Huge Conductance Peak Caused by Symmetry in Double Quantum Dots

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We predict a huge interference effect contributing to the conductance through large ultraclean quantum dots of chaotic shape. When a double-dot structure is made such that the dots are the mirror image of each other, constructive interference can make a tunnel barrier located on the symmetry axis effectively transparent. We show (via theoretical analysis and numerical simulation) that this effect can be orders of magnitude larger than the well-known universal conductance fluctuations and weak localization (both less than a conductance quantum). A small magnetic field destroys the effect, massively reducing the double-dot conductance; thus a magnetic field detector is obtained, with a similar sensitivity to a SQUID, but requiring no superconductors.

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In the 1990s, interference effects (universal conductance fluctuations and weak localization) were observed for electrons flowing through clean quantum dots [1,2]. The chaotic shape of such dots makes these effects analogous to speckle patterns in optics rather than to the regular interference patterns observed with Young's slits or Fabry-Perot etalons. While such interference phenomena are beautiful, they have only a small effect on the properties of quantum dots coupled to multimode leads. Here we provide a theoretical analysis and numerical simulations showing that a much larger interference effect occurs in systems which are mirror symmetric but otherwise chaotic [3–6], see Fig. 1. We show that the mirror symmetry induces interference that greatly enhances tunneling through a barrier located on the symmetry axis; it can make the barrier become effectively transparent. Thus an open double-dot system with an almost opaque tunnel barrier between the two dots will exhibit a huge peak in conductance when the two dots are the mirror image of each other, see Fig. 2. This effect could be used to detect anything that breaks the mirror symmetry. For example, current 2D electron gas (2DEG) technology [7] could be used to construct a device whose resistance changes by a factor of 10, when an applied magnetic flux goes from zero to a fraction of a flux quantum. This sensitivity is similar to that of a SQUID, but is achieved without superconductivity, facilitating integration with other 2DEG circuitry.

Origin of the conductance peak.—The origin of the effect can be intuitively understood by looking at Fig. 1. Assume that electrons follow only the two paths shown (instead of an infinite number of different paths). Path 1 does not tunnel the first time it hits the barrier, but does tunnel the second time it hits it. Path 2 tunnels the first time it hits the barrier, but not the second time. Quantum mechanics gives the probability to go from the left lead to the right lead as $|r(\theta)t(\theta')e^{iS_1/\hbar} + t(\theta)r(\theta')e^{iS_2/\hbar}|^2$, where the scattering matrix of the tunnel barrier has amplitudes $r(\theta)$ and $t(\theta)$ for reflection and transmission at angle θ. If there is no correlation between the classical actions of the two paths (S_1 and S_2), then the cross term cancels upon averaging over energy, leaving the probability as $|r(\theta)t(\theta')|^2 + |t(\theta)r(\theta')|^2$. In contrast, if there is a perfect mirror symmetry, then $S_2 = S_1$, and the probability is $|r(\theta)t(\theta') + t(\theta)r(\theta')|^2$, which is significantly greater than $|r(\theta)t(\theta')|^2 + |t(\theta)r(\theta')|^2$. Indeed, if we could drop the θ dependence of r and t, the probability would be doubled by the constructive interference induced by the mirror symmetry. A path that hits the barrier $(n + 1)$ times has 2^n partners with the same classical action (each path segment that begins and ends on the barrier can be reflected with respect to the barrier axis). However, the conductance is *not* enhanced by 2^n, because (due to the scattering matrix of the barrier) there is also destructive interference when one path tunnels $(4j - 2)$ times more than the other (for integer j).

The effect looks superficially like resonant tunneling. However, that only occurs when dots are weakly coupled to the leads, so that each dot has a peak for each level of the

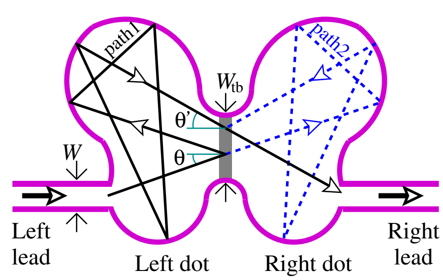

FIG. 1 (color online). A mirror-symmetric double dot, where the classical dynamics is highly chaotic. We call it a "butterfly double dot" to emphasize the left-right symmetry. Every classical path from the left lead to the right lead (solid line) which hits the barrier more than once is part of a family of paths which are related to it by the mirror symmetry (dashed line).

closed dot and the current flow is enhanced when two peaks are aligned. Instead, in our case each dot is well coupled to a lead (with $N \gg 1$ modes), so the density of states in each dot is featureless (the broadening of each level is about N times the level spacing). Furthermore, resonant tunneling occurs at discrete energies, while our effect is largely energy independent. Another similar effect, called "reflectionless tunneling," occurs when electrons are *retroreflected* as holes, due to Andreev reflection from a superconductor [8,9]. However, this retroreflection transforms the classical dynamics in the dot from chaotic to integrable [10], and large interference effects in integrable systems are not uncommon (consider a Fabry-Perot etalon). Here, the mirror symmetry induces a large interference effect without any retroreflection and without a change in the nature of the classical dynamics (chaotic motion remains chaotic).

Semiclassical theory.—Our analysis uses the semiclassical theory of transport through clean chaotic quantum dots [11]. The conductance through a system whose dimensions are much greater than a Fermi wavelength can be written as a double sum over classical paths γ and γ', which both start at a point y_0 on the cross section of the left lead and end at y on the right lead:

$$G = (2\pi\hbar)^{-1} G_0 \sum_{\gamma,\gamma'} A_\gamma A^*_{\gamma'} \exp[i(S_\gamma - S_{\gamma'})/\hbar], \quad (1)$$

where $G_0 = 2e^2/h$ is the quantum of conductance, and S_γ is the classical action of path γ. A tunnel barrier with left-right symmetry has a scattering matrix

$$\mathcal{S}_{\rm tb}(\theta) = e^{i\phi_{r(\theta)}} \begin{pmatrix} |r(\theta)| & \pm i|t(\theta)| \\ \pm i|t(\theta)| & |r(\theta)| \end{pmatrix}, \quad (2)$$

where $r(\theta)$ and $t(\theta)$ are the reflection and transmission amplitudes for a plane wave at angle of incidence θ. Keeping only the upper sign in $\mathcal{S}_{\rm tb}(\theta)$ [12],

$$A_\gamma = \left(\frac{dp_{y_0}}{dy}\right)^{1/2}_\gamma \prod_{j=1}^{m_T(\gamma)} i|t(\theta_{Tj})| \prod_{k=1}^{m_R(\gamma)} |r(\theta_{Rk})| \quad (3)$$

where path γ starts with a momentum across the left lead p_{y_0} and a total momentum given by the Fermi momentum p_F [13]. This path reflects off the barrier $m_R(\gamma)$ times (with the kth reflection at angle θ_{Rk}) and transmits $m_T(\gamma)$ times (at angles θ_{Tk}) before hitting the right lead at y. The factor $(dp_{y_0}/dy)_\gamma$ is the stability of the path that would exist if the barrier were absent for each transmission and impenetrable for each reflection. For most pairs with $\gamma \neq \gamma'$, the exponent in Eq. (1) varies fast with energy, so that averaging over energy removes such pairs from the double sum. We keep only the main contributions surviving such averaging: those where γ' can be constructed from γ by means of the reflection with respect to the barrier axis (symmetry axis) of any path segment that begins and ends on the barrier, for which $S_{\gamma'} = S_\gamma$ at all energies [the paths thereby have the same stability $(dp_{y_0}/dy)_\gamma$]. Dropping weak-localization effects [3,14], the average conductance reads

$$\langle G\rangle = \frac{G_0}{2\pi\hbar}\int_L dy_0 \int_R dy \sum_\gamma \left|\frac{dp_{y_0}}{dy}\right|_\gamma \left[\prod_{m=1}^{n(\gamma)} \mathbb{C}_{\gamma,m}\mathbb{S}\right]_{41}, \quad (4)$$

where the product is ordered, and path γ hits the barrier $n(\gamma)$ times. The four-by-four matrix $\mathbb{S} = \mathcal{S}_{\rm tb} \otimes \mathcal{S}^\dagger_{\rm tb}$ gives the scattering of the two paths at the barrier. Thus $\mathbb{S}_{ij}$ gives the weight to go from state j to state i, where we define state 1 as both paths in the left dot, state 2 as path γ in the left dot and path γ' in the right dot, state 3 as path γ in the right dot and path γ' in the left dot, and state 4 as both paths in the right dot. The matrix $\mathbb{C}_{\gamma,m}$ is diagonal with elements: $[\mathbb{C}_{\gamma,m}]_{11} = [\mathbb{C}_{\gamma,m}]_{44} = 1$ and $[\mathbb{C}_{\gamma,m}]_{22} = [\mathbb{C}_{\gamma,m}]^*_{33} = \exp[i\delta S_{\gamma,m}/\hbar]$. The action difference $\delta S_{\gamma,m}$ is that between path γ in the left dot and its mirror image in the right dot between the $(m-1)$th and mth collision with the barrier. For perfect symmetry $\mathbb{C}_m = \mathbb{I}$ and the product equals $[\mathbb{S}^n]_{41}$.

We assume that the classical dynamics is sufficiently mixing that paths uniformly explore the dot between subsequent collisions with the barrier (or leads). Defining $\delta S_0/\hbar$ as the phase difference acquired in one time of flight across the dot, we have $\mathbb{C}_{\gamma,m} \simeq \exp[-\Gamma t_{\gamma,m}]$ where Γ is a complex number, with $\mathrm{Im}[\Gamma] \simeq \langle\delta S_0\rangle/(\tau_0\hbar)$ and $\mathrm{Re}[\Gamma] \simeq \mathrm{var}[\delta S_0]/(\tau_0\hbar^2)$. The probability that a path survives in the dot for a time t without hitting either the barrier or the lead is e^{-t/τ'_D}. Using this, we replace $\mathbb{C}_{\gamma,m}$ by its time average $\mathbb{C} = \langle\mathbb{C}_{\gamma,m}\rangle$; its only nonzero elements are $\mathbb{C}_{11} = \mathbb{C}_{44} = 1$ and $\mathbb{C}_{22} = \mathbb{C}^*_{33} = [1 + \Gamma\tau'_D]^{-1}$. Thus the product in Eq. (4) reduces to $(\mathbb{C}\mathbb{S})^n$. The sum is over all γs that hit the barrier n times, and is independent of y_0, y. To proceed, we define $\tilde{\mathbb{S}} \equiv \mathbb{C}^{1/2}\mathbb{S}\mathbb{C}^{1/2}$; it is simple to show that $[(\mathbb{C}\mathbb{S})^n]_{41} = [\tilde{\mathbb{S}}^n]_{41}$ for all n. Then, defining $P = W_{\rm tb}/(W_{\rm tb} + W)$ as the probability for a path to hit the $W_{\rm tb}$-wide barrier before escaping into the W-wide lead, we find that $\langle G\rangle = G_0 N(1-P)\sum_{n=1}^\infty P^n[\tilde{\mathbb{S}}^n]_{41}$, where $N = p_F W/(\pi\hbar)$ is the number of modes in a lead. Upon finding the matrix $\mathbb{U}$, which diagonalizes $\tilde{\mathbb{S}}$, one can evaluate the geometric series in n.

This analysis gives the following average conductance of the symmetric double dot ($\Gamma = 0$),

$$\langle G_{\rm sym}\rangle = G_0 NP(1+P)T_{\rm tb}/[(1-P)^2 + 4PT_{\rm tb}], \quad (5)$$

where $T_{\rm tb}$ is the tunneling probability $|t(\theta)|^2$ averaged over all θ. For $T_{\rm tb} < (1-P)/2$ (i.e., for $G_{\rm tb}$, the conductance of a barrier with transmission $T_{\rm tb}$ in a waveguide of width $W_{\rm tb}$, less than P times the conductance of the series of the two constrictions), one finds that $\langle G_{\rm sym}\rangle$ is greater (often much greater) than the tunnel barrier conductance $G_{\rm tb}$. Thus symmetrically placing constrictions on either side of the barrier can strongly *enhance* its conductance, a stark example of the fact that quantum conductances in series are not additive. In contrast, for the asymmetric double dot (large Γ)

$$\langle G_{\rm asym}\rangle = G_0 NPT_{\rm tb}/[1 - P + 2PT_{\rm tb}], \tag{6}$$

which is always less than $G_{\rm tb}$. The ratio $\langle G_{\rm sym}\rangle/\langle G_{\rm asym}\rangle$ is plotted in Fig. 3. For any finite $T_{\rm tb}$, the ratio is maximal at $P = (1 - 2T_{\rm tb}^{1/2})/(1 - 4T_{\rm tb})$. This choice of P gives $\langle G_{\rm sym}\rangle = G_0N/4$ and (for small $T_{\rm tb}$) $\langle G_{\rm asym}\rangle \simeq T_{\rm tb}^{1/2} G_0 N/2$. Thus the conductance ratio can be arbitrarily large for a highly opaque barrier.

Peak shape with symmetry-breaking.—The effect of the mirror symmetry is suppressed by (a) a perpendicular magnetic field B, (b) moving the boundary of one dot by a distance δL, (c) disorder with a mean free time between subsequent disorder-scatterings $\tau_{\rm mf}$, or (d) decoherence on a time scale τ_φ. The suppression is given in terms of the following parameters:

$$\Gamma_B = \eta (eB\mathcal{A}/h)^2/\tau_0, \tag{7}$$

$$\Gamma_{\rm boundary} = \tau_0^{-1}({\rm var}[\delta L]/\lambda_F^2 + i\langle \delta L\rangle/\lambda_F), \tag{8}$$

$$\Gamma_{\rm mf} = \tau_{\rm mf}^{-1}, \qquad \Gamma_\varphi = \tau_\varphi^{-1}, \tag{9}$$

where e is the electronic charge, $\mathcal{A}$ is the area of one dot, and τ_0 is the time to cross the dot. In Γ_B, the constant η is of order one, but is hard to estimate [15]. For $\Gamma_{\rm boundary}$, we have $\langle \delta L\rangle \sim x\xi$ and ${\rm var}[\delta L] \sim x^2(\xi - \xi^2)$, if a fraction ξ of the left dot is deformed outwards by a distance x. For multiple asymmetries, the total Γ is the sum of the individual Γs given above. For real Γ,

$$\langle G(\Gamma)\rangle = \langle G_{\rm asym}\rangle + \frac{\langle G_{\rm sym}\rangle - \langle G_{\rm asym}\rangle}{1 + F(P, T_{\rm tb}) \times \Gamma\tau'_D}, \tag{10}$$

where $F(P, T_{\rm tb}) = \langle G_{\rm sym}\rangle/[\langle G_{\rm asym}\rangle(1 + P)]$, and $\tau'_D \sim \pi L\tau_0/(W + W_{\rm tb})$ is the typical time a path spends in one dot before hitting a lead or the barrier. For the large conductance ratio [see below Eq. (6)], $F(P, T_{\rm tb})\tau'_D$ is about half the dwell time in the double dot, $\tau_D \sim (1 - P)^{-1}\tau'_D$. Thus the conductance is a Lorentzian function of the B field, with similar width to the weak-localization dip in the same system with no barrier [14]. This makes the system an extremely sensitive detector of magnetic fields and deformations of the confining potential (for example, due to charges moving near one dot). Intriguingly, if lead positions break the symmetry the peak remains, it is only suppressed with asymmetry parameter $\Gamma_{\rm lead} = (1 - P)/\tau'_D$.

For complex Γ, as in Fig. 2(b), we have no analytic result for $\langle G(\Gamma)\rangle$, but we can get it by numerically diagonalizing the 4-by-4 matrix $\tilde{\mathbb{S}}$. In Fig. 2(b), the data and the theory curve drop below $\langle G_{\rm asym}\rangle = 0.23G_0$. We will show elsewhere that this is due to destructive interference. The

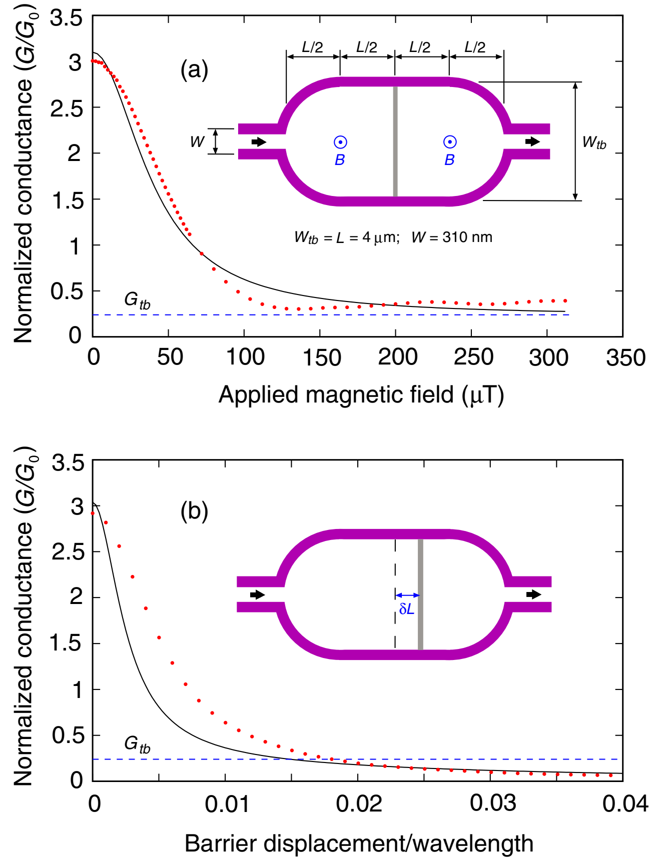

FIG. 2 (color online). Average conductance as (a) a function of applied B field (with the barrier on the symmetry axis), and as (b) a function of the barrier position (for zero B field). The latter mimics the effect of gates that reduce the size of one dot relative to the other. The data points come from simulations performed for the structures shown in the insets. The curve comes from the semiclassical theory; in (b) there is no fitting parameter, while in (a) an unknown parameter (of order one) is adjusted to fit the data. The conductance of the tunnel barrier alone is $G_{\rm tb}$.

conductance rises back up to $\langle G_{\rm asym}\rangle$ when the barrier is moved a wavelength or so.

Proposal for experimental observation.—Consider making such a double dot in an ultraclean two-dimensional electron gas (2DEG) at the lowest achievable temperatures [7]. A finger gate could define the barrier [16], with split gates controlling the lead widths. To maximize the effect for a 2DEG with a mean free path [7] of order 500 μm, each dot (see Fig. 2) can have size $L = 4\ \mu$m (circumference $\sim 3.6L \sim 15\ \mu$m) with 12 mode leads ($W = 310$ nm $\sim 6\lambda_F$). A barrier with $T_{\rm tb} = 1.48 \times 10^{-3}$ and width $W_{\rm tb} = L$ gives $P = 0.93$ and $\tau'_D \sim 3.5\tau_0$. In this case, $\langle G_{\rm sym}\rangle \simeq 14\langle G_{\rm asym}\rangle \simeq 3.2G_0$ (resistance $R_{\rm sym} \sim 5$ kΩ). The crossover from $\langle G_{\rm sym}\rangle$ to $\langle G_{\rm asym}\rangle$ happens for $\Gamma \simeq 0.14/\tau'_D \sim 0.04/\tau_0$. At low temperatures ($\tau_\varphi > \tau_{\rm mf}$), disorder will suppress the peak to about 83% of $\langle G_{\rm sym}\rangle$, since $F(P, T_{\rm tb})\Gamma_{\rm mf}\tau'_D \sim 0.2$. Thus the double-dot conductance will drop by an order of magnitude if 10% of the

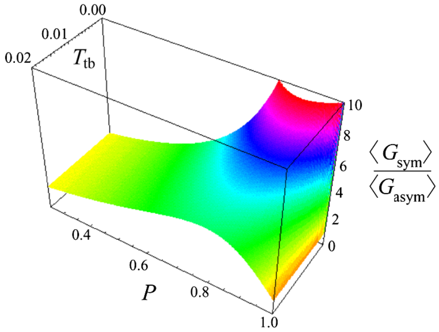

FIG. 3 (color online). Plot of the ratio $\langle G_{sym} \rangle / \langle G_{asym} \rangle$, given by Eqs. (5) and (6). The ratio grows as $T_{tb} \to 0$ for all P (although $\langle G_{sym,asym} \rangle$ shrink). For given T_{tb}, the ratio is maximal at $P = (1 - 2T_{tb}^{1/2})/(1 - 4T_{tb})$.

boundary of one dot is moved by $\lambda_F/2$, or if a B field is applied such that a fifth of a flux-quantum threads each dot. The latter is a B-field sensitivity similar to that of a SQUID.

The main experimental challenge will be to define dots that are mirror symmetric on a scale significantly less than $\lambda_F \sim 50$ nm. We suggest that each dot should be defined by means of multiple gates (made as symmetric as possible); their voltages can then be tuned to maximize the symmetry. We propose the following protocol for this maximization. Starting with very wide leads, in such a way that P is far from unity and the conductance peak is very broad, one scans the dot-defining gate voltages over a broad range to reveal the approximate symmetry point (maximal conductance). One then narrows the leads (increasing P) so the peak in $\langle G \rangle$ is higher and narrower, and adjusts the dot-defining gate voltages to maximize $\langle G \rangle$. Repeating this will give the symmetry point with increasing accuracy, up to the limit imposed by inherent asymmetries (disorder, etc.).

Numerical simulations.—For the above maximization we took $W_{tb} = L$ and only 12 lead modes. This calls into question two assumptions in the theory. First, we can no longer assume that paths in the dot will be well randomized between collisions with the barrier, since $\tau'_D \sim 3.6\tau_0$. Second, we may not be able to neglect other interference effects (weak-localization, etc.), since $\langle G \rangle$ is at most a few G_0. Thus, to verify that the effect is as expected in such a parameter regime, we numerically simulated a stadium billiard containing a barrier with $T_{tb} = 1.48 \times 10^{-3}$, see Fig. 2. We use the recursive Green's function technique [17] working in real space for the direction of current propagation (cut into multiple slices) and in mode space for the transverse direction. Magnetic fields are in a Landau gauge where the vector potential is oriented in the transverse direction [18]. The number of longitudinal slices and transverse modes was increased until the results converged. The data shown here are for 836 longitudinal slices (200 of which are in the outer leads) and 200 transverse modes. We mimic thermal smearing, at a temperature of 23 mK, by averaging over 44 energies uniformly distributed over an interval of 0.02 meV around the Fermi energy of 9.02 meV. We use the effective mass in GaAs of $0.067m_0$. The simulation (data points in Fig. 2) clearly shows that the effect exists in this regime. Indeed, despite the assumptions in its derivation, the theory (solid curve) agrees surprisingly well with the numerical data.

Concluding comment.—The conductance peak is *not* destroyed by bias voltages or temperatures greater than $\hbar/\tau_D$, because the mirror symmetry is present at all energies and not just at the chemical potential (unlike the electron-hole symmetry for reflectionless tunneling into a superconductor). Large biases or temperatures should still be avoided, as they increase the decoherence.

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