Noise Induces Partial Annihilation of Colliding Dissipative Solitons

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Partial annihilation of two counterpropagating dissipative solitons, with only one pulse surviving the collision, has been widely observed in different experimental contexts, over a large range of parameters, from hydrodynamics to chemical reactions. However, a generic picture accounting for partial annihilation is missing. Based on our results for coupled complex cubic-quintic Ginzburg-Landau equations as well as for the FitzHugh-Nagumo equation we conjecture that noise induces partial annihilation of colliding dissipative solitons in many systems.

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Solitonic behavior is a hallmark of nonlinear macroscopic systems almost free of dissipation, in fields such as nonlinear optics, fluid dynamics and plasma physics [1]. More recently, solitontype behavior has also been observed in strongly dissipative systems including chemical reactions [2] and nonlinear optics [3] as well as prototype equations [4–8]. In binary fluid convection [9,10], a dispersive-dissipative system, collisions between counterpropagating pulses lead to bound states of pulses for low approach velocity, and to partial annihilation of pulses—with only one pulse surviving the collision—for high velocity. In a dissipative-excitable chemical system [2], apart from solitonic behavior, collisions of pulses lead mostly to partial annihilation. This has been modeled using a defect zone in [11].

Interpenetration, annihilation, and bound states of pulses could be accounted for in modeling to a very large extent by using coupled complex cubic-quintic Ginzburg-Landau (GL) [4–7,12,13] and order parameter (Swift-Hohenberg) [8] equations as they arise as prototype equations in the vicinity of a weakly inverted bifurcation to traveling waves. In Ref. [5] it has been shown that such an outcome can be obtained from deterministic coupled complex cubic-quintic GL equations if one of the pulses prepared as an initial condition has not reached its final state yet. However, a generic picture accounting for the experimentally observed partial annihilation of pulses, which are in their asymptotic state before the interaction starts, is missing. It is the goal of the present Letter to present such a framework in terms of two coupled cubic-quintic GL equations with noise. In addition, we present results on the stochastic version of a complementary prototype equation for excitable media as it arises frequently in chemical reactions, nerve pulse propagation as well as for other biological systems, the FitzHugh-Nagumo equation [14,15]. Based on our results for these two vastly different prototype equations we conjecture that partial annihilation of pulses is a noise-induced effect.

We investigate two coupled complex subcritical cubic-quintic Ginzburg-Landau equations for counterpropagating waves with noise:

$$\partial_t A - v\partial_x A = \mu A + (\beta_r + i\beta_i)|A|^2 A + (\gamma_r + i\gamma_i)|A|^4 A + (c_r + ic_i)|B|^2 A + (D_r + iD_i)\partial_{xx} A + \eta\xi_A, \quad (1)$$

$$\partial_t B + v\partial_x B = \mu B + (\beta_r + i\beta_i)|B|^2 B + (\gamma_r + i\gamma_i)|B|^4 B + (c_r + ic_i)|A|^2 B + (D_r + iD_i)\partial_{xx} B + \eta\xi_B, \quad (2)$$

where $A(x, t)$ and $B(x, t)$ are complex fields and where we have discarded quintic cross-coupling terms for simplicity. A and B are slowly varying envelopes and where the stochastic forces $\xi_A(x, t)$ and $\xi_B(x, t)$ denote white noise with the properties $\langle \xi_{A,B} \rangle = 0$, $\langle \xi_A(x,t)\xi_A(x',t')\rangle = \langle \xi_B(x,t)\xi_B(x',t')\rangle = \langle \xi_A(x,t)\xi_B(x',t')\rangle = 0$ and $\langle \xi_A(x, t) \times \xi_A^*(x', t')\rangle = \langle \xi_B(x, t)\xi_B^*(x', t')\rangle = 2\delta(x - x')\delta(t - t')$, where ξ_A^* denotes the complex conjugate of ξ_A. The fast spatial and temporal variations have already been split off when writing down the coupled envelope equations. To compare with measurable quantities such as, for example, temperature variations in fluid dynamics, these rapid variations must be taken into account [16–20].

We have carried out our numerical studies for the following values of the parameters, which we kept fixed for the present purposes: $\mu = -0.112$, $\beta_r = 1$, $\beta_i = 0.2$, $\gamma_r = -1$, $\gamma_i = 0.15$, $D_r = 1$, $D_i = -0.1$, and $c_i = 0$. We note that these parameter values have also been used in previous studies [7,12]. To get an overview where partial annihilation can occur as a function of the parameter values in the model under study, we have plotted in Fig. 1 the phase diagram using the approach velocity, v, of the pulses and the strength of the cubic cross-coupling of counterpropagating waves, c_r, as the axes. Here we focus on stabilizing (negative) values of c_r. The corresponding phase diagram for positive values of c_r revealing a rich variety of outcomes of collisions including various types of bound states of pulses and holes as well as counterpropagating holes has been given elsewhere [7,12].

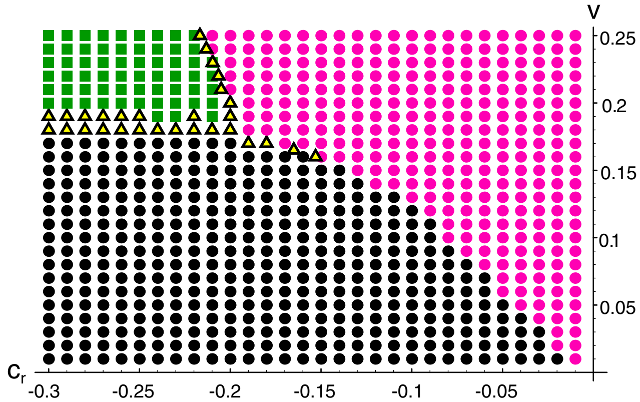

FIG. 1 (color online). Phase diagram in the plane approach velocity v versus strength of cubic cross-coupling of counterpropagating waves, c_r, for stabilizing (negative) values of c_r. Marked in green (grey solid squares) is the annihilation of two pulses, a result, which dominates for strongly stabilizing cross-coupling. Marked as black solid circles are bound states of pulses and in pink (grey solid circles) the interpenetration of two pulses. As open triangles we have depicted the outcome of partial annihilation, for which either the right- or the left-traveling pulse survives, but not the other. Note that partial annihilation due to numerical error appears in the vicinity of the boundary between two different results.

Figure 1 has been obtained numerically with $\eta = 0$. Thus the simulation shown in the above mentioned figure contains only the numerical error. We see immediately that for this very small amount of noise—a quantitative analysis of this statement is given below—partial annihilation occurs at the boundaries between the different outcomes of collisions: (a) stationary bound states—interpenetration, (b) stationary bound states—annihilation, and (c) interpenetration—annihilation.

In Fig. 2 we show x-t plots of the four results obtained as outcome of collisions along the diagonal of the phase diagram presented in Fig. 1 for which all outcomes are present. We note that there is, when averaged over many runs, an equal number of pulses traveling to the left and to the right for the case of partial annihilation.

To investigate the influence of noise on the parameter range over which one can observe partial annihilation, we have superposed on the deterministic coupled complex cubic-quintic GL equations white noise of variable strength η. The numerical integration of the pulse collisions was performed using the Heun method [21] applied to a spatially discretized version of the stochastic partial differential equations (1) and (2). The applied noise strength of additive noise has been varied over 6 orders of magnitude and we found three different classes of behavior. In Fig. 3 we have plotted one representative of each class.

For an applied noise strength of $\eta = 10^{-7}$ we obtain the result plotted in Fig. 3(a). A comparison with the data

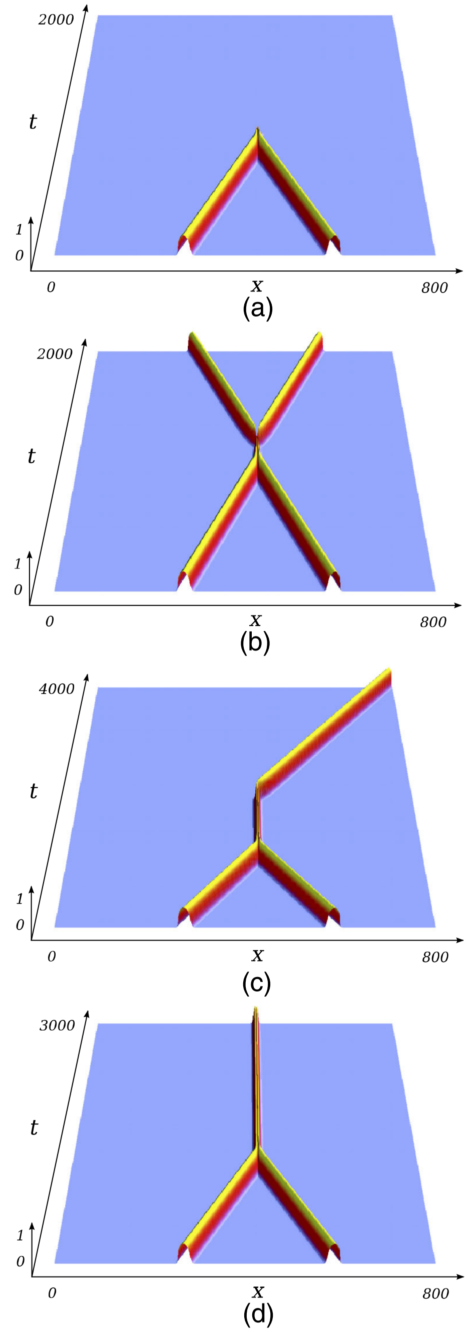

FIG. 2 (color online). The figures show the results of collisions as x-t plots, as the parameter c_r, characterizing the strength of the cross-coupling between counterpropagating waves, is varied. The parameter values for $c_r = v$ were: $c_r = -0.21$ [Fig. 2(a), annihilation]; $c_r = -0.18$ [Fig. 2(b), interpenetration]; $c_r = -0.165$ [Fig. 2(c), partial annihilation]; and $c_r = -0.15$ [Fig. 2(d), bound state of two pulses]. The parameter values used are given in the main text.

obtained with those given in the diagonal of Fig. 1 shows that there is no change for the range of the occurrence of partial annihilation. We also note that the boundaries between the different outcomes are sharp: there is always only one possible outcome of the interaction of

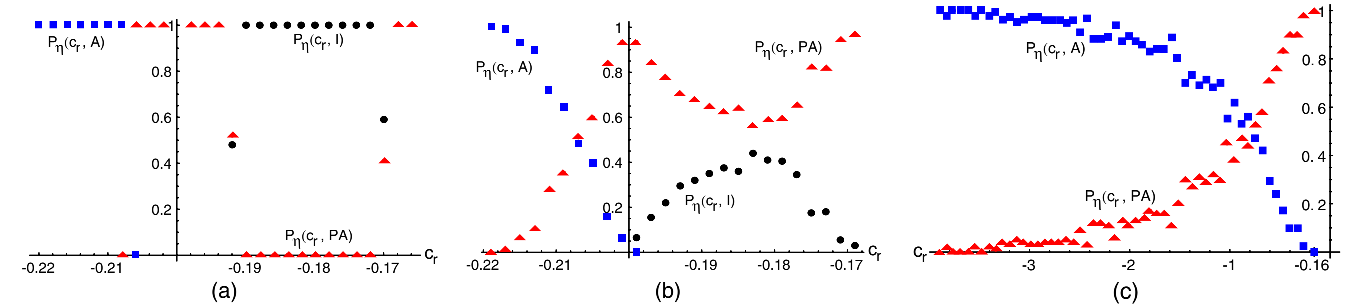

FIG. 3 (color online). The probabilities $P_\eta(c_r, PA)$ (solid triangles), $P_\eta(c_r, A)$ (solid squares) and $P_\eta(c_r, I)$ (solid circles) for partial annihilation (PA), annihilation (A) and interpenetration (I) have been plotted as functions of c_r, the strength of the stabilizing interaction of counterpropagating waves, for three different values of the noise strength η: (a) $\eta = 10^{-7}$, (b) $\eta = 10^{-5}$, and (c) $\eta = 10^{-2}$ for the case $c_r = v$. We note the large difference in the interval shown for c_r demonstrating that increasing the noise strength enhances drastically the parameter interval over which partial annihilation can be induced by noise.

pulses for fixed values of c_r and v. As the noise strength is increased, the behavior of the system changes qualitatively. Figure 3(b) shows the result for a noise strength that is 2 orders of magnitude higher: that is, $\eta = 10^{-5}$. Two central features emerge: (a) the range over which partial annihilation is observed as an outcome of the collision between two pulses increases by a factor of about two; and (b) for the increased noise strength the outcome of collisions is no longer unique. For fixed values of the parameters it can be either partial or complete annihilation on the one hand, or partial annihilation or complete interpenetration on the other, with varying proportions as the parameters are changed. We note that the boundary to bound states of pulses, occurring at higher values of c_r, remains fixed and therefore the basin of attraction for partial annihilation is not invading that of bound pulses.

As the applied noise strength is increased further, even more dramatic changes in the results of collisions arise. This is brought out in Fig. 3(c), for which we have increased the noise strength by 3 orders of magnitude compared to Fig. 3(b): $\eta = 10^{-2}$. First of all we see that the basin of attraction for interpenetration has disappeared completely. Even more remarkably the range over which partial annihilation can occur increases by almost 2 orders of magnitude (by a factor of $\sim$70). This means that the basin of attraction of complete annihilation is shrinking considerably over a large parameter range making room for the expansion of the range over which partial annihilation can occur. We also note that the proportions for which partial or complete annihilation occur vary perfectly smoothly. Finally, we mention that even for this fairly large noise strength of $\eta = 10^{-2}$, the boundary in c_r to bound states remains essentially unchanged.

The intuitive picture that emerges is as follows. Deterministically a stabilizing cross-coupling leads to a reduction of the amplitude and the width of the pulses during the interaction. If the reduction is smaller by a certain amount than two critical values of amplitude and width, the pulses grow back up to the shape and size before the collision: interpenetration. If the reduction is larger than to a certain limiting area, complete annihilation results [4]. In the presence of noise one of the two fields might be reduced in amplitude and width to values below a critical value while the other field stays above and grows back up in amplitude and width to the value before the collision: partial annihilation results as a consequence of the noise-induced breaking of left-right symmetry.

In addition to the coupled stochastic complex cubic-quintic GL equations we have investigated a stochastic version of the FitzHugh-Nagumo equation, which is a prototype equation to describe excitable media [14,15] and thus of a qualitatively different nature compared to

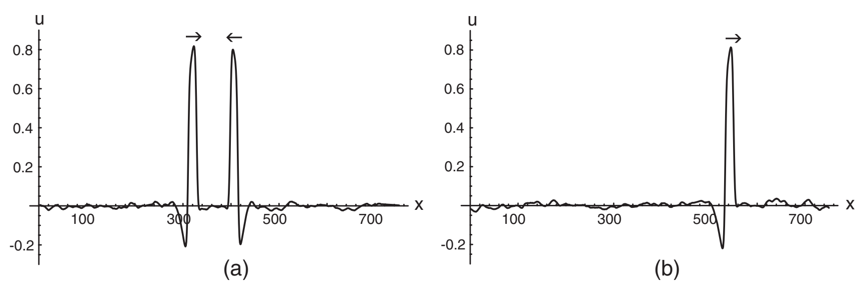

FIG. 4. The process of partial annihilation for the stochastic FitzHugh-Nagumo equation: (a) before the interaction two excitable waves approach each other, (b) partial annihilation after a transient: the parity symmetry is broken and only one excitable wave has survived. Noise strength $\eta = 0.0095$, $a = -0.044$; other parameter values are given in Ref. [15].

cubic-quintic complex GL equations. We find that for the FitzHugh-Nagumo equation noise-induced partial annihilation arises close to the border [15] separating interpenetration and complete annihilation of counterpropagating excitable waves: Fig. 4.

In summary we have presented a noise-induced mechanism for the occurrence of the partial annihilation of pulses in the framework of a CGL model, a pattern-forming system with dissipation and dispersion. We have demonstrated that noise plays a crucial role for the range of existence of partial annihilation (breaking the symmetry $A \rightarrow B$ and $x \rightarrow -x$) and thus can be used to control the outcome of collisions. We conjecture that noise can induce the partial annihilation of pulses. This conjecture has been validated for a prototype equation of a very different type: the FitzHugh-Nagumo equation. It will be most interesting to see experimental tests of the predictions of the model studied for nonequilibrium systems ranging from strongly dissipative fluid convection [9,10] to pattern-forming chemical reactions in their excitable or oscillatory regimes [2,15,22]. For example, one can superpose noise in a controlled way on the gas flux of the components in chemical reactions on surfaces such as Ir(111) [23,24].

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