# Role of the Final-State Interaction and Three-Body Force on the Longitudinal Response Function of $^4$He

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We present an *ab initio* calculation of the longitudinal electron scattering response function off $^4$He with two- and three-nucleon forces and compare to experimental data. The full four-body continuum dynamics is considered via the Lorentz integral transform method. The importance of the final-state interaction is shown at various energies and momentum transfers $q$. The three-nucleon force reduces the quasielastic peak by 10% for $q$ between 300 and 500 MeV/$c$. Its effect increases significantly at lower $q$, up to about 40% at $q = 100$ MeV/$c$. At very low $q$, however, data are missing.

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Inelastic electron scattering off nuclei provides important information on nuclear dynamics. Varying the momentum $q$, transferred by the electron to the nucleus, one can focus on different dynamical regimes. At lower $q$, the collective behavior of nucleons is studied. As $q$ increases, one probes properties of the single nucleon in the nuclear medium and its correlations to other nucleons from long to short range. Thus the *inclusive* longitudinal $R_L$ and transverse $R_T$ response functions are of particular importance. Different from $R_T$, in a nonrelativistic framework $R_L$ does not require the knowledge of implicit degrees of freedom (exchange currents), providing a clean leptonic probe of the nuclear Hamiltonian. In addition, the theoretical study of *inclusive* processes is important to help planning further investigations, for selected kinematics, via *exclusive* scattering experiments.

In the 1980s and 1990s, an intense experimental activity was devoted to *inclusive* electron scattering $(e, e')$, in the so-called quasielastic (q.e.) regime, corresponding to $q$ values of several hundred MeV/$c$ and energy transfers $\omega$ around the q.e. peak ($\omega \simeq q^2/2m$). Here one can envisage that the electron has scattered elastically with a single nucleon of mass $m$. Various nuclear targets have been considered, from very light to heavy ones [1]. At these $q$, one enters a very challenging regime, where nuclear and subnuclear degrees of freedom intertwine. A very lively debate has taken place about the interpretation of those data. The two most discussed topics have been (i) short-range correlations, i.e., the dynamical properties of nucleons at short distances, and (ii) in medium modifications of the nucleon form factor. To date the debate is still open. More experiments are planned at Jefferson Laboratory (E05.110 at Hall A) which will contribute to those issues, and a theoretical effort is needed to help interpret old and new experimental results.

The reason for concentrating on the q.e. regime has been the conviction that for such a kinematics the plane wave impulse approximation (PWIA) might be a reliable framework to describe the reaction. The neglect of the final-state interaction (FSI) has the advantage to allow a simple interpretation of the cross section in terms of the dynamical properties of the nucleons in the ground state. Thus it is important to clarify the reliability of the PWIA (as well as of further refinements). The Euclidean approach [2] has already shown that the PWIA is rather poor; however, this method does not easily allow one to obtain the $\omega$ dependence of the FSI effects.

The aim of this Letter is twofold. On the one hand, we study the role of FSI on $R_L$ of $^4$He at 300 MeV/$c \leq q \leq$ 500 MeV/$c$, where by now only calculations with central two-nucleon forces exist [3,4]. Here we use a realistic two-body potential augmented by a three-nucleon force (3NF) and compare the PWIA to results obtained via the Lorentz integral transform (LIT) method [5,6]. The LIT method is an *ab initio* approach, which allows the full treatment of the four-body problem. It has already been applied to various realistic calculations of electroweak reactions in three- [7–11] and four-body systems [12,13]. Different from the Euclidean approach, the LIT method allows a comparison with the PWIA regarding the $\omega$ dependence of $R_L$. Our second focus lies on the study of the role of 3NFs. We contribute to this much-debated issue investigating 3NF effects on initial and final states by studying $R_L$ in various kinematical regions.

The choice of $^4$He as a target is of particular interest. In fact, $^4$He has quite a large average density. Moreover, its binding energy per particle is similar to that of heavier systems. Therefore, $^4$He results can serve better as guidelines for investigating heavier nuclei than results for two- and three-body systems. Various *inclusive* $^4$He $(e, e')$ experiments have been performed in the past (see [14] for a summary of the world data), and a comparison theory experiment is possible without the ambiguities, created by the Coulomb distortions, which affect heavier systems.

 

The longitudinal response function is given by

$$R_L(\omega, q) = \sum\!\!\!\!\!\!\int_f |\langle\Psi_f|\hat{\rho}(q)|\Psi_0\rangle|^2 \delta\Big(E_f + \frac{q^2}{2M} - E_0 - \omega\Big),$$

where $M$ is the target mass and $|\Psi_{0/f}\rangle$ and $E_{0/f}$ denote initial- and final-state wave functions and energies, respectively. The charge density operator $\hat{\rho}$ is defined as

$$\hat{\rho}(q) = \frac{e}{2}\sum_i (1 + \tau_i^3) \exp[i\mathbf{q}\cdot\mathbf{r}_i], \quad (1)$$

where $e$ is the proton charge and $\tau_i^3$ the isospin third component of nucleon $i$. The $\delta$ function ensures energy conservation. $R_L$ contains a sum over all possible final states, which are excited by the electromagnetic probe, including also continuum states. Thus, in a straightforward evaluation one would need to calculate both bound and continuum states. The latter constitute the major obstacle for many-body systems if one wants to treat the nuclear interaction rigorously. In the LIT method [5,6], this difficulty is circumvented by considering instead of $R_L(\omega, q)$ an integral transform $\mathcal{L}_L(\sigma, q)$ with a Lorentzian kernel defined for a complex parameter $\sigma = \sigma_R + i\sigma_I$ by

$$\mathcal{L}_L(\sigma, q) = \int d\omega \frac{R_L(\omega, q)}{(\omega - \sigma_R)^2 + \sigma_I^2} = \langle\tilde{\Psi}^\rho_{\sigma,q}|\tilde{\Psi}^\rho_{\sigma,q}\rangle. \quad (2)$$

The parameter $\sigma_I$ determines the resolution of $\mathcal{L}_L$ and is kept at a constant finite value ($\sigma_I \neq 0$). The basic idea of considering $\mathcal{L}_L$ lies in the fact that it can be evaluated from the norm of a function $\tilde{\Psi}^\rho_{\sigma,q}$, which is the unique solution of the inhomogeneous equation

$$(\hat{H} - E_0 - \sigma)|\tilde{\Psi}^\rho_{\sigma,q}\rangle = \hat{\rho}(q)|\Psi_0\rangle. \quad (3)$$

Here $H$ denotes the nuclear Hamiltonian. Because of the presence of the imaginary part $\sigma_I$ in (3) and the fact that its right-hand side is localized, one has a bound-state-like asymptotic boundary condition. Thus, one can apply bound-state techniques for its solution. Finally, $R_L(\omega, q = \mathrm{const})$ is obtained by inverting the LIT (2). Subsequently, the isoscalar and isovector parts of $R_L$ are multiplied by the proper nucleon form factors. For the LIT inversion, various methods have been devised [15,16].

The PWIA result is obtained under the hypothesis of one outgoing free proton with mass $m$ and a spectator $(A - 1)$ system with mass $M_s$:

$$R_L^{\mathrm{PWIA}}(\omega, q) = \int d\boldsymbol{p}\, n(\boldsymbol{p})\delta\Big(\omega - \frac{(\boldsymbol{p} + \boldsymbol{q})^2}{2m} - \frac{\boldsymbol{p}^2}{2M_s} - \epsilon\Big).$$

Here $n(\boldsymbol{p})$ represents the proton momentum distribution and $\epsilon$ the proton separation energy. In the following, we present results obtained with the Argonne V18 (AV18) [17] and the Urbana IX (UIX) [18] two- and three-body forces. As nucleon form factors, we use the proton dipole fit and the neutron electric form factor from Ref. [19]. The solution of (3), as well as the ground state $|\Psi_0\rangle$, is expanded in hyperspherical harmonics (HH). The HH expansion is truncated beyond a maximum value $K_{\max}$ of the HH grand-angular momentum quantum number. The convergence of the HH expansion is improved by introducing a $K_{\max}$-dependent effective interaction (EIHH method) [20,21]. In order to evaluate $\mathcal{L}_L$, we have calculated the norm $\langle\tilde{\Psi}^\rho_{\sigma,q}|\tilde{\Psi}^\rho_{\sigma,q}\rangle$ directly, using the Lanczos algorithm [22]. The operator $\hat{\rho}$ is expanded in Coulomb multipoles of order $J$. The LIT is calculated for each isoscalar ($T = 0$) and isovector ($T = 1$) multipole separately up to a maximal value of $J_{\max}$ where convergence of the expansion is reached. The values of $J_{\max}$ vary from 2 to 7 for $q$ ranging from 50 to 500 MeV/$c$.

The accuracy of the results is determined mainly by the convergence of the HH expansion and the stability of the inversion. In the calculations we used a ground state hyperspherical momentum value $K^0_{\max} = 16$ (14) for the AV18 + UIX (AV18) case, leading to a binding energy of 28.4 (24.3) MeV. Since a multipole-dependent convergence pattern has been encountered and each multipole contributes differently to the total strength, the $K_{\max}$ used for the LIT evaluation vary according to the value of $J$, namely, $K^{JT}_{\max} = 12$–16 for even $J$ and $K^{JT}_{\max} = 13$–17 for odd $J$ have been considered. Our LIT results converge at a percentage level. In Fig. 1, the accuracy of the results for $R_L$ regarding both the HH expansion and the inversion stability aspects is illustrated exemplarily for the isoscalar and isovector parts at $q = 500$ MeV/$c$. The figures contain three curves: The full line is obtained when the single multipole contributions $\mathcal{L}^{JT}_L$, calculated up to $K^{JT}_{\max}$, are first inverted and then summed up. The dashed line represents the results where the various multipole contributions $\mathcal{L}^{JT}_L$ are calculated only up to $K^{JT}_{\max} - 2$. The comparison between these two results illustrates the quality of the HH convergence. The dotted line reflects the inversion of the total $\mathcal{L}_L(\sigma, q)$, where the various multipole contributions

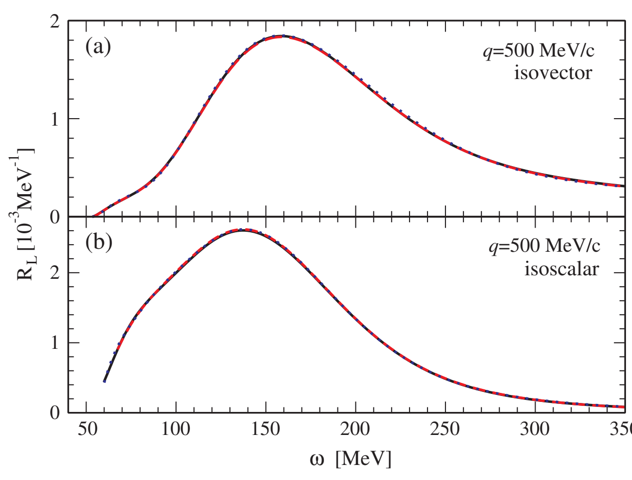


FIG. 1 (color online). Isovector (a) and isoscalar (b) parts of $R_L(\omega, q)$ at $q = 500$ MeV/$c$. Single multipole contributions with $K^{JT}_{\max}$ (solid line) and with $K^{JT}_{\max} - 2$ (dashed line) first inverted and then summed up; single multipole contributions with $K^{JT}_{\max}$ first summed up and then inverted (dotted line).

$\mathcal{L}_L^{JT}$, calculated up to $K_{\max}^{JT}$, are first summed up and then inverted. The comparison between the dotted and full lines shows the accuracy of the inversion. In Fig. 1, one finds very satisfying results for both isospin channels for the HH convergence and the accuracy of the inversion as well. We should mention that we do not show the low-energy isoscalar response, where a narrow $0^+$ resonance with a width of a few hundred keV is present at $E_r$ very close to threshold [23]. To get accurate results for such a resonance, a convergent LIT calculation with a $\sigma_I$ much smaller than the presently used values (smallest value $\sigma_I = 5$ MeV) should be carried out, which then leads to a very slow asymptotically falloff of the solution $|\tilde{\Psi}^{\rho}_{\sigma,q}\rangle$ (see [24]). Such a calculation requires a considerable additional computational effort, and thus the threshold region is excluded from our present work. Allowing a narrow resonance in the inversion [24], we have checked that our results are stable for energies above $E_r + 2\sigma_I$.

In Fig. 2, the results of $R_L(\omega, q)$ at various $q$ are shown and compared to data. In all cases one finds that the FSI effects are very large and essential for reaching agreement with experiment. The PWIA fails particularly in the q.e. peak and at low $\omega$. With growing $q$, FSI effects decrease in the peak region but not at low $\omega$. One may also consider a more refined PWIA, where a spectral function is used instead of a momentum distribution (see, e.g., [28]). In Ref. [28], it was shown that such an improved PWIA modifies the simple PWIA result by only 10%–20%.

In Fig. 2, one also sees the 3NF effects on the full calculation. For $q = 300$ MeV/$c$, one notes a good agreement of the data with the AV18 + UIX result. This is true for $q = 400$ MeV/$c$ as well, if one does not consider the data of Ref. [26], which exhibit larger error bars. At $q = 500$ MeV/$c$, some discrepancies between theory and experiment are present in the low- and high-energy range, while there is a fairly good agreement in the peak region. However, investigations on the three-body systems [10] have shown that a consideration of relativistic effects becomes important at such a momentum transfer.

Table I illustrates the 3NF effect on peak position and peak height also for lower $q$. One notes that there is no unique 3NF effect on the position, while one has a reduction of the height due to the 3NF at any $q$. The size of the reduction amounts to 10% for the higher $q$, whereas below $q = 300$ MeV/$c$ the reduction grows with decreasing $q$, reaching almost 30% at $q \leq 100$ MeV/$c$. In Fig. 3, the results at lower $q$ are shown. The important role of the 3NF is evident in the whole peak region, leading to a strong decrease of $R_L$ of up to 40% for some $\omega$ values. Recently, also some new data at $q \simeq 200$ MeV/$c$ have been published [29] [see Fig. 3(a)]. While one finds a satisfactory agreement between the AV18 + UIX result and data beyond the peak, one observes a non-negligible discrepancy in the peak itself. In Figs. 3(a) and 3(b), we also illustrate $R_L$ for a calculation [4] with a central two-nucleon potential [the Malfliet Tjon (MTI-III) model [30]]. Results are more similar to the AV18 than to the AV18 + UIX curves, showing that the 3NF effect is not simply explained by the binding energy difference ($^4$He binding energy with AV18, AV18 + UIX, and MTI-III is 24.3, 28.4, and 30.6 MeV, respectively).

We summarize our results as follows. We have carried out an *ab initio* calculation of the longitudinal ($e$, $e'$) response function $R_L(\omega, q)$ of $^4$He for various kinematics up to $q = 500$ MeV/$c$. The full dynamics of the four-body

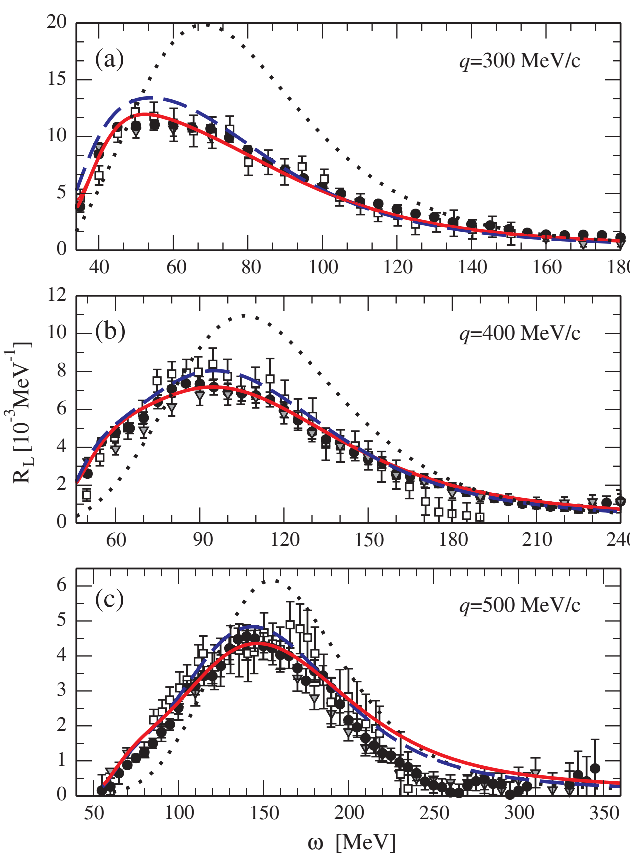


FIG. 2 (color online). $R_L(\omega, q)$ at various $q$: PWIA using $n(\boldsymbol{p})$ of AV18 + UIX [25] (dotted line); full calculation with AV18 (dashed line) and AV18 + UIX (solid line). Data from Bates [26] (squares), Saclay [27] (circles), and world-data set from Ref. [14] (triangles).

TABLE I. $R_L$ peak position $\omega_p$ and $R_L$ peak height without 3NF (AV18), with 3NF (AV18 + UIX), and relative 3NF effect $\Delta R = 100 \times [R_L(\text{AV18}) - R_L(\text{AV18 + UIX})]/R_L(\text{AV18})$.

| $q$ [MeV/$c$] | AV18 $\omega_p$ [MeV] | AV18 + UIX $\omega_p$ [MeV] | AV18 $R_L(\omega_p, q)$ [$10^{-3}$ MeV$^{-1}$] | AV18 + UIX $R_L(\omega_p, q)$ [$10^{-3}$ MeV$^{-1}$] | $\Delta R$ [%] |
|---|---|---|---|---|---|
| 50 | 26 | 28 | 2.96 | 2.15 | −27 |
| 100 | 28 | 30 | 9.56 | 7.11 | −26 |
| 200 | 36 | 38 | 17.5 | 14.5 | −17 |
| 300 | 54 | 52 | 13.4 | 12.0 | −10 |
| 350 | 73 | 70 | 10.3 | 9.20 | −11 |
| 400 | 95 | 95 | 8.04 | 7.18 | −11 |
| 500 | 143 | 146 | 4.84 | 4.36 | −10 |

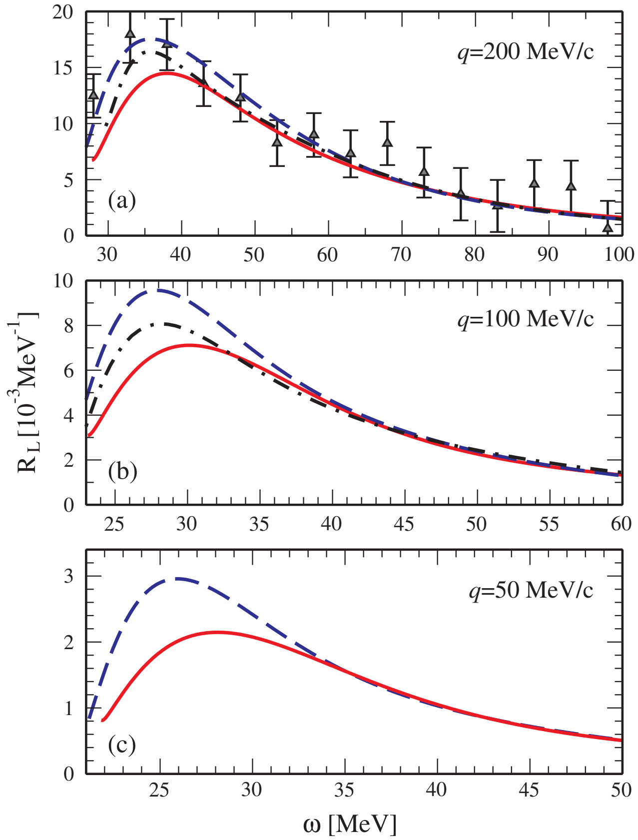


FIG. 3 (color online). $R_L(\omega, q)$ at various $q$ with the AV18 (dashed line), AV18 + UIX (solid line), and MTI-III (dashed-dotted line) potential. Data in (a) from Ref. [29].

system has been taken into account for the $^4$He ground state and the four-body continuum states as well. The rigorous inclusion of FSI has been achieved by use of the LIT method. Our work has been mainly focused on two points, namely, the study of the importance of FSI and of 3NF. We have shown that both ingredients play an important role and need to be considered in a calculation of $R_L$. A particularly important finding is the very large 3NF effects of up to 40% in the $R_L$ peak region at $q \leq 200$ MeV/$c$. Thus it is becoming apparent that there exists an electromagnetic observable, complementary to the purely hadronic ones, where one can learn more about the not yet well established 3NF. In view of our findings, we hope for a revival of the experimental interest in electron scattering, especially on light nuclei and at lower energies and momenta.

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