



Jumps in Current-Voltage Characteristics in Disordered Films

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We argue that giant jumps of current at finite voltages observed in disordered films of InO, TiN, and YSi manifest a bistability caused by the overheating of electrons. One of the stable states is overheated and thus low resistive, while the other, high-resistive state is heated much less by the same voltage. The bistability occurs provided that cooling of electrons is inefficient and the temperature dependence of the equilibrium resistance $R(T)$ is steep enough. We use experimental $R(T)$ and assume phonon mechanism of the cooling taking into account its strong suppression by disorder. Our description of the details of the I - V characteristics does not involve adjustable parameters and turns out to be in quantitative agreement with the experiments. We propose experiments for more direct checks of this physical picture.

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Recent experiments on the I - V characteristics of disordered films of InO [1,2] and TiN [3,4] (similar findings were reported earlier for amorphous YSi films [5]) are quite intriguing. The Ohmic resistance R of these films at temperatures $T < 1$ K demonstrates the insulating behavior fit by the Arrhenius law ($k_B = 1$)

$$R(T) = R_0 \exp[(\Delta/T)^\gamma], \quad \gamma = 1, \quad (1)$$

with R_0 and $\Delta \simeq 1$ –5 K being T independent. Though this behavior is quite interesting by itself [6], it is the I - V characteristics at low temperatures which turned out to be most unusual. When the voltage V is increased from $V = 0$, the current I first increases gradually, remaining rather small [highly resistive (HR) state]. At a certain voltage V_{HL} , I jumps up by several orders of magnitude and a low resistive (LR) state arises. When the voltage is decreased from $V > V_{HL}$, an inverse jump between the LR and HR states occurs at a voltage $V_{LH} < V_{HL}$. These HR-LR switches persist in a wide range of magnetic fields $B \simeq 0$ –10T, with the threshold voltages $V_{LH(HL)}$ increasing with B . Authors of Refs. [4,7] assumed that the HR state is a new collective state—“superinsulator.” Their calculation was vigorously disputed in Ref. [8].

In this Letter we show that these I - V characteristics can be explained without using new concepts. Our phenomenological approach predicts exactly this behavior when T dependence of the resistance is steep as in Eq. (1) and the electron-phonon (e -ph) thermalization is inefficient at low T . We assume that (similar assumptions for a system without the switches were made in Ref. [9])

1. The electron-electron (e - e) interaction is strong enough for electrons being mutually thermalized; i.e., one can introduce their temperature T_{el} although the system is driven out of equilibrium by a finite voltage;

2. The e -ph interaction is weak, so that electrons can be out of equilibrium with phonons (or any other thermal bath) of temperature T_{ph} , i.e., $T_{el} > T_{ph}$;

3. $R(T)$ dependence at a finite voltage is the same as in the Ohmic regime, but $T_{el}(V)$ is substituted for T .

These assumptions are sufficient to explain qualitatively the experimental I - V characteristics, most carefully investigated in the accompanying Letter [2].

Phenomenological analysis.—Temperature T_{el} acquired by the electron due to a fixed external voltage V is determined by the balance between the Joule heating and the cooling by the phonon bath (electron-electron collisions conserve the energy and do not affect the heat balance): Before specifying the model for the electron-phonon (e -ph) coupling we analyze the cooling using model-independent arguments. The heat balance equation for a sample of volume \mathcal{V} and the electron density of states ν (assumed to be smooth at the Fermi level) is

$$\frac{V^2}{R(T_{el})} = \frac{\mathcal{E}(T_{el})}{\tau_{e-ph}(T_{el})} - \frac{\mathcal{E}(T_{ph})}{\tau_{e-ph}(T_{ph})}, \quad (2a)$$

$$\frac{1}{\tau_{e-ph}(T)} = \left(\frac{T}{\hbar}\right) \left(\frac{T}{\Theta}\right)^{\beta-3}, \quad (2b)$$

where $\mathcal{E} = \pi^2 \nu \mathcal{V} T^2 / 6$. The right-hand side (rhs) of Eq. (2a) obeys the following requirements: (i) it vanishes in the thermal equilibrium; (ii) it provides the entropy growth; (iii) at $T_{el} \gg T_{ph}$, it ceases to depend on T_{ph} as under such condition the radiation of the hot phonons by hot electrons dominates. Factor T/\hbar provides the proper dimensionality for the relaxation rate. The second factor in rhs of Eq. (2b) describes a suppression of the e -ph coupling at low T . As the phonons are gapless, the suppression is a power law. The energy scale Θ encodes the strength of e -ph coupling. We will specify β and Θ later.

For a steep enough $R(T)$, there is a region on the V - T_{ph} plane (for $T_{\text{ph}} < T_{\text{ph}}^{\text{cr}}$ and $V^< < V < V^>$) where Eqs. (2) have two stable solutions: a “hot” LH state with relatively high T_{el} and a “cold” HR state with $T_{\text{el}} \approx T_{\text{ph}}$. Such a situation was discussed in Ref. [10] in connection with the many-body localization [11].

To analyze Eqs. (1) and (2) we use dimensionless variables

$$t_{\text{el,ph}} = \frac{T_{\text{el,ph}}}{\Delta}, \quad v \equiv \frac{V}{V_0}, \quad (3)$$

and rewrite Eq. (2a) in the form

$$F(t_{\text{el}}, t_{\text{ph}}) = v; \quad F(x, y) \equiv [e^{1/x^\gamma}(x^\beta - y^\beta)]^{1/2}. \quad (4)$$

The new characteristic voltage, V_0 emerging in Eq. (3), is natural to define through the electric field scale V_0/L , independent of the system size:

$$\frac{V_0}{L} = \left(\frac{\pi^2 \nu \Delta^3}{6\sigma_0 \hbar} \right)^{1/2} \left(\frac{\Delta}{\Theta} \right)^{(\beta-3)/2}; \quad \sigma_0 = \frac{L^2}{R_0 \mathcal{V}}. \quad (5)$$

The first factor in the rhs of Eq. (5) is uniquely determined as a combination of local characteristics of an isolated system, ν , σ_0 , and Δ , with a correct dimensionality. The problem of nonlinear dissipative transport is meaningless without coupling of the electrons with a thermal bath, i.e., when $\Theta \rightarrow \infty$. This is reflected by the second factor in Eq. (5).

At the bistability boundaries $t_{\text{el}}^{h,c}$, the derivatives with respect to t_{el} of both sides of Eq. (4) are equal, i.e.,

$$(\beta/\gamma)t_{\text{el}}^\gamma = 1 - (t_{\text{ph}}/t_{\text{el}})^\beta. \quad (6)$$

This equation has two positive solutions if $t_{\text{ph}} \leq t_{\text{ph}}^{\text{cr}}(\beta)$. Here, the critical phonon temperature $t_{\text{ph}}^{\text{cr}} = T_{\text{ph}}^{\text{cr}}/\Delta$ is

$$t_{\text{ph}}^{\text{cr}} = (1 + \beta/\gamma)^{-(\beta+\gamma)/\gamma\beta} < 1. \quad (7)$$

The two solutions $t_{\text{el}}^{c(h)}(t_{\text{ph}})$ of Eq. (6), see Fig. 1, define the region of voltages $v^< < v < v^>$ where two states are locally stable, $v^{<,>} = F(t_{\text{el}}^{h,c}, t_{\text{ph}})$. The hot branch t_{el}^h turns out to only slightly depend on t_{ph} :

$$\left(\frac{\gamma}{\beta + \gamma} \right)^{1/\gamma} = t_{\text{el}}^h(t_{\text{ph}}^{\text{cr}}) \leq t_{\text{el}}^h(t_{\text{ph}}) \leq t_{\text{el}}^h(0) = \left(\frac{\gamma}{\beta} \right)^{1/\gamma}. \quad (8a)$$

As the result, $v^<(t_{\text{ph}})$ is almost independent of T_{ph} ,

$$\sqrt{e} \left(\frac{\beta}{\beta + \gamma} \right)^{(\beta+\gamma)/2\gamma} \left(\frac{e\gamma}{\beta} \right)^{\beta/2\gamma} \leq v^<(t_{\text{ph}}) \leq \left(\frac{e\gamma}{\beta} \right)^{\beta/2\gamma}. \quad (8b)$$

Contrarily, for $\beta \gg 1$, the cold branch may be well approximated (except narrow vicinity of $t_{\text{ph}}^{\text{cr}}$) by

$$t_{\text{el}}^c - t_{\text{ph}} \approx t_{\text{ph}}^{\gamma+1}/\gamma < t_{\text{ph}}/\beta. \quad (8c)$$

For the upper bistability boundary, we find

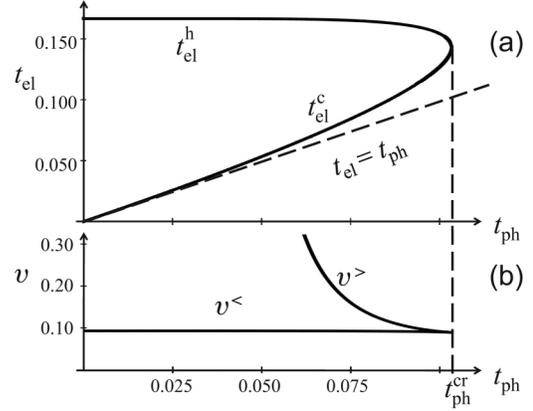


FIG. 1. Dependence of the dimensionless electronic temperature (a) and voltage (b) bistability boundaries on the bath temperature t_{ph} . Here t_{el}^c and $v^>$ are the boundaries for the HR (cold) state and t_{el}^h and $v^<$ for the LR (hot) state. Both plots are for $\beta = 6$, $\gamma = 1$.

$$v^>(t_{\text{ph}}) \approx [\beta/(e\gamma)]^{1/2} t_{\text{ph}}^{(\beta+\gamma)/2} \exp[1/(2t_{\text{ph}}^\gamma)]. \quad (8d)$$

At each voltage in the bistability interval $v^< < v < v^>$ one of the states is metastable. As it is usual for the first order phase transition the voltages, where the switches between HR and LR states happen (V_{HL} for HR \rightarrow LR and V_{LH} for LR \rightarrow HR switches), are determined by kinetics of the decay of metastable states. Theoretical analysis of this decay and evaluation of $V_{\text{HL,LH}}$ is beyond the scope of this Letter. Here we can predict only their bounds

$$V_0 v^< = V^< < V_{\text{LH}} < V_{\text{HL}} < V^> = V_0 v^>. \quad (9)$$

The difference between the local instability and the metastable state decay transition can be ascertained from the slope dI/dV near the transition, see Fig. 2.

Finally, it is important to emphasize that Eqs. (3)–(5) imply a nontrivial scaling of the bistability bounds,

$$V^{<,>} = \Delta^{\beta/2} f^{<,>}(T_{\text{ph}}/\Delta), \quad (10)$$

which should apply to $V_{\text{HL,LH}}$ provided that the switches are close to (a) type, see Fig. 2.

The I - V characteristics is most conveniently visualized by calculating the nonlinear conductance G

$$I = G(V, T_{\text{ph}})V; \quad G(T_{\text{ph}}, V) = \frac{\exp[-t_{\text{el}}(v)^{-\gamma}]}{R_0}, \quad (11)$$

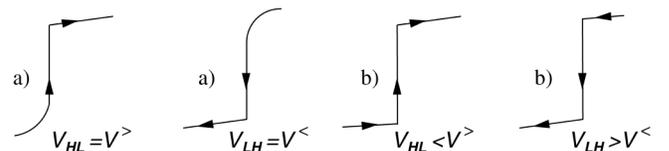


FIG. 2. The schematic I - V characteristics for the switches occurring through the local instability [type (a)] and through the decay of metastable state [type (b)].

see Eq. (1), where $t_{\text{el}}(v)$ is found from Eq. (4). The I - V curves obtained by substituting the numerical solution of Eq. (4) into Eq. (11) are plotted on Fig. 3. The HR and LR branches are connected by unstable branches shown by dotted lines. The HR \rightarrow LR (cold to hot state) and LR \rightarrow HR (hot to cold state) switches are depicted by vertical dashed lines.

If T_{ph} is not too close to the critical temperature (7), we can use the fact $t_{\text{el}} - t_{\text{ph}} \ll t_{\text{ph}}$, see Eq. (8c), to obtain an analytic description of $G(V, T_{\text{ph}})$ for the HR state:

$$\left(\frac{v}{v^>}\right)^2 = \frac{e \ln[\text{GR}(T_{\text{ph}})]}{\text{GR}(T_{\text{ph}})}; \quad 1 \leq \text{GR}(T_{\text{ph}}) \leq e. \quad (12)$$

Note, that the independence is universal; i.e., it holds for arbitrary values of exponents β , γ .

Microscopic input.—To quantify the developed phenomenology we rely on conventional theory of normal disordered metals neither seeking a microscopic explanation for, e.g., the Arrhenius law (1) nor involving physics of the insulator-superconducting transition. The current jumps occur in the insulating regime $T < \Delta$. However, the thermal balance equation is valid also for $T \gtrsim \Delta$ and the e -ph coupling should be a continuous function. Thus, our strategy is to use the theory of the e -ph interaction in dirty metals [12,13] for $T > \Delta$ and extrapolate to the lower temperatures. The cooling rate is determined by the material mass density ρ and the transverse sound velocity c_s . Disorder is known to suppress the e -ph coupling if the wavelength of a thermal phonon exceeds the electron elastic mean free path $\hbar c_s/T \gg \ell$ (as it does for films of Refs. [1–5] where $c_s \sim 3 \times 10^5$ cm/s, $T \lesssim 1$ K and

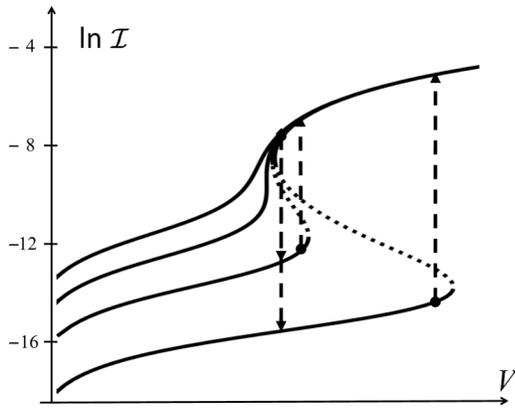


FIG. 3. Universal I - V characteristics for $\beta = 6$, $\gamma = 1$ (going from the top) $t_{\text{ph}} = 1.15 t_{\text{ph}}^{\text{cr}}$, and $t_{\text{ph}} = 0.90 t_{\text{ph}}^{\text{cr}}$, and $t_{\text{ph}} = 0.75 t_{\text{ph}}^{\text{cr}}$. Instable regions on the S -shaped curves are shown by dotted lines which join the upper and lower solid lines at the stability boundaries of the LR (hot) and HR (cold) state, respectively. The vertical dashed lines with arrows show possible positions for LR \rightleftharpoons HR current jumps. The units of I and V are arbitrary, but the values of the current switches in the log scale are universal and in a good agreement with the experimental values.

$\ell \lesssim 10$ nm). The result is [13]

$$\frac{\mathcal{E}(T)}{\tau_{e\text{-ph}}(T)} = \frac{\alpha^2 k_F \ell \mathcal{V} n_{\text{el}} T^6}{\hbar^4 \rho c_s^5}, \quad \alpha = \frac{2\pi^2}{\sqrt{315}} \approx 1.1, \quad (13)$$

where n_{el} is the conduction electrons density, and $k_F = (3\pi^2 n_{\text{el}})^{1/3}$ is the Fermi momentum [14,17]. Comparing Eqs. (13) and (2b) we obtain

$$\beta = 6, \quad \Theta = \left(\frac{\pi^2 \hbar^3 \nu \rho c_s^5}{6\alpha^2 n_{\text{el}} k_F \ell} \right)^{1/3}. \quad (14)$$

Substituting Eq. (14) into Eq. (5), and using, at $T > \Delta$, the Drude formula $\sigma_0 = (e^2/\hbar)(k_F \ell)(n_{\text{el}}/k_F^2)$, we find

$$\frac{eV_0}{L} = \alpha \frac{k_F \Delta^3}{(\rho c_s^5 \hbar^3)^{1/2}}. \quad (15)$$

Remarkably, the elastic mean free path ℓ —the only quantity, which becomes meaningless in the insulator, does not enter electric field scale V_0/L . It makes us believe that Eq. (15) derived for the metallic regime $T > \Delta$ may be extended to the insulator at $T < \Delta$.

Qualitative comparison with experiment.—Several conclusions of our phenomenological consideration can be compared directly with the experimental results:

1. The I - V curves in [1–5] look like those on Fig. 3 for both $T > T_{\text{ph}}^{\text{cr}}$ and $T < T_{\text{ph}}^{\text{cr}}$. At $T \rightarrow T_{\text{ph}}^{\text{cr}} + 0$ the inflection point becomes increasingly pronounced. At T below $T_{\text{ph}}^{\text{cr}}$, hysteretic HR \rightarrow LR and LR \rightarrow HR current jumps occur. These jumps indeed reach several orders of magnitude;

2. At a rather large interval of voltages the dependence of $\log I$ on V looks like linear;

3. V_{LH} only slightly changes with T_{ph} , while V_{HL} substantially increases when phonon temperature is reduced. This is what Eqs. (8b) and (8d) predict.

4. Observed T_{ph} dependence of V_{HL} is still weaker than the exponential dependence (8d). This is consistent, however, with bound (9) and the type (b) HR \leftrightarrow LR transition [see discussion before Eq. (9), and Fig. 2]. Another possible reason for a relatively slow dependence of V_{HL} on T_{ph} is a deviation from the Arrhenius law of Eq. (1).

5. The assumption that $V_{\text{HL}} < V^>$ and $V_{\text{LH}} \approx V^<$ implies the shapes of the I - V characteristics to be of type (a) close to LR \rightarrow HR switch and of type (b) in the vicinity of HR \rightarrow LR switch (see Fig. 2). This is exactly what was observed.

Further analysis of data of Ref. [2] uncovers an intriguing discrepancy: the observed ratio of the nonlinear conductances $G(V_{\text{HL}})/G(V \rightarrow 0)$, see Eq. (11), is noticeably bigger than the bounds of Eq. (12) and Fig. 3, valid for any single scale dependence $R(T)$. This discrepancy can be resolved within our approach only by involving an extra temperature scale $T < T_{\text{ph}}^{\text{cr}}$ in addition to Δ . While the origin of this scale is yet to be understood, the many-body localization [11] is a possibility.

Quantitative discussion.—The magnitude of the LR-HR jump is almost independent of other experimental parame-

ters. Since locations of the jumps between the upper and lower stability boundaries are ill dened, we can only estimate the order of magnitude. For $T_{\text{ph}} = 0.75T_{\text{ph}}^{\text{cr}}$ the predicted 4–5 orders in magnitude of the current jump (Fig. 3) agrees reasonably with experiment. Experimental cooling is well fitted with $\beta = 6$ in Eq. (2), in agreement with the above description of the electron-phonon mechanism. The critical bath temperature can be estimated from the temperature scale $\Delta \approx 1.2$ K: although the Arrhenius law does not give a very good fit, this scale can be rather reliably extracted from the data [2]. Then for $\gamma = 1$ and $\beta = 6$, we find $T_{\text{ph}}^{\text{cr}} \approx 0.1\Delta \approx 0.1$ K in agreement with the experimental values [2].

More quantitative comparison is hindered by the strong sensitivity of the critical temperature and switching voltages to the choice of γ , see Eqs. (7) and (8). For a consistent quantitative comparison with experimental observation, one should fit the experimentally observed equilibrium resistance $R(T)$ into the left-hand side of the heat balance equation Eq. (2a) and solve the resulting equation numerically. This is done in the accompanying experimental Letter [2].

Conclusion.—There are a number of strong evidences in favor of the electron overheating being the main cause of the giant HR \leftrightarrow LR current jumps observed in [1–5]. Direct detection of the electron overheating through, e.g., noise measurements would be an unambiguous proof of this. As the cooling is a rather slow process it looks plausible to perform the time-resolved studies of electron transport, e.g., measure the current caused by a train of voltage pulses as a function of the pulse duty cycle. Such a measurement could also shed some light on the kinetics of the switches. Also, the scaling relation (10) is very characteristic for the overheating mechanism (it can be verified by tuning Δ , e.g., by gentle annealing).

Finally, we emphasize the importance of our interpretation of the data [1–5] in the context of the general theory of the electron transport. Overheating of the electrons is quite usual in low resistive metals [18]. As to insulators (resistance far in excess of h/e^2) the overheating was rarely [9] considered quantitatively, because the conventional mechanisms of the low temperature charge transport are based on phonon-assisted hopping (see, e.g., [19]). We fully realize that our explanation of the current jumps contradicts the standard picture [as well as the Arrhenius law Eq. (1), $\gamma = 1$, though]. At the same time, we find the arguments in favor for the overheating being observed in Refs. [1–5] to be quite convincing. If our explanation is confirmed, these experiments should be considered as the first (to the best of our knowledge) reliable evidence of the strong overheating in the insulating state and, thus, of the existence of phonon-independent transport in insulators.

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