QCD Corrections to $e^+e^- \to J/\psi + gg$ at B Factories

Yan-Qing Ma,[1] Yu-Jie Zhang,[1] and Kuang-Ta Chao[1,2]
[1]Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[2]Center for High Energy Physics, Peking University, Beijing 100871, China
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In heavy quarkonium production, the measured ratio $R_{c\bar{c}} = \sigma[J/\psi + c\bar{c} + X]/\sigma[J/\psi + X]$ at B factories is much larger than existing theoretical predictions. To clarify this discrepancy, in nonrelativistic QCD we find the next-to-leading-order (NLO) QCD correction to $e^+e^- \to J/\psi + gg$ can enhance the cross section by about 20%. Together with the calculated NLO result for $e^+e^- \to J/\psi + c\bar{c}$, we show that the NLO corrections can significantly improve the fit to the ratio $R_{c\bar{c}}$. The effects of leading logarithm resummation near the end point on the J/ψ momentum distribution and total cross section are also considered. Comparison of the calculated cross section for $e^+e^- \to J/\psi + gg$ with the observed cross section for $e^+e^- \to J/\psi$ + non-($c\bar{c}$) is expected to provide unique information on the issue of color-octet contributions.

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In recent years, a number of challenging problems in heavy quarkonium production have appeared [1]. Aside from the J/ψ production cross sections and polarizations in hadron collisions at the Tevatron, charmonium production in e^+e^- annihilation at B factories [2,3] also conflicted with theoretical predictions. The observed double charmonium production cross sections for $e^+e^- \to J/\psi \eta_c(\chi_{c0})$ were larger than the LO calculations in nonrelativistic QCD (NRQCD) [4] by an order of magnitude [5], and later it was found that these discrepancies could be largely resolved by the next-to-leading-order (NLO) QCD corrections (see [6,7] for $J/\psi\eta_c$ and [8] for $J/\psi\chi_{c0}$) with relativistic corrections [9,10]. For the J/ψ production associated with an open charm pair $e^+e^- \to J/\psi + c\bar{c}$, the NLO QCD correction [11] was also found to significantly enhance the cross section (see also [12]) and reduce the large gap between experiment and the LO calculations [13].

Another important issue concerns the ratio

$$R_{c\bar{c}} = \frac{\sigma[e^+e^- \to J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \to J/\psi + X]}. \tag{1}$$

Belle found first $R_{c\bar{c}} = 0.59^{+0.15}_{-0.13} \pm 0.12$ [2] and later $R_{c\bar{c}} = 0.82 \pm 0.15 \pm 0.14$ [14]. On the contrary, LO NRQCD [13,15] and light-cone perturbative QCD predictions [16] for the ratio are only about 0.1–0.3. The color evaporation model gives a value of only 0.06 [17].

In NRQCD, $\sigma[J/\psi + X]$ includes color-singlet contributions $\sigma[J/\psi(^3S_1^{[1]}) + c\bar{c}]$ and $\sigma[J/\psi(^3S_1^{[1]}) + gg]$ and color-octet contributions $\sigma[J/\psi(^3P_J^{[8]}, {}^1S_0^{[8]}) + g]$. Contributions of other Fock states are suppressed by either α_s, the strong coupling constant, or v, the relative velocity between quark and antiquark in heavy quarkonium. $\sigma[J/\psi(^3P_J^{[8]}, {}^1S_0^{[8]}) + g]$ was calculated at LO in α_s [18], and an apparent enhancement at the J/ψ maximum energy was predicted. But experiments did not show any enhancement at the end point. The resummations of the v expansion and $\log(1-z)$, where $z = E_{c\bar{c}}/E_{c\bar{c}}^{\max}$, are considered [19], but the theoretical results rely heavily on the phenomenological shape function. It is possible that the observed end point behavior of J/ψ and the large ratio $R_{c\bar{c}}$ might indicate that the color-octet matrix elements are much smaller than previously expected. To test this thought we assume the color-octet contribution to be ignored and consider only the color-singlet contributions. Under this assumption, the ratio becomes

$$R_{c\bar{c}} = \sigma[J/\psi + c\bar{c}]/(\sigma[J/\psi + c\bar{c}] + \sigma[J/\psi + gg]). \tag{2}$$

In the following, we concentrate on $\sigma(J/\psi + gg)$ in NRQCD. Aside from the LO calculations in NRQCD (see related references in [13,15]), Ref. [20] considered $\sigma[J/\psi + gg]$ within the framework of soft collinear effective theory, and Ref. [21] summed over the leading and next-to-leading logarithms in the end point region of $\sigma[J/\psi + gg]$. However, considering the crucial importance of the NLO QCD corrections found in many heavy quarkonium production processes [6–8,11,12,22], it is necessary to carry out the calculation of NLO QCD correction to $e^+e^- \to J/\psi + gg$.

We now present this calculation. We use FEYNARTS [23] to generate Feynman diagrams and amplitudes, FEYNCALC [24] to handle amplitudes, and LOOPTOOLS [25] to evaluate the infrared-finite scalar Passarino-Veltman integrals. Feynman diagrams for the Born, virtual correction, and real correction are shown in Figs. 1–3. Note that $e^+e^- \to J/\psi gc\bar{c}$ is excluded in the real correction, because it should be included in the J/ψ production associated with open charm $e^+e^- \to J/\psi + c\bar{c} + X$. Moreover, we include ghost diagrams in the real correction because we choose unphysical polarizations for the gluons in the final

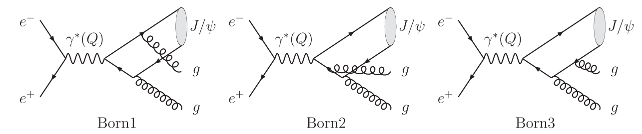

FIG. 1. Three of the six Born diagrams for $e^+e^- \to J/\psi gg$.

state. There are generally ultraviolet (UV), infrared (IR), and Coulomb singularities. Conventional dimensional regularization with $D = 4 - 2\epsilon$ is adopted to regularize them.

The UV divergences from self-energy and triangle diagrams are removed by renormalization. The renormalization constants Z_m, Z_2, and Z_3, which correspond, respectively, to the charm quark mass m, charm field ψ_c, and gluon field A^a_μ, are defined in the on-mass-shell (OS) scheme, while Z_g corresponding to the coupling α_s is defined in the modified-minimal-subtraction ($\overline{\rm MS}$) scheme:

$$\delta Z_m^{\rm OS} = -3C_F \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{\rm UV}} + \frac{4}{3}\right],$$
$$\delta Z_2^{\rm OS} = -C_F \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{\rm UV}} + \frac{2}{\epsilon_{\rm IR}} + 4\right],$$
$$\delta Z_3^{\rm OS} = \frac{\alpha_s}{4\pi} N_\epsilon \left[[\beta_0(n_{lf}) - 2C_A]\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right) - \frac{4T_f}{3\epsilon_{\rm UV}}\right],$$
$$\delta Z_g^{\overline{\rm MS}} = -\frac{\beta_0(n_f)}{2} \frac{\alpha_s}{4\pi} N_\epsilon \left[\frac{1}{\epsilon_{\rm UV}} + \ln\frac{m^2}{\mu^2}\right], \qquad (3)$$

where $N_\epsilon = (\frac{4\pi\mu^2}{m^2})^\epsilon \Gamma(1+\epsilon)$ is an overall factor in our calculation, $\beta_0(n_f) = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ is the one-loop coefficient of the QCD beta function, $n_f = 4$ is the number of active quark flavors, $n_{lf} = 3$ is the number of light quark flavors, and μ is the renormalization scale.

IR singularities coming from loop integration and phase space integration of the real correction are found to cancel each other. We use the method in Ref. [26] to separate the soft and collinear singularities in the virtual corrections and then treat the singular part analytically and the finite part numerically.

We use the phase space slicing method [27] to extract poles in the real correction. The method introduces a soft cut δ_s and a hard collinear cut δ_c to the phase space. Then the cut region can be partly integrated and becomes some color connected Born cross sections multiplied by singular factors. The remaining region, the hard noncollinear region, which is nonsingular, can be integrated using the standard Monte Carlo techniques. In order to make the method effective, $\delta_c \ll \delta_s$ is needed as mentioned in Ref. [27]. With a careful treatment for the two cuts, we verified that our result is independent of the two cuts in a large range.

We then find that by considering all NLO virtual and real corrections, and factoring the Coulomb singular term into the J/ψ wave function, we get an UV and IR finite cross section for $e^+e^- \to J/\psi + gg$.

In the numerical calculation we use $\sqrt{s} = 10.6$ GeV, $\Lambda^{(4)}_{\overline{\rm MS}} = 338$ MeV, and the J/ψ wave function squared at the origin $|R_{J/\psi}(0)|^2 = 1.01\,{\rm GeV}^3$, which is extracted from the J/ψ leptonic width [28] at NLO in α_s: $|R_{J/\psi}(0)|^2 = \frac{9M^2_{J/\psi}}{16\alpha^2[1-(16/3)\alpha_s/\pi]}\Gamma_{J/\psi\to e^+e^-}$.

Taking $M_{J/\psi} = 2m$ (at LO in v) and $m = 1.4$ GeV, we get $\alpha_s(\mu) = 0.267$ for $\mu = 2m$, and the cross section at NLO in α_s is $\sigma(e^+e^- \to J/\psi gg) = 0.496$ pb, which is a factor of 1.19 larger than the LO cross section 0.418 pb. If we set $\mu = \sqrt{s}/2$, then $\alpha_s = 0.211$, and the cross section is 0.394 pb. Since the experimental data correspond to the J/ψ prompt production, in addition to the direct production discussed above, we should also include the feeddown contributions from higher charmonium states which decay into J/ψ. Since for the P-wave states χ_{cJ} the direct production rates in the non-$c\bar{c}$ associated process are suppressed ($e^+e^- \to \gamma^* \to \chi_{cJ}gg$ are forbidden due to charge parity conservation), and for the $\psi(nS)(n > 2)$ states the decay branching ratios into $J/\psi + X$ are negligible, we need only to consider the $\psi(2S)$ feeddown contribution. This implies that an additional enhancing factor of 1.355 should be multiplied [11]. In Fig. 4, we show the prompt production cross sections at LO and NLO as functions of the renormalization scale μ. We see that NLO QCD correction substantially reduces the μ dependence and enhances the cross section by about 20%.

The NLO cross sections $\sigma(e^+e^- \to J/\psi c\bar{c})$ were calculated in Ref. [11]. We list the values of prompt cross sections [11] in Table I, together with the prompt cross sections $\sigma(e^+e^- \to J/\psi gg)$ obtained above. Then we can get the ratio $R_{c\bar{c}}$ at LO and NLO in α_s. The dependence of $R_{c\bar{c}}$ on the renormalization scale μ is shown in Fig. 5,

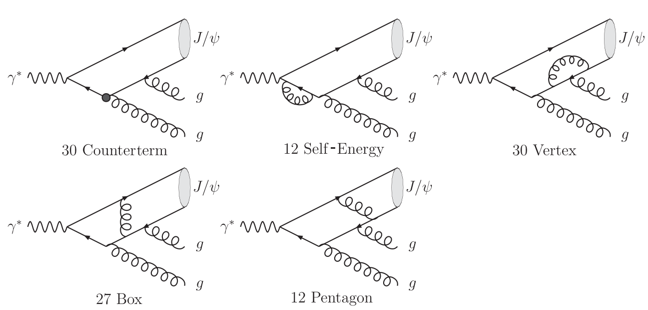

FIG. 2. Feynman diagrams for the virtual correction to $e^+e^- \to J/\psi gg$.

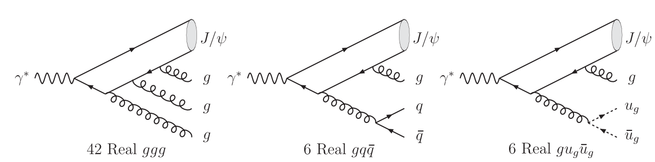

FIG. 3. Feynman diagrams for the real correction to $e^+e^- \to J/\psi gg$.

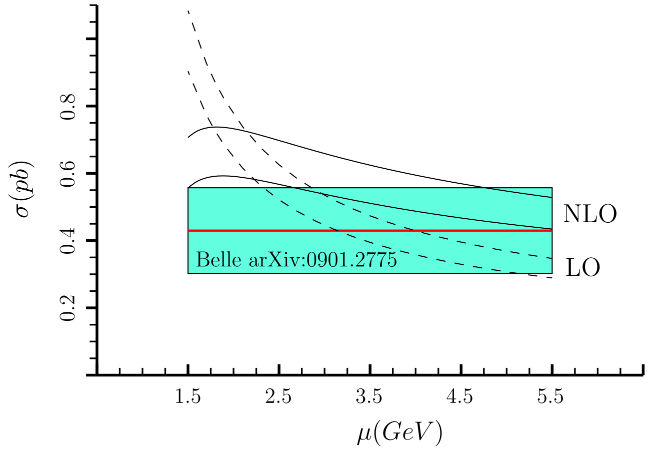

FIG. 4 (color online). Prompt cross sections of $e^+e^- \to J/\psi gg$ as functions of the renormalization scale μ at LO and NLO in α_s. The upper curves correspond to $m = 1.4$ GeV, and the lower ones correspond to $m = 1.5$ GeV.

where $m = 1.4$ GeV is fixed. The μ dependence for $\sigma(e^+e^- \to J/\psi gg)$ is mild, while for $\sigma(e^+e^- \to J/\psi c\bar{c})$ it is strong. A reasonable choice should be between $\mu = 2m_c$ and $\mu = \sqrt{s}/2$, more preferably the latter.

Finally, we note that the large logarithms of $\log(1-z)$ appear at the end point in NLO calculation, where $z = E_{J/\psi}/E_{J/\psi}^{\max}$. The leading logarithms (LLs) have been resumed in Refs. [20,21]. Using a similar approach, we define the differential cross section (and other quantities) as [21]

$$d\sigma_{\mathrm{LO(NLO)+LL}} = d\sigma_{\mathrm{LO(NLO)}} + P[r,z]d\sigma_{\mathrm{resum}} - P[r,z](d\sigma_{\mathrm{resum}})_{\mathrm{LO(NLO)}}, \quad (4)$$

where $(d\sigma_{\mathrm{resum}})_{\mathrm{LO(NLO)}}$ means expanding $d\sigma_{\mathrm{resum}}$ in α_s to LO (NLO). To be consistent with our previous calculation, we choose $\mu_c = \sqrt{2(1-z)}\mu_H$, and $\mu_H = \mu = 2m$. In Fig. 6, we show the cross sections of $e^+e^- \to J/\psi gg$ as functions of the J/ψ momentum $P_{J/\psi}$. The correction of LL resummation to the total cross section is about -6.6% at LO and 0.5% at NLO. But it becomes large at the end point region when $z \to 1$, suppressing the LO cross section and enhancing the NLO cross section. The LL resummation changes the J/ψ momentum distribution near the end point but has only a little effect on the total cross section. It is interesting to note that, with the NLO correction, the J/ψ momentum spectrum becomes much softer than the LO result.

TABLE I. Cross sections of prompt (feeddown included) $J/\psi gg$ (this Letter) and $J/\psi c\bar{c}$ (Ref. [11]) production in e^+e^- annihilation at B factories in units of picobarns.

	$\mu = 2.8$ GeV LO	$\mu = 2.8$ GeV NLO	$\mu = 5.3$ GeV LO	$\mu = 5.3$ GeV NLO
$\sigma(gg)$	0.57	0.67	0.36	0.53
$\sigma(c\bar{c})$	0.38	0.71	0.24	0.53
$R_{c\bar{c}}$	0.40	0.51	0.40	0.50

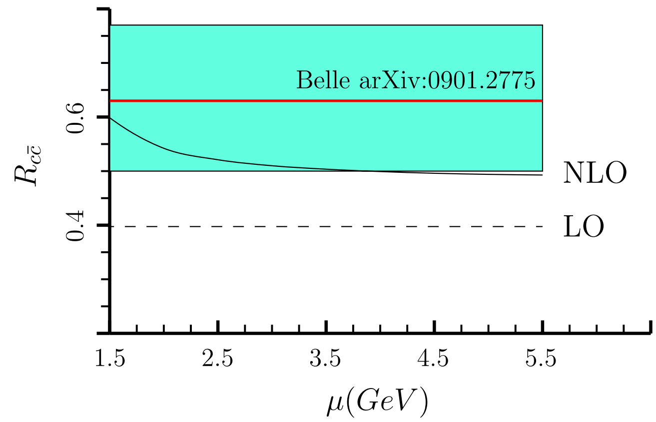

FIG. 5 (color online). $R_{c\bar{c}}$ as a function of the renormalization scale μ at LO and NLO in α_s. Here $m_c = 1.4$ GeV.

In summary, we find that, by considering all NLO virtual and real corrections and factoring the Coulomb singular term into the $c\bar{c}$ bound state wave function, we get an ultraviolet, infrared, and collinear finite cross section for the direct production of $e^+e^- \to J/\psi + gg$ at $\sqrt{s} = 10.6$ GeV, which enhances the cross section by about 20%. By adding the feeddown contribution from $\psi(2S)$, the prompt production cross section of $e^+e^- \to J/\psi + gg$ at NLO in α_s is found to be $(0.67^{-0.13}_{+0.17})$ pb for $\mu = 2m$ and $(0.53^{-0.09}_{+0.12})$ pb for $\mu = \sqrt{s}/2$, with $m = (1.4 \pm 0.1)$ GeV. Together with the calculated $\sigma(e^+e^- \to J/\psi c\bar{c})$ at NLO in α_s [11], we get $R_{c\bar{c}} \approx 0.50$. The result significantly reduces the discrepancy between theory and experiment. The effect of the leading logarithm resummation near the end point on the $J/\psi + gg$ total cross section is found to be small.

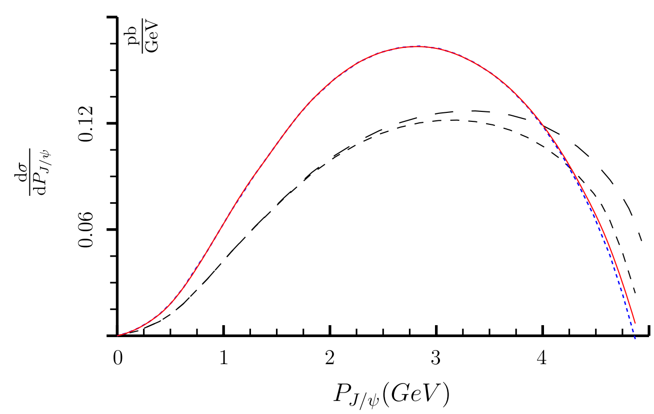

FIG. 6 (color online). The cross section of $e^+e^- \to J/\psi gg$ as functions of the J/ψ momentum $P_{J/\psi}$. Here $\mu = 2.8$ GeV and $m = 1.4$ GeV. The solid curve is the NLO + LL prediction, and the dotted, short-dashed, and long-dashed curves are the NLO, LO + LL, and LO predictions, respectively.

Note.—Very recently, Belle reported a new measurement with higher statistics [29]:

$$\sigma(e^+e^- \to J/\psi + c\bar{c}) = (0.74 \pm 0.08^{+0.09}_{-0.08})\ \text{pb}, \quad (5)$$

$$\sigma[e^+e^- \to J/\psi + \text{non-}(c\bar{c})] = (0.43 \pm 0.09 \pm 0.09)\ \text{pb}. \quad (6)$$

The observed cross section of $e^+e^- \to J/\psi + \text{non-}(c\bar{c})$ and $R_{c\bar{c}}$ are displayed in Figs. 4 and 5 with central values and error bands in comparison with theoretical predictions. We see that our predictions (NLO with feeddown) for $\sigma(e^+e^- \to J/\psi + gg)$ are consistent with the new measurement of $\sigma[e^+e^- \to J/\psi + \text{non-}(c\bar{c})]$ within certain uncertainties. Moreover, the predicted J/ψ momentum spectrum also agrees with the experiment [29]. Importantly, our result of $\sigma[e^+e^- \to J/\psi + \text{non}(c\bar{c})]$ indicates that the calculated $\sigma(e^+e^- \to J/\psi + gg)$ has already saturated the observed $\sigma[e^+e^- \to J/\psi + \text{non-}(c\bar{c})]$, hence leaving little room for the color-octet contributions. These are also confirmed by a similar study [30], which agrees with ours.

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