Collective-Interaction Control and Reduction of Optical Frequency Shot Noise in Charged-Particle Beams

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We present a theoretical model for longitudinal collective Coulomb interactions in a charged-particle beam. It suggests a possibility to control and reduce optical frequency shot-noise current in accelerated electron beams. For short interaction lengths, the model describes well coherent optical transition radiation effects observed in SLAC LCLS and in other laboratories. For longer interaction lengths (quarter plasma oscillation period) the model predicts the possibility to reduce the beam current noise below the classical shot-noise level, an effect not yet observed experimentally at optical frequencies.

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The current shot noise in charged-particle beams is the current fluctuation due to the corpuscular nature of the particles and their random arrival at the current measurement point. The shot noise is the source of incoherent radiation in electron beam radiation sources, including self-amplified spontaneous emission (SASE) free electron lasers (FELs) [1]. The particle beam current shot noise is the source of radiation in beam diagnostics schemes such as ‘‘optical transition radiation’’ (OTR), used to measure the particle beam transverse current distribution under the assumption that the current shot noise is proportional to the current density at the OTR screen [2].

In a number of laboratories it was observed recently that the transverse distribution of OTR emission from accelerated electron beams deviates substantially from the beam current distribution [3–6]. This effect of ‘‘coherent OTR’’ (COTR) is related to the process of longitudinal space-charge random ‘‘microbunching’’ [7–9], which is a transversely-coherent, longitudinally-random current (or density) noise in the e beam. Of relevance to our work are the COTR observations in the photocathode rf linac injector of the SLAC LCLS experiment. In this experiment effects of coherent OTR diffraction patterns and nonlinear growth of the integrated OTR power vs beam current were observed, and were termed ‘‘unexpected physics’’ when first discovered [3,4]. Now there is an understanding that these observations are evidence of a longitudinal collective interaction process, which took place in the high current-density drift section along a focused beam waist formed after the 135 MeV rf linac injector [5]. This interaction produced transversely coherent longitudinally-random beam-energy modulation (microbunching), which transformed into current (density) microbunching after beam passage through an energy dispersive element (a magnetic dipole bend). The observed transverse coherence and the intensity enhancement of the measured COTR radiation were the result of the transverse coherence and intensity enhancement of this current microbunching noise.

In this Letter we present a theoretical model for longitudinal collective Coulomb interactions and noise evolution dynamics. For a short collective interaction region our model predicts COTR power enhancement as observed in the LCLS experiments; it is in good agreement with the observed transverse coherence features. For long collective interaction lengths our model predicts a possibility of controlling the current-noise level, and in particular, the possibility of decreasing it below the normal shot-noise level, an effect not yet observed at optical frequencies.

It should be noted that current shot-noise reduction was studied and demonstrated some 60 years ago in the microwave tube frequency range [10,11]. The possibility to extend such noise reduction schemes to the optical frequency regime and to relativistic electron beams is still controversial. We show in this Letter that despite the longitudinal plasma frequency reduction at highly relativistic acceleration energies, the high-current–high-quality beams attainable nowadays with photocathode injectors [12], may allow attainment of conditions for optical frequency current shot-noise reduction via the longitudinal collective interaction process.

Our theory is based on an axial interaction model (in the beam flow direction z), and a linear expansion of the beam fluid plasma equations in terms of time dependent, small signal modulation parameters (charge density, current density, velocity, space-charge field). The interdependence of these parameters enables expression of the variables in terms of two parameters: the beam axial current-density modulation $j_z(\underline{r}_\perp, z, t)$and the beam axial velocity modulation $v_z(z, t)$ [13].

After Fourier transformation, it is convenient to describe the dynamic axial interaction evolution process in terms of the spectral axial current: $\check{i}(z, \omega) = \iint \check{j}_z(\underline{r}_\perp, z, \omega) d^2 r_\perp$ and the spectral kinetic voltage $\check{V}(z, \omega)$:

$$\check{V}(\omega) = -\gamma_0{}^3 v_0 (m/e)\check{v}(\omega) = -(mc^2/e)\check{\gamma}(\omega) \qquad (1)$$

where $\check{v}(\omega)$, $\check{\gamma}(\omega)$ are the Fourier transformed beam velocity and energy modulation parameters, and $\breve{V}(\omega)$ is a relativistic re-definition of Chu's "kinetic voltage" [10].

In a free-drift beam transport section with constant beam parameters, our *relativistically extended* theory results in the following dynamic equations for the spectral noise parameters [13]:

$$\breve{i}(L_d, \omega) = [\cos\phi_p \breve{i}(0, \omega) - i(\sin\phi_p/W_d)\breve{V}(0, \omega)]e^{i\phi_b(L_d)}, \quad (2)$$

$$\breve{V}(L_d, \omega) = [-iW_d \sin\phi_p \breve{i}(0, \omega) + \cos\phi_p \breve{V}(0, \omega)]e^{i\phi_b(L_d)}, \quad (3)$$

where $\phi_b = L_d\omega/v_0$, $\phi_p = \theta_{\rm pr}L_d$, $\theta_{\rm pr} = r_p\omega_{\rm pl}/v_0$, $\omega_{\rm pl} = (e^2n_0/m\varepsilon_0\gamma^3)^{1/2}$, $W_d = \sqrt{\mu_0/\varepsilon_0}/k\theta_{\rm prd}A_e$ define the plasma wave optical-phase ϕ_b, the plasma longitudinal oscillation phase ϕ_p, the plasma longitudinal oscillation wave number $\theta_{\rm pr}$, the relativistic longitudinal plasma oscillation frequency $\omega_{\rm pl}$ and the beam modulation impedance W_d, respectively. These frequency domain relations, if viewed in the beam rest frame, are the well-known equations of longitudinal wave oscillation in a plasma column. The plasma reduction factor [14] $r_p < 1$ is used to account for reduction of the longitudinal space-charge field due to the fringing of the field lines in a finite cross-section beam of radius r_b and cross-section area $A_e = \pi r_b{}^2$ (for a flat-top transverse current distribution model).

The complex initial current and kinetic-voltage modulations are not known in the incoherent noise problem. Their averaged square values are derived from the classical one-dimensional shot-noise theory:

$$\overline{|\breve{i}(0, \omega)|^2} = eI_b \quad (4)$$

$$\overline{|\breve{V}(0, \omega)|^2} = \frac{(\delta E_c)^2}{eI_b}. \quad (5)$$

Here δE_c is the longitudinal energy spread in the beam. In low-noise vacuum tube guns it is ideally limited by the cathode temperature ($\delta E_c \approx k_BT_c$), but in rf gun injectors and accelerators it is significantly increased during the acceleration processes. The statistical averaging symbol corresponds to averaging over the initial entrance times of the electrons and their velocity distribution.

This model is essentially a coasting beam modified one-dimensional longitudinal interaction model. In photocathode-gun injectors (before compression) the beam duration is 1–10 ps. Considering the γ_0 factor length extension the beam bunch length in the beam rest frame is not much longer than the beam width, and the use of the coasting beam model is justified. The 3D effect of "field fringing" is negligible ($r_p \approx 1$) if the beam modulation wavelength, viewed in the electron beam rest frame is smaller than the beam diameter: $\lambda\gamma_0\beta_0 \ll 2r_b$ (here $\lambda = 2\pi c/\omega$, where ω is the Fourier frequency component of the current in the lab frame). For a Gaussian transverse current distribution with standard deviation parameter $\sigma_x = r_b/\sqrt{2}$, the condition is:

$$2\pi\sigma_x/\gamma_0\beta_0\lambda\rangle 1. \quad (6)$$

At *optical frequencies*, even with $\gamma_0 \gg 1$, inequality (6) may be satisfied. However, for our single-mode Langmuir plasma wave model it is desirable to keep (6) near equality, where transverse density fluctuations are negligible [15]. From Eqs. (2) and (3) one can note that for cases where the current shot-noise amplitude dominates —

$$\overline{|\breve{i}(0, \omega)|^2}\rangle\rangle\overline{|\breve{V}(0, \omega)|^2}/W_d^2 \quad (7)$$

and the collective interaction length is short ($\phi_p\langle\langle 1$), then:

$$\overline{|\breve{i}(L_d, \omega)|^2} = \overline{|\breve{i}(0, \omega)|^2}\cos^2\phi_d \cong \overline{|\breve{i}(0, \omega)|^2} \quad (8)$$

$$\overline{|\breve{V}(L_d, \omega)|^2} = \overline{|\breve{i}(0, \omega)|^2}\sin^2\phi_d \cong \overline{|\breve{i}(0, \omega)|^2}W_d^2\theta_p^2L_d^2, \quad (9)$$

namely, the current noise does not change, and the velocity noise amplitude$(\overline{|\breve{V}(L_d, \omega)|^2})^{1/2}$ grows in proportion to the interaction length. By use of the Poisson and continuity equations, it can be shown that Eq. (9) is equivalent to linear modulation of the beam energy $mc^2\delta\gamma_m$ by the axial space-charge field: $mc^2\delta\gamma_m(L_d) \approx -eE_zL_d$. This approximation is essentially the one used in the current theories of microbunching instability development, where a "space-charge impedance" concept is used to describe the proportionality relation between the energy and current modulations in the Coulomb interaction energy modulation stage [5,7–9,15]. This approximation is only valid for the common case of short interaction length ($\phi_p\langle\langle 1$).

We shall use these expressions to describe an example based on the LCLS experimental parameters [4,5]: $\gamma_0 = 265$, $I_b = 40$ A, $L_d = 2.5$ m, $\sigma_x = 67.3$ μm, $\lambda = 1$ μm, $\delta E_c = 3$ keV. We find that $\phi_p = 0.35 < 1$, which enables use of the approximate expressions (8) and (9), and justifies the use of the conventional linear modulation model. The spectral energy-noise enhancement factor in the collective interaction region is:

$$f_v = \overline{|\breve{V}(L_d, \omega)|^2}/\overline{|\breve{V}(0, \omega)|^2} = \left(\frac{\sqrt{\mu_0/\varepsilon_0}I_b}{\delta E_c/e}\frac{L_d}{kA_e}\right)^2. \quad (10)$$

We find $f_v = 5 \times 10^3$.

For our chosen example, if the e beam is, consequently passed through an ideal energy dispersive electron-optic element having only a momentum-compaction coefficient $R_{56} \neq 0$ (for the other coefficients: $R_{i\neq j} = 0$, $R_{ii} = 1$) the energy noise transforms into a current noise according to:

$$\frac{\overline{|\breve{i}_{\rm out}(\omega)|^2}}{I_0^2} = \left(\frac{e}{mc^2}\frac{k}{\gamma_0}R_{56}\right)^2\overline{|\breve{V}_{\rm in}(\omega)|^2} \quad (11)$$

and the overall [relative to (4)] spectral current-noise enhancement factor after the dispersive element becomes:

$$f_i = \frac{\overline{|\breve{i}_{\rm out}(\omega)^2|}}{\overline{|\breve{i}(0,\omega)|^2}} = \left(\frac{\sqrt{\mu_0/\varepsilon_0} I_b}{mc^2/e} \frac{R_{56} L_d}{\gamma_0 A_e}\right)^2. \qquad (12)$$

For $R_{56} = 6.3$ mm (as in the magnetic bend of the LCLS experiment): $f_i = 3.8 \times 10^3$. In the LCLS experiment, a much smaller current-noise enhancement factor was measured (but still with $f_i \gg 1$). The reasons are the degrading effects due to the finite energy spread and emittance of the beam and nonvanishing of the $R_{i\neq j}$ coefficients [5].

We shall now consider the unexplored limit of long collective interaction, where the quarter plasma oscillation condition is satisfied:

$$\phi_p = \theta_p L_d = \pi/2. \qquad (13)$$

In this case, there is full transformation of velocity noise into density noise and vice versa:

$$\overline{|\breve{i}(L_d,\omega)|^2} = \overline{|\breve{V}(0,\omega)|^2}/W_d^2 \qquad (14)$$

$$\overline{|\breve{V}(L_d,\omega)|^2} = \overline{|\breve{i}(0,\omega)|^2} W_d^2 \qquad (15)$$

and the current-noise enhancement factor in the collective-interaction section is given by:

$$f_i = \frac{\overline{|\breve{i}(L_d,\omega)|}^2}{\overline{|\breve{i}(0,\omega)|^2}} = \frac{\overline{|\breve{V}(0,\omega)|}^2}{\overline{|\breve{i}(0,\omega)|^2} W_d^2} = \left(\frac{\delta E_c/e}{\sqrt{\mu_0/\varepsilon_0} I_b} k\theta_{\rm pr} A_e\right)^2. \qquad (16)$$

For the common case of a current shot-noise dominated beam (7), Eq. (16) results in current-noise reduction ($f_i < 1$). For example, if we consider an experiment with all of the LCLS parameters as above, but with a beam drifting through a longer interaction length to satisfy (13), Eq. (16) yields a huge current-noise reduction: $f_i = 2.5 \times 10^{-5}$. Concurrently, the energy noise (10) is enhanced to its maximal value $f_v = 1/f_i = 4 \times 10^4$.

The possibility of reducing the e beam current-noise seems counterintuitive (although no thermodynamic principles are violated, since the current-noise reduction is accompanied by an energy-noise enhancement). The longitudinal plasma oscillation and current-velocity modulation exchange process (2) and (3) of a *coherent* plasma wave is well known. For a random density modulation case the exchange process can be understood as dilation of randomly formed bunches due to Coulomb repulsion of charges within the bunches. For the purpose of illustration, we show in Fig. 1 the homogenization process that takes place in a model of a beam composed of finite cross-section charge discs, randomly distributed in the axial dimension, interacting with each other by Coulomb repulsion.

In the following we analyze limitations to the validity of our fluid plasma model in the optical frequency range, for which we propose to apply the current-noise reduction scheme. To justify our longitudinal charge bunching model, a multitude of particles must be present per bunching wavelength $\lambda\beta_0$:

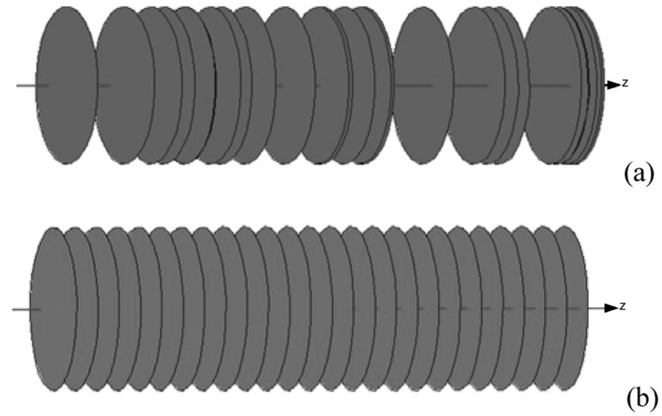

FIG. 1. (a) A model of finite diameter charge disks randomly positioned in the axial dimension in the beam rest frame. (b) The homogenized distribution of the charged disks, obtained by simulation of the redistribution dynamics in response to the Coulomb forces applied mutually by all disks.

$$n_0 A_e \lambda \beta_0 \gg 1. \qquad (17)$$

Another restriction to our "cold beam" model is, that the optical-phase spread $\Delta\varphi_b$ of the electrons along the interaction length L_d due to the beam velocity spread should satisfy $\Delta\varphi_b = kL_d\Delta(1/\beta_z) \ll \pi$. This corresponds to restrictions on the beam-energy spread:

$$\Delta\gamma/\gamma_0 \ll \beta_0^3\gamma_0^2\lambda/2L_d \qquad (18)$$

and on the emittance:

$$\varepsilon_n \ll \beta_0^2\gamma_0\sigma_{x0}(\lambda/L_d)^{1/2}. \qquad (19)$$

In the Appendix we show that in order to realize a quarter plasma period oscillation interaction length (13) by focusing the beam to a waist in free space, a sufficient condition, besides (18) and (19) is that the transported beam expansion is space-charge dominated. To satisfy this condition the emittance must also satisfy:

$$\varepsilon_n \langle\langle (2I_0/I_A\gamma_0\beta_0)^{1/2} r_{b0} = (2^{5/2}/\pi)(I_0/I_A\beta_0^2\gamma_0^2)L_d, \qquad (20)$$

where L_d is the quarter plasma wavelength length (13). Condition (20) needs to be satisfied only for a beam propagating in free space. It is not necessary if the beam is guided without expansion by means of focusing elements ($k_0 \neq 0$) or possibly by partial beam charge neutralization.

Considering again the previous example based on the LCLS parameters, and given the reported emittance $\varepsilon_n = 1\ \mu$m, one can verify that all inequalities (17) and (19) are satisfied but not (20). Therefore, to satisfy (13) with the design example, it would be necessary to achieve improved emittance or extend the interaction length to more than $L_d = 15$ m. Alternatively, condition (13) may be satisfied at shorter lengths, if one avoids free-space propagation, and provides additional focusing means along the interaction length.

The current-noise reduction scheme would have an important application if the "quieted" e beam could be used as the current input of the coherent seed-injected FEL schemes: seed radiation [16], prebunched beam [17] and high gain harmonic generation (HGHG) [18]. In these seeding schemes temporal coherence of the radiation would be attained only if the seed power exceeds the incoherent beam-noise power [19]. High harmonic generation (HHG) seed-radiation injection has already been demonstrated in the uv-wavelength regime [20]. Great efforts are devoted worldwide to extend these schemes to the extreme-uv (xuv) regime.

The possibility of attaining input beam-noise reduction at frequencies beyond the ir-uv depends on technological development and a proper electron-optical design that satisfies the theoretical restrictions derived above. It should be borne in mind that the collective beam-noise interaction region in the LCLS experiment is right after the rf linac injector and the noise effects were measured in the visible-IR regime. In practical xuv FEL designs the beam is transported through bends, chicanes and other dispersive elements that degrade the beam parameters, and may give rise to destructive microbunching instabilities (one may consider their control by *minimizing* the energy noise entering the dispersive section). Further theoretical and experimental studies are needed in order to understand the noise dynamics in these elements and evaluate the feasibility of the proposed scheme at frequencies beyond the visible. Note that in *seed-radiation* injection xuv FEL schemes, one would need to diminish the beam-noise at xuv frequencies in order to enhance the FEL radiation coherence. However in HGHG FELs the noise reduction needs to be accomplished only at the frequency of the first laser buncher.

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Appendix: Space-charge dominated beam expansion theorem.—The validity of the model is limited by conditions (17)–(19). Our noise reduction analysis required also a beam drift length L_d satisfying (13) with a uniform cross section. Can such uniform drift be maintained without continuous guiding by means of focusing elements? We prove a simple and useful theorem: quarter plasma wavelength oscillation (13) takes place within the waist of a free-space drifting beam under the condition that the beam expansion away from the waist is space-charge dominated.

The proof is based on the Kapchinsky-Vladimirsky (KV) beam envelope equation [21]

$$r_b''(z) + k_0^2 r_b(z) - K/r_b(z) - \bar{\varepsilon}^2/r_b^3(z) = 0 \qquad \text{(A1)}$$

which is valid for a flat-top current-density distribution of a beam of radius $r_b(z)$. It is approximately valid for a Gaussian distribution with $r_b = \sqrt{2}\sigma_x$, $\bar{\varepsilon} = 2\varepsilon$ where $\varepsilon = \varepsilon_n/\beta_0\gamma_0 = \sigma_{x0}\sigma_{x'0}$ is the conventional emittance definition, k_0 is the external focusing betatron wave number and $K = (2/\gamma_0^3\beta_0^3)I_0/I_A = \theta_p^2 r_b^2/2$ is the relativistic perviance definition, where $I_A = 17$ kA is the Alfvén current.

The solution of (A1) for $k_0 = \bar{\varepsilon} = 0$, namely—space-charge-dominated transport beam expansion in free space, is given in [21]. The envelope expansion of the solution within the section $-L_d/2 < z < L_d/2$, where $L_d = \pi/2\theta_p$ (13), is only $r(\pm L_d/2) = 1.15 r_{b0}$. This satisfies the condition of uniform cross section drift.

To be in the space-charge dominated beam expansion regime, the fourth term in (A1) must be negligible relative to the third. This sets a condition on the emittance or the beam radius [Eq. (20)].

The physical significance of this theorem is that the process of dilation of longitudinal density bunches occurs at the same rate as the beam envelope expansion due to the Coulomb forces. In the beam frame both processes seem the same.

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