is obtained by substituting the leading terms of the solution into the subleading terms in (11). The explicit form of $h(\frac{U_0}{U})$ can be obtained once $\mathcal{S}_{(3)}$ is given. It is important that this last term has an extra factor of $\lambda^{-(3/2)}$ compared with the other terms, which is understandable since the effective expansion parameter is given by $\alpha'/\rho^2 \sim (U_0^3/\lambda)^{1/2}$ as mentioned above. Note, for instance, that the fourth term in (11) is estimated as $\alpha'^3 \mathcal{R}^4 \sim \alpha'^3 \times (\alpha'^{-1} \lambda^{-(1/2)})^4 = \alpha'^{-1} \lambda^{-2}$ using $R \sim \alpha'^{-1} \lambda^{-(1/2)}$ deduced from (8). (We have also checked by explicit calculations that this kind of estimate is true for all possible subleading terms.) Since the other equations of motion have the same structure, we conclude that $H_{(3)}$ and $f_{(3)}$ can be written as

$$H_{(3)} = \left(\frac{U_0}{\lambda^{1/3}} \right)^{9/2} \tilde{H} \left(\frac{U_0}{U} \right), \quad f_{(3)} = \left(\frac{U_0}{\lambda^{1/3}} \right)^{9/2} \tilde{f} \left(\frac{U_0}{U} \right) \quad (14)$$

with some functions $\tilde{H}(\frac{U_0}{U})$ and $\tilde{f}(\frac{U_0}{U})$.

The location of the horizon U_H is shifted from U_0 due to the α' corrections, and it should be determined from $f(U_H) = 0$, which reads

$$\frac{U_0}{U_H} = 1 + \frac{\tilde{f}(1)}{7} \left(\frac{U_H}{\lambda^{1/3}} \right)^{9/2}. \quad (15)$$

The Hawking temperature is obtained as

$$T = \frac{1}{4\pi} H^{-(1/2)} f' |_{U=U_H} \\ = c_2 \lambda^{1/3} \left(\frac{U_H}{\lambda^{1/3}} \right)^{5/2} \left\{ 1 + c_4 \left(\frac{U_H}{\lambda^{1/3}} \right)^{9/2} \right\}, \quad (16)$$

where $c_4 = \tilde{f}(1) - \frac{1}{7} \tilde{f}'(1) - \frac{1}{2} \tilde{H}(1)$. By solving this equation for U_H iteratively, we obtain

$$\frac{U_H}{\lambda^{1/3}} = \left(\frac{T}{c_2 \lambda^{1/3}} \right)^{2/5} \left\{ 1 - \frac{2}{5} c_4 \left(\frac{T}{c_2 \lambda^{1/3}} \right)^{9/5} \right\}. \quad (17)$$

The Bekenstein-Hawking entropy formula is no longer valid in the presence of higher derivative terms, and we need to use the Wald formula [18,19]. For spherically symmetric black holes, it reads

$$S = -8\pi \int d\Omega_8 \frac{\delta S}{\delta R_{tUtU}} \sqrt{-g_{tt} g_{UU}} |_{U=U_H}, \quad (18)$$

where the variation of the action should be taken by regarding the Riemann tensor as an independent variable. Explicit calculations yield

$$\frac{\delta S}{\delta R_{tUtU}} = \frac{\sqrt{-g} e^{-2\phi} g^{tt} g^{UU}}{32\pi G_N} \{1 + \alpha'^3 (\mathcal{R}^3 + \dots)\}, \quad (19)$$

where we define $\mathcal{R}^3 \equiv 2g_{tt} g_{UU} (\delta \mathcal{R}^4 / \delta R_{tUtU})$. Inserting the leading supergravity solution to the $O(\alpha'^3)$ terms in Eq. (19), we obtain $\alpha'^3 (\mathcal{R}^3 + \dots) = s(\frac{U_0}{U}) (U/\lambda^{1/3})^{9/2}$, similarly to the argument that led to (14). Therefore the entropy (18) is evaluated as

$$\frac{S}{N^2} = \frac{1}{N^2} \frac{\tilde{\mathcal{A}}}{4G_N} \left\{ 1 + s(1) \left(\frac{U_H}{\lambda^{1/3}} \right)^{9/2} \right\} \\ = c_3 \left(\frac{T}{\lambda^{1/3}} \right)^{9/5} \left\{ 1 + c_5 \left(\frac{T}{c_2 \lambda^{1/3}} \right)^{9/5} \right\}, \quad (20)$$

where $c_5 = -\frac{9}{5} c_4 + s(1)$ and the horizon area $\tilde{\mathcal{A}}$ includes α' corrections through Eqs. (14) and (15). The internal energy is obtained as (1), where $C = -\frac{28}{23} c_1 c_5 (c_2)^{-(9/5)}$.

The worldvolume theory.—The worldvolume theory of N D0-branes is given by the $U(N)$ supersymmetric MQM defined by the action

$$S_{\text{MQM}} = \frac{N}{\lambda} \int_0^\beta dt \text{tr} \left\{ \frac{1}{2} (D_t X_i)^2 - \frac{1}{4} [X_i, X_j]^2 + \frac{1}{2} \psi_\alpha D_t \psi_\alpha \right. \\ \left. - \frac{1}{2} \psi_\alpha (\gamma_i)_{\alpha\beta} [X_i, \psi_\beta] \right\}, \quad (21)$$

where $D_t = \partial_t - i[A(t), \cdot]$ represents the covariant derivative with the gauge field $A(t)$ being an $N \times N$ Hermitian matrix. The model describes the open string degrees of freedom attached to the branes, which are decoupled from the bulk degrees of freedom in the decoupling limit $\alpha' \rightarrow 0$ with fixed λ . Note that N and λ can be arbitrary for this statement.

Monte Carlo results.—In simulating the model (21), we fix the gauge by the static diagonal gauge $A(t) = \frac{1}{\beta} \text{diag}(\alpha_1, \dots, \alpha_N)$, where $-\pi < \alpha_a \leq \pi$, and introduce a UV cutoff Λ as $X_i^{ab}(t) = \sum_{n=-\Lambda}^\Lambda \tilde{X}_{in}^{ab} e^{2\pi i n t / \beta}$. Integration over the fermionic matrices yields a complex Pfaffian, which is replaced by its absolute value following the argument in Ref. [20] based on the large- N factorization.

The effective coupling constant is given by $\lambda_{\text{eff}} \equiv \lambda/T^3$, and we set $\lambda = 1$ in actual simulations without loss of generality. In Fig. 1 we plot $7.41T^{14/5} - \frac{1}{N^2} E$ against T in the log-log scale. Indeed the plot reveals a clear power-law behavior of the subleading term with the power $\frac{23}{5} = 4.6$ as predicted in Eq. (1). The data points for $\Lambda = 6$ show small discrepancies at $T \lesssim 0.55$, which can be understood as finite Λ effects by comparing them with the data points for $\Lambda = 8$. Figure 2 shows a linear plot of the energy as a function of T . Fitting the data within $0.5 \leq T \leq 0.7$ (with largest Λ at each T) to $\frac{1}{N^2} E = 7.41T^{14/5} - CT^p$, we obtain $p = 4.58(3)$ and $C = 5.55(7)$. If we instead make a one-

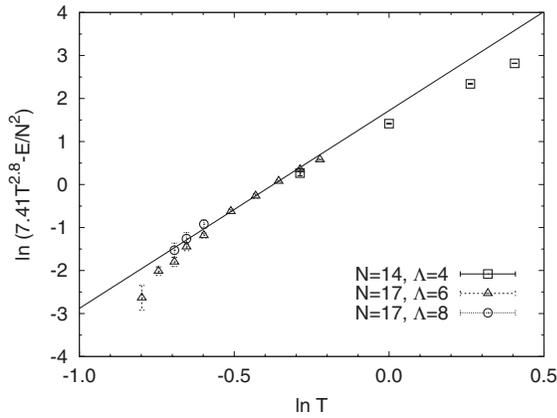


FIG. 1. The deviation of the internal energy $\frac{1}{N^2}E$ from the leading term $7.41T^{14/5}$ is plotted against the temperature in the log-log scale for $\lambda = 1$. The solid line represents a fit to a straight line with the slope 4.6 predicted from the α' corrections on the gravity side.

parameter fit with $p = 4.6$ fixed, we obtain $C = 5.58(1)$. This value, in turn, provides a prediction for the α' corrections on the gravity side.

Summary.—We have discussed the α' corrections to the black hole thermodynamics, which enable us to determine the power of the subleading term in (1). This power is then found to be reproduced precisely by Monte Carlo data in gauge theory. Let us emphasize that the subleading term is crucial for the precision test of the gauge-gravity duality. It is intriguing that our results in gauge theory can tell us the absence of $O(\alpha')$ and $O(\alpha'^2)$ corrections to the supergravity action.

Recently [20], Monte Carlo data for the Wilson loop were also shown to reproduce a prediction obtained by estimating the disk amplitude in the dual geometry. Unlike

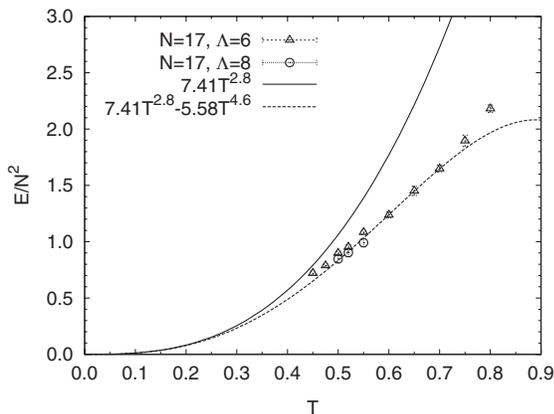


FIG. 2. The internal energy $\frac{1}{N^2}E$ is plotted against T for $\lambda = 1$. The solid line represents the leading asymptotic behavior at small T predicted by the gauge-gravity duality. The dashed line represents a fit to the behavior (1) including the subleading term with $C = 5.58$.

the present case, α' corrections to that quantity start at $O(\alpha')$ due to the fluctuation of the string world sheet and its coupling to the background dilaton field.

While it is certainly motivated to obtain the coefficient C of the subleading term from gravity, our results already provide a strong evidence that the gauge-gravity duality holds including α' corrections. This, in particular, implies that we can understand the microscopic origin of the black hole thermodynamics including α' corrections in terms of the open strings attached to the $D0$ -branes.

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*masanori.hanada@weizmann.ac.il

†hyakutake@cep.osaka-u.ac.jp

‡jnishi@post.kek.jp

§shingo@apctp.org

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