Particle-Accelerator Constraints on Isotropic Modifications of the Speed of Light

Michael A. Hohensee,¹ Ralf Lehnert,^{2,3} David F. Phillips,⁴ and Ronald L. Walsworth^{1,4}

¹Department of Physics, Harvard University, Cambridge, Massachusetts, 02138, USA

²Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany

³Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. Postal 70-543, 04510 México D.F., Mexico

⁴Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

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The absence of vacuum Cherenkov radiation for 104.5 GeV electrons and positrons at the LEP collider at CERN combined with the observed stability of 300 GeV photons at the Tevatron constrains deviations of the speed of light relative to the maximal attainable speed of electrons. Within the standard-model extension, the limit $-5.8 \times 10^{-12} \le \tilde{\kappa}_{tr} - \frac{4}{3}c_e^{00} \le 1.2 \times 10^{-11}$ is extracted, which sharpens previous bounds by more than 3 orders of magnitude. The potential for further refinements of this limit with terrestrial experiments and astrophysical observations is discussed.

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The speed of light, c, has played a crucial role in both the conception of special and general relativity and its experimental tests. The confidence we place in relativity theory is embodied in the fact that $c \equiv 299792458 \text{ ms}^{-1}$ is set to be a constant and provides the basis for the definition of length in the International System of Units.

Currently, relativity tests, including precise searches for modifications of the speed of light, are experiencing a revival of interest, motivated by theoretical studies that identify minute violations of Lorentz symmetry as imprints of Planck-scale physics [1]. A general theoretical description of weak Lorentz-symmetry breaking at low energies is provided by the standard-model extension (SME), which contains the usual standard model and general relativity as limiting cases [2,3]. The SME has served as the basis for relativity tests in many physical systems [4–6].

The majority of potential relativity violations in electrodynamics is governed by the dimensionless $(k_F)^{\alpha\beta\gamma\delta}$ coefficient in the SME, which causes the speed of light to be direction and polarization dependent. The birefringent components of k_F are bounded down to 10^{-37} with spectropolarimetric studies of astrophysical sources [7]. The remaining components $\tilde{\kappa}_{e-}$, $\tilde{\kappa}_{o+}$, and $\tilde{\kappa}_{tr}$ are, respectively, the symmetric, antisymmetric, and trace pieces of a 3 × 3 matrix; they lead to polarization-independent shifts of the speed of light. These parameters can be bounded with Michelson–Morley experiments [8], for which effects of $\tilde{\kappa}_{e-}$ are unsuppressed, $\tilde{\kappa}_{o+}$ effects are suppressed by β , and $\tilde{\kappa}_{tr}$ effects are suppressed by β^2 [9], where $\beta \approx 10^{-4}$ is the Earth's orbital speed. The corresponding limits are 10^{-17} , 10^{-13} , and 10^{-8} , respectively.

These results indicate that improvements of limits on $\tilde{\kappa}_{tr}$ assume particular urgency. Here, we use the analogy

$$n = 1 + \tilde{\kappa}_{\rm tr} + \mathcal{O}(\tilde{\kappa}_{\rm tr}^2) \tag{1}$$

between $\tilde{\kappa}_{tr}$ and a conventional frequency-independent refractive index *n* to reduce the sensitivity gap between $\tilde{\kappa}_{tr}$ and $\tilde{\kappa}_{o+}$. A positive $\tilde{\kappa}_{tr}$ would imply n > 1, so that the

maximal attainable speed (MAS) of other particles can exceed the speed of light. This allows charges in a Lorentz-violating ($\tilde{\kappa}_{tr} > 0$) vacuum to move faster than the modified speed of light c/n and to become unstable against the emission of Cherenkov photons. A negative $\tilde{\kappa}_{tr}$ would imply that the MAS of charges is now less than the speed of light [10]. In this respect, the roles of the photon and the charge are reversed relative to the $\tilde{\kappa}_{tr} > 0$ case, suggesting that then the photon is unstable. Indeed, a simple argument shows that above some energy threshold photon decay into a charge-anticharge pair is now kinematically allowed. We employ the absence of these two effects for electrons at CERN's LEP collider and photons at the Tevatron to derive improved limits at the 10^{-11} level on $\tilde{\kappa}_{tr} - \frac{4}{3}c_e^{00}$, a quantity that describes differences between the speed of light and the electron's MAS.

The quality of such an analysis rests on various requirements. First, the nature of the charge must be known: its MAS serves as the reference relative to which the speed of light is constrained [11]. Second, the total rates for vacuum Cherenkov radiation and photon decay must be known. Purely kinematical analyses of energy-momentum conservation in these processes are not themselves sufficient to draw conclusions based on their absence [12,13]. Third, other Lorentz-violating effects must be considered: vacuum Cherenkov radiation and photon decay could also be generated by, e.g., the electron's SME coefficients. Note that SME coefficients can typically not be set to zero because they may be generated by quantum effects. Finally, the bound should be extracted with minimal modeling.

The analysis reported here incorporates all four of the above requirements, providing a clean and conservative bound on $\tilde{\kappa}_{tr} - \frac{4}{3}c_e^{00}$. Our study takes advantage of the high-quality data collected at the world's highest-energy accelerators, as well as their superbly controlled laboratory environments. Thus, we both improve upon previous constraints, and highlight new avenues for exploring SME physics at existing and future colliders.

Although related tests using observations of ultrahighenergy cosmic rays (UHECRs) have also sought to constrain Lorentz-violating modifications of the fermionphoton vertex [14–21], their conclusions are not directly comparable to our result. Many UHECR studies do not estimate the rate of vacuum Cherenkov radiation or photon decay, an issue that is nontrivial even for propagation over cosmological distances [13]; or they focus on dispersionrelation parameters whose relation to $\tilde{\kappa}_{tr}$ is unknown or unclear [14-17]. More recent studies [20,21] involve atomic nuclei as well as electrons, and thus constrain $\tilde{\kappa}_{tr}$ – $\frac{4}{3}c_X^{00}$ for the generally undetermined nucleus X that serves as the UHECR primary scatterer. Quantitative interpretation of such studies in the broader context of the SME is complicated by both the composite nature of the nuclei as well as the contribution of numerous other SME coefficients that cannot be ignored at UHECR energy scales. Finally, some UHECR investigations of Lorentz-symmetry violation require various mild assumptions regarding astrophysical processes [18,19].

The physical system we will consider consists of photons and electrons, so we begin by recalling the singleflavor QED limit of the flat-spacetime SME [2]:

$$\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{4}(k_F)^{\kappa\lambda\mu\nu}F_{\kappa\lambda}F_{\mu\nu} + (k_{AF})^{\mu}A^{\nu}\tilde{F}_{\mu\nu} + \frac{1}{2}i\bar{\psi}\Gamma^{\nu}\overset{}{D}_{\nu}\psi - \bar{\psi}M\psi, \qquad (2)$$

where

$$\Gamma^{\nu} \equiv \gamma^{\nu} + c_e^{\mu\nu} \gamma_{\mu} + d_e^{\mu\nu} \gamma_5 \gamma_{\mu},$$

$$M \equiv m_e + b_e^{\mu} \gamma_5 \gamma_{\mu} + \frac{1}{2} H_e^{\mu\nu} \sigma_{\mu\nu}.$$
(3)

Here, $F^{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the electromagnetic fieldstrength tensor and $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ denotes its dual. The spinor ψ describes electrons of mass m_e , and the usual U(1)-covariant derivative is denoted by $D^{\mu} = \partial^{\mu} + ieA^{\mu}$. The spacetime-independent SME coefficients $(k_F)^{\mu\nu\rho\lambda}$, $(k_{AF})^{\mu}$, b_e^{μ} , $c_e^{\mu\nu}$, $d_e^{\mu\nu}$, and $H_e^{\mu\nu}$ control the extent of Lorentz and *CPT* violation.

In what follows, we are primarily interested in the $\tilde{\kappa}_{tr}$ component of the *CPT*-even $(k_F)^{\mu\nu\rho\lambda}$. The k_F coefficient exhibits the symmetries of the Riemann tensor, and its double trace $(k_F)^{\mu\nu}{}_{\mu\nu}$ vanishes. This leaves 19 independent components. In a given coordinate system, which is conventionally chosen to be the Sun-centered celestial equatorial frame, k_F can be decomposed as follows [22]. Ten components are associated with birefringence and can be grouped into the two dimensionless 3×3 matrices $\tilde{\kappa}_{o-}$ and $\tilde{\kappa}_{e+}$. The remaining nine components

$$\tilde{k}^{\mu\nu} \equiv (k_F)_{\alpha}{}^{\mu\alpha\nu} \tag{4}$$

give rise to $\tilde{\kappa}_{e-}$, $\tilde{\kappa}_{o+}$, and $\tilde{\kappa}_{tr}$, as explained earlier. In particular, $\tilde{\kappa}_{tr} = -\frac{2}{3}(k_F)^{0j0j}$, where the index *j* runs from 1 to 3 and is summed over in this expression.

All of the SME coefficients in Lagragian (2) modify either the photon's or the electron's dispersion relation and therefore also the kinematics of the electron–photon vertex. It follows that $\tilde{\kappa}_{tr}$ cannot be singled out in studies of vacuum Cherenkov radiation and photon decay; the other SME coefficients for the photon and the electron must in general also be taken into account. However, a careful analysis of previous experiments reveals stringent limits on these additional SME parameters [11]. The scale of these limits is governed by

$$S = \max\left(\tilde{\kappa}_{e^{-}}, \tilde{\kappa}_{e^{+}}, \tilde{\kappa}_{o^{+}}, \tilde{\kappa}_{o^{-}}, \frac{k_{AF}}{m_{e}}, \frac{b_{e}}{m_{e}}, c_{e}, d_{e}, \frac{H_{e}}{m_{e}}\right), \quad (5)$$

where the absolute values of the individual components listed here are implied. At present, $S \sim 10^{-13}$, dominated by $\tilde{\kappa}_{o+}$ [5]. This value is more than 5 orders of magnitude smaller than present limits on $\tilde{\kappa}_{tr}$, and about a factor of 10^2 smaller than the bound on $\tilde{\kappa}_{tr}$ we derive here. We therefore can safely ignore other SME coefficients in our analysis and retain $\tilde{\kappa}_{tr}$ only.

A related issue concerns the physical equivalence of the photon's $\tilde{k}^{\mu\nu}$ and electron's $c_e^{\mu\nu}$. These two coefficients are associated with the same phenomenology, and they can therefore not be distinguished within the framework of Lagragian (2). In the present context, our $\tilde{\kappa}_{tr}$ model is physically equivalent to a model with

$$c_e^{00} = -\frac{3}{4}\tilde{\kappa}_{\text{tr}}$$
 and $c_e^{jj} = -\frac{1}{4}\tilde{\kappa}_{\text{tr}}.$ (6)

(In this expression, there is no sum over j = 1, 2, 3.) This equivalence can be established rigorously with a coordinate rescaling [22,23], which implies that only $\tilde{\kappa}_{tr} - \frac{4}{3}c_e^{00}$ is observable in the context of Eq. (2). This rescaling can be used to remove either $\tilde{\kappa}_{tr}$ or c_e^{00} from the model. We often select coordinates such that $c_e^{00} = 0$, but undo this rescaling and reinstate c_e^{00} when quoting results.

At leading order, the photon dispersion relation in the presence of $\tilde{\kappa}_{tr}$ is given by [22]

$$E_{\gamma}^2 - (1 - \tilde{\kappa}_{\rm tr})\vec{p}^2 = 0,$$
 (7)

where $p^{\mu} \equiv (E_{\gamma}, \vec{p})$ is the photon's four-momentum. Thus, the speed of light is $(1 - \tilde{\kappa}_{tr})$. In the present context, the fermion dispersion relation is unaffected by Lorentz violation, and Cherenkov radiation can only occur for positive $\tilde{\kappa}_{tr}$. Using Eq. (7) and energy-momentum conservation for the emission of a Cherenkov photon yields the energy threshold [18]

$$E_{\rm VCR} = \frac{1 - \tilde{\kappa}_{\rm tr}}{\sqrt{(2 - \tilde{\kappa}_{\rm tr})\tilde{\kappa}_{\rm tr}}} m_e = \frac{m_e}{\sqrt{2\tilde{\kappa}_{\rm tr}}} + \mathcal{O}(\sqrt{\tilde{\kappa}_{\rm tr}}).$$
(8)

For electrons with energies above $E_{\rm VCR}$, vacuum Cherenkov radiation is kinematically allowed. Equation (8) can alternatively be derived from the usual Cherenkov condition that the electron must be faster than the speed of light $(1 - \tilde{\kappa}_{\rm tr})$.

We extract a limit on $\tilde{\kappa}_{tr}$ through the absence of observed vacuum Cherenkov radiation. This requires the emission to be efficient enough that charges with energies above E_{VCR}

are rapidly decelerated below threshold. Near E_{VCR} , the dominant deceleration process is single-photon emission with the estimated rate [18]

$$\Gamma_{\rm VCR} = \alpha m_e^2 \frac{(E_e - E_{\rm VCR})^2}{2E_e^3},\tag{9}$$

where α is the fine-structure constant, and E_e denotes the electron energy. This expression shows that the emission process is quite efficient, and we now use it to derive limits on $\tilde{\kappa}_{tr}$ from the energies attained by primary electrons at the LEP collider. The highest laboratory-frame particle energy reached at LEP was $E_{\text{LEP}} = 104.5 \text{ GeV}$ with a relative uncertainty in the center-of-mass energy $\Delta E_{c.m.}/E_{c.m.}$ below 2.0×10^{-4} [24]. Using Eq. (9), we find that if $E_{\text{VCR}} = 104 \text{ GeV}$, electrons initially accelerated to 104.5 GeV would be rapidly slowed by emission of Cherenkov photons to an energy below E_{VCR} over a 1/e length of about 95 cm. The total energy lost to the Cherenkov effect in such a scenario would far exceed the value allowed by measurements [11]. With Eq. (8), the requirement that E_{VCR} be greater than 104 GeV becomes

$$\tilde{\kappa}_{\rm tr} - \frac{4}{3}c_e^{00} \le 1.2 \times 10^{-11},$$
 (10)

where we include the dependence on c_e^{00} . This bound is significantly smaller than previous laboratory limits on $\tilde{\kappa}_{tr}$. Note also that the scale S defined in Eq. (5) is not yet reached, which justifies the exclusion of other photon or electron SME coefficients in our analysis.

For negative $\tilde{\kappa}_{tr}$, the dispersion relation (7) remains valid, and photons may travel faster than the MAS of electrons [10]. This precludes vacuum Cherenkov radiation at the cost of eliminating photon stability: for E_{γ} above the threshold

$$E_{\text{pair}} = \frac{2m_e}{\sqrt{\tilde{\kappa}_{\text{tr}}(\tilde{\kappa}_{\text{tr}}-2)}} = \sqrt{\frac{2}{-\tilde{\kappa}_{\text{tr}}}}m_e + \mathcal{O}(\sqrt{\tilde{\kappa}_{\text{tr}}}), \quad (11)$$

photon decay into an electron-positron pair is kinematically allowed [11,15]. The corresponding leading-order decay rate is given by [11,21]

$$\Gamma_{\text{pair}} = \frac{2}{3} \alpha E_{\gamma} \frac{m_e^2}{E_{\text{pair}}^2} \sqrt{1 - \frac{E_{\text{pair}}^2}{E_{\gamma}^2}} \left(2 + \frac{E_{\text{pair}}^2}{E_{\gamma}^2}\right).$$
(12)

Paralleling the Cherenkov case, this process is also highly efficient. For example, a 40 GeV photon with energy 1% above threshold would decay after traveling about 30 μ m.

The above reasoning establishes that the existence of long-lived photons with high energies constrains negative values of $\tilde{\kappa}_{tr}$. Photons generated in terrestrial laboratories provide a clean, well-characterized source for bounding $\tilde{\kappa}_{tr}$. Although the accessible energies are lower than those in cosmic rays, terrestrial tests offer larger data samples and a better control of the experimental conditions. Hadron colliders produce the highest-energy photons and therefore yield tight Earth-based experimental limits on $\tilde{\kappa}_{tr}$. Thus,

we consider Fermilab's Tevatron $p\bar{p}$ collider with centerof-mass energies up to 1.96 TeV. Isolated-photon production with an associated jet is important to QCD studies and has been investigated with the D0 detector. The recorded photon spectrum extended up to a single event at 442 GeV [25], which would imply $\tilde{\kappa}_{tr} \gtrsim -3 \times 10^{-13}$. While such a single event is insufficient to draw conservative conclusions regarding Lorentz symmetry, it is indicative of the sensitivity of this method.

We restrict our analysis to lower-energy D0 photon data with good statistics, where photons with energies up to 340.5 GeV were observed [26], and comparisons to QCD theory were made. The 340.5 GeV bin extended from 300 GeV to 400 GeV; the measured flux deviated by a factor of 0.52 ± 0.26 from QCD predictions [26], so that at most 74% of the produced photon flux can be lost due to hypothetical photon decay. We continue by conservatively assuming that all events in this bin were 300 GeV photons, and we take $E_{pair} = 300$ GeV. This is again justified by the rapid photon-decay rate (12): if E_{pair} were just 0.1 keV below the lowest observed 300 GeV photon energy, the photon deficit would be larger than the allowed 74% [11]. In other words, the uncertainty in E_{pair} is essentially determined by the resolution of the photon-energy measurement. This reasoning gives the limit

$$-5.8 \times 10^{-12} \le \tilde{\kappa}_{\rm tr} - \frac{4}{3} c_e^{00},\tag{13}$$

where we again include the contribution of c_e^{00} . Like the Cherenkov constraint, the photon-stability limit (13) is larger than the scale S, so other photon- or electron-sector coefficients are not further constrained by this argument. At the same time, this justifies the exclusion of these additional coefficients from our study.

Combining the bounds (10) and (13), we obtain

$$-5.8 \times 10^{-12} \le \tilde{\kappa}_{\rm tr} - \frac{4}{3} c_e^{00} \le 1.2 \times 10^{-11}$$
(14)

on isotropic deviations of the phase speed of light from the electron's MAS. This represents an improvement of more than 3 orders of magnitude upon previous laboratory-based constraints. We obtain this limit by exploiting the threshold effects of vacuum Cherenkov radiation and photon decay for positive and negative $\tilde{\kappa}_{tr}$, respectively.

An independent constraint on $\tilde{\kappa}_{tr}$ may be obtained by future low-energy laboratory tests with estimated sensitivities at the level of 10^{-11} or better [27]. Another possibility for improvements may come from photon triple splitting, because the amplitude for this effect is nonzero in the presence of $c^{\mu\nu}$ -type SME coefficients [28]. As opposed to vacuum Cherenkov radiation and photon decay, photon triple splitting is not a threshold effect, so that it may not necessitate high photon energies.

Significantly improved terrestrial bounds using the same reasoning as presented here may be obtained at the prospective International Linear Collider. Accelerating electrons to laboratory-frame energies of 500 GeV, the International Linear Collider may provide a one-sided Cherenkov bound with a sensitivity at the level of 5×10^{-13} . Similarly, the LHC is scheduled to attain roughly 7 times the energy of the Tevatron. Assuming that the energy E_{γ} of the produced photons scales by the same factor, the bound of Eq. (13) can be sharpened by a factor of 50. Additional improvements of the photon-decay limit may be achieved with a dedicated D0 or LHC study: for instance, the highest-energy data not analyzed for QCD tests could be used for the present purposes. Moreover, the high-energy tail of the photon-energy spectrum could be utilized more efficiently by avoiding large energy binning.

The $\tilde{\kappa}_{tr}$ limits from both vacuum Cherenkov radiation and photon decay scale quadratically with the energy of the primary particle. At present, UHECRs offer the highest possible energies in the Sun-centered celestial equatorial frame; the spectrum is limited only by the opacity of the universe to cosmic rays above certain energy thresholds (e.g., Greisen-Zatsepin-Kuzmin suppression [29] or pair creation on IR-photon background). For example, particles with energies up to about 2×10^{11} GeV have been observed at the Pierre Auger Observatory. Assuming these particles are iron nuclei, and that the neutron $c_n^{\mu\nu}$ coefficients are insignificant, bounds at the 10^{-20} level can be extracted [21]. Although this limit is not directly comparable to our results (it does not measure the photon speed relative to the MAS of the electron), it does illustrate the potential of cosmic-ray tests. Primary photons from the Crab nebula are another example: Energies up to 8×10^4 GeV have been reported by HEGRA [30]. Equation (11) would then give one-sided limits on $\tilde{\kappa}$ coefficients at the level of 10^{-16} . We mention that at UHECR scales some of the nonbirefringent $\tilde{\kappa}$ matrices and certain electron SME coefficients can no longer be neglected [see Eq. (5)]. In any case, the interpretation of future UHECR analyses of Lorentz violation would greatly benefit from a more reliable identification of the primary particle.

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