Quark Forces from Hadronic Spectroscopy

Dan Pirjol[1] and Carlos Schat[2]

[1]*Department of Particle Physics, National Institute for Physics and Nuclear Engineering, 077125 Bucharest, Romania*

[2]*CONICET and Departamento de Física, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria, Pab. 1, (1428) Buenos Aires, Argentina*

(Received 1 December 2008; published 16 April 2009)

We consider the implications of the most general two-body quark-quark interaction Hamiltonian for the spin-flavor structure of the negative parity $L = 1$ excited baryons. Assuming the most general two-body quark interaction Hamiltonian, we derive two correlations among the masses and mixing angles of these states, which constrain the mixing angles, and can be used to test for the presence of three-body quark interactions. We find that the pure gluon-exchange model is disfavored by data, independently of any assumptions about hadronic wave functions.

DOI: 10.1103/PhysRevLett.102.152002 PACS numbers: 12.39.−x, 12.38.−t, 14.20.−c

The constituent quark model (CQM) [1–4] is a popular and time-tested approach used for modeling hadron properties. The basic assumption is that the quarks inside the hadron can be approximated as nonrelativistic point particles with constituent masses, interacting through two-body potentials.

In a recent paper [5], we presented a general method for relating the quark interaction Hamiltonian to the spin-flavor structure of the hadronic mass operator. Consider a given two-body interaction Hamiltonian $V_{qq} = \sum_{i>j} O_{ij} R_{ij}$, where O_{ij} acts only on the spin-flavor indices of the quarks i, j, and R_{ij} acts only on their orbital degrees of freedom. Then the hadronic matrix elements of the Hamiltonian V_{qq} on a baryon state $|B\rangle$ contain only the projections $\mathcal{O}_\alpha$ of O_{ij} onto irreducible representations of S_3, the permutation group of 3 objects acting on the spin-flavor degrees of freedom

$$\langle B|V_{qq}|B\rangle = \sum_\alpha C_\alpha \langle B|\mathcal{O}_\alpha|B\rangle. \quad (1)$$

The coefficients C_α are related to the reduced matrix elements of the orbital operators R_{ij}, and are given by overlap integrals of the quark model wave functions. The relation Eq. (1) allows a general study of the hadronic spin-flavor structure independently of the orbital structure of the interaction and wave functions. An application of the S_3 group in a similar context was discussed in Ref. [6], where S_3 refers to permutations of the quarks' orbital degrees of freedom. The present analysis makes crucial use of the transformation properties of operators and states under S_3 acting on the spin-flavor degrees of freedom.

One of the basic assumptions of the constituent quark model is the dominance of two-body quark interactions. However, three-body quark interactions may be present as well. For example, it is known that in QCD with $N_f = 3$ light quark flavors, three-body interactions are induced by instanton effects ('t Hooft interaction) [7]. We point out that the system of the negative parity excited nucleons is a possible testing ground for the presence of three-body quark forces.

In this Letter, we use the representation Eq. (1) to obtain information about the spin-flavor structure of the quark forces from the system of the negative parity excited baryons. We derive universal correlations among masses and mixing angles which are valid in any model for quark interactions containing only two-body interactions. Deviations from these predictions can be used to test for the presence of three-body quark interactions. We obtain constraints on the strength of spin-orbit interactions, which can be used to distinguish between two popular models for quark interactions: the one-gluon exchange model [1], and the Goldstone boson exchange model [8]. This gives information about the relative importance of these two interactions in generating the effective quark forces in the low energy regime.

The most general two-body quark interaction Hamiltonian in the constituent quark model can be written in generic form as $V_{qq} = \sum_{i<j} V_{qq}(ij)$ with

$$V_{qq}(ij) = \sum_k f_{0,k}(r_{ij}) O_{S,k}(ij) + f^a_{1,k}(r_{ij}) O^a_{V,k}(ij) + f^{ab}_{2,k}(r_{ij}) O^{ab}_{T,k}(ij), \quad (2)$$

where O_S, O^a_V, O^{ab}_T act on spin-flavor, and $f_k(r_{ij})$ are functions of $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. Their detailed form is unimportant for our considerations. $a, b = 1, 2, 3$ denote spatial indices.

We list in Table I a complete set of spin-flavor two-body operators with all possible Lorentz structures allowed by the orbital angular momentum $L = 1$. Columns 3 and 4 of Table I list the projections of the spin-flavor operators O_S, O^a_V, O^{ab}_T onto the irreducible representations of the S_3

TABLE I. The most general two-body spin-flavor quark interactions and their projections onto irreducible representations of S_3, the permutation group of three objects acting on the spin-flavor degrees of freedom. $C_2(F) = \frac{F^2-1}{2F}$ is the quadratic Casimir of the fundamental representation of SU(F). The last column shows the projection of each two-body operator onto the basis of 10 operators in Eq. (3).

Operator	$\mathcal{O}_{ij}$	O_S	$O_{\rm MS}$	
Scalar	1	1	$\cdots$	1
	$t_i^a t_j^a$	$T^2 - 3C_2(F)$	$T^2 - 3t_1T_c - 3C_2(F)$	$O_1,\ O_1 - 3O_2$
	$\vec{s}_i \cdot \vec{s}_j$	$\vec{S}^2 - \frac{9}{4}$	$\vec{S}^2 - 3\vec{s}_1 \cdot \vec{S}_c - \frac{9}{4}$	$O_2 + 2O_3,\ O_2 - O_3$
	$\vec{s}_i \cdot \vec{s}_j t_i^a t_j^a$	$G^2 - \frac{9}{4}C_2(F)$	$3g_1G_c - G^2 + \frac{9}{4}C_2(F)$	$\frac{F}{4}O_1 + \frac{1}{2}O_2 + O_3,\ O_1 - (3 + \frac{4}{F})O_2 + \frac{4}{F}O_3$
Vector (symm)	$\vec{s}_i + \vec{s}_j$	$\vec{L} \cdot \vec{S}$	$3\vec{L} \cdot \vec{s}_1 - \vec{L} \cdot \vec{S}$	$O_4 + O_5,\ 2O_5 - O_4$
	$(\vec{s}_i + \vec{s}_j)t_i^a t_j^a$	$\frac{1}{2}L^i\{G^{ia}, T^a\} - C_2(F)L^iS^i$	$2\frac{1-F}{F}L^iS_c^i + L^ig_1^{ia}T_c^a + L^it_1^aG_c^{ia}$	$O_6 + O_7 + \frac{F-1}{2F}O_4,\ O_6 + O_7 - 2\frac{F-1}{F}O_4$
Vector (anti)	$\vec{s}_i - \vec{s}_j$	$\cdots$	$3\vec{L} \cdot \vec{s}_1 - \vec{L} \cdot \vec{S}$	$2O_5 - O_4$
	$(\vec{s}_i - \vec{s}_j)t_i^a t_j^a$	$\cdots$	$L^ig_1^{ia}T_c^a - L^it_1^aG_c^{ia}$	$O_6 - O_7$
Tensor (symm)	$\{s_i^a, s_j^b\}$	$L_2^{ij}\{S^i, S^j\}$	$3L_2^{ij}\{s_1^i, S_c^j\} - L_2^{ij}\{S^i, S^j\}$	$O_8 + 4O_9,\ O_8 - 2O_9$
	$\{s_i^a, s_j^b\}t_i^c t_j^c$	$L_2^{ij}\{G^{ia}, G^{ja}\}$	$L_2^{ij}g_1^{ia}G_c^{ja} - \frac{F-1}{4F}L_2^{ij}\{S_c^i, S_c^j\}$	$\frac{F-1}{2F}O_8 + 4O_{10},\ \frac{F-1}{F}O_8 - 4O_{10}$
Tensor (anti)	$[s_i^a, s_j^b]$	$\cdots$	0	$\cdots$
	$[s_i^a, s_j^b]t_i^c t_j^c$	$\cdots$	0	$\cdots$

permutation group, computed as explained in Ref. [5]. The representation content depends on the symmetry of O_{ij} under the permutation $[ij]$: the symmetric operators O_{ij} are decomposed as $S + \text{MS}$, and antisymmetric O_{ij} as $\text{MS} + A$.

The symmetric S projection depends only on quantities acting on the entire hadron S^i, T^a, G^{ia}, while the mixed-symmetric MS operators depend on operators acting on the core and excited quarks. We express them in a form commonly used in the application of the $1/N_c$ expansion [9], according to which their matrix elements are understood to be evaluated on the spin-flavor state $|\Phi(SI)\rangle$ constructed as a tensor product of an excited quark with a symmetric core with spin-flavor $S_c = I_c$. The antisymmetric operators contain also an A projection; its orbital matrix element vanishes for $N_c = 3$ because of T-invariance [5,6], such that these operators do not contribute, and are not shown in Table I.

The orbital matrix elements yield factors of L^i, $L_2^{ij} = \frac{1}{2}\{L^i, L^j\} - \frac{1}{3}\delta^{ij}L(L+1)$, which are the only possible structures which can carry the spatial index.

From Table I, one finds that the most general form of the mass operator in the presence of two-body quark interactions is a linear combination of 10 operators

$$
\begin{aligned}
&O_1 = T^2, \quad O_2 = \vec{S}_c^2, \quad O_3 = \vec{s}_1 \cdot \vec{S}_c, \quad O_4 = \vec{L} \cdot \vec{S}_c, \\
&O_5 = \vec{L} \cdot \vec{s}_1, \quad O_6 = L^i t_1^a G_c^{ia}, \quad O_7 = L^i g_1^{ia} T_c^a, \\
&O_8 = L_2^{ij}\{S_c^i, S_c^j\}, \quad O_9 = L_2^{ij} s_1^i S_c^j, \quad O_{10} = L_2^{ij} g_1^{ia} G_c^{ja}.
\end{aligned} \tag{3}
$$

This gives the most general form of the hadronic mass operator Eq. (1) of the negative parity $L = 1$ states allowing only two-body quark operators.

The $L = 1$ quark model states include the following SU(3) multiplets: two spin-$1/2$ octets $8_{1/2}$, $8'_{1/2}$, two spin-$3/2$ octets $8_{3/2}$, $8'_{3/2}$, one spin-$5/2$ octet $8'_{5/2}$, two decuplets $10_{1/2}$, $10_{3/2}$ and two singlets $1_{1/2}$, $1_{3/2}$. States with the same quantum numbers mix, and we define the relevant mixing angles in the nonstrange sector as

$$
\begin{aligned}
N(1535) &= \cos\theta_{N1} N_{1/2} + \sin\theta_{N1} N'_{1/2} \\
N(1650) &= -\sin\theta_{N1} N_{1/2} + \cos\theta_{N1} N'_{1/2}
\end{aligned} \tag{4}
$$

and analogous for the $J = 3/2$ states with the replacements $[N(1535), N(1650), N_{1/2}, N'_{1/2}, \theta_{N1}] \to [N(1520), N(1700), N_{3/2}, N'_{3/2}, \theta_{N3}]$.

The quark model basis states ($N_{1/2}$, $N'_{1/2}$) and ($N_{3/2}$, $N'_{3/2}$) have quark spin $S = (1/2, 3/2)$ which adds up together with the orbital angular momentum $L = 1$ to give $J = 1/2$ and $J = 3/2$, respectively. The mixing angles can be chosen to lie in the range $(0°, 180°)$ by appropriate phase redefinitions of the hadron states.

The hadronic mass operator in the quark model basis can be written as a linear combination of the 11 coefficients $\hat{M}_{ij}C_j = N_i^*$, where we represent the octets and decuplets by their nonstrange members $N^* = (N_{1/2}, N'_{1/2}, N_{1/2} - N'_{1/2}, N_{3/2}, N'_{3/2}, N_{3/2} - N'_{3/2}, N'_{5/2}, \Delta_{1/2,3/2}, \Lambda_{1/2,3/2})^T$. The coefficients $\hat{M}_{ij}$ can be extracted from the tables in Ref. [9].

The rank of the matrix $\hat{M}_{ij}$ is 9, which implies the existence of two universal relations among the 11 hadronic parameters (the masses of the 9 multiplets plus the two mixing angles) which must hold in any quark model containing only two-body quark interactions.

The first universal relation involves only the nonstrange hadrons, and requires only isospin symmetry. It can be expressed as a correlation among the two mixing angles θ_{N1} and θ_{N3} (see Fig. 1 left)

$$\frac{1}{2}[N(1535)+N(1650)]+\frac{1}{2}[N(1535)-N(1650)](3\cos 2\theta_{N1}+\sin 2\theta_{N1})-\frac{7}{5}[N(1520)+N(1700)]$$
$$+[N(1520)-N(1700)]\left[-\frac{3}{5}\cos 2\theta_{N3}+\sqrt{\frac{5}{2}}\sin 2\theta_{N3}\right]=-2\Delta_{1/2}+2\Delta_{3/2}-\frac{9}{5}N_{5/2}. \quad (5)$$

This expresses a correlation among the mixing angles (θ_{N1}, θ_{N3}) which is universal for any quark model containing only two-body interactions. This correlation holds also model independently in the $1/N_c$ expansion, up to corrections of order $1/N_c^2$, since for nonstrange states, the mass operator to order $O(1/N_c)$ [9,10] is generated by the operators in Eq. (3). An example of an operator which violates this correlation is $L^i g^{ja}\{S_c^j, G_c^{ia}\}$, which can be introduced by three-body quark forces.

On the same plot, we show also the values of the mixing angles obtained in several analyses of the $N^* \rightarrow N\pi$ strong decays and N^* hadron masses. The two black dots correspond to the mixing angles $(\theta_{N1}, \theta_{N3}) = (22.3°, 136.4°)$ and (22.3°, 161.6°) obtained from a study of the strong decays in Ref. [11]. The second point is favored by a $1/N_c$ analysis of photoproduction amplitudes Ref. [12]. The yellow square corresponds to the values used in Ref. [9,10] $(\theta_{N1}, \theta_{N3}) = (35.0°, 174.2°)$, and the triangle gives the angles corresponding to the solution 1′ in the large N_c analysis of Ref. [13] $(\theta_{N1}, \theta_{N3}) = (114.6°, 80.2°)$. All these determinations (except the triangle) are compatible with the ranges $\theta_{N1} = 0°$–35°, $\theta_{N3} = 135°$–180°. They are also in good agreement with the correlation Eq. (5) and provide no evidence for the presence of three-body quark interactions.

The second universal relation expresses the spin-weighted SU(3) singlet mass $\bar{\Lambda} = \frac{1}{6}(2\Lambda_{1/2} + 4\Lambda_{3/2})$ in terms of the nonstrange hadronic parameters

$$\bar{\Lambda}=\frac{1}{6}[N(1535)+N(1650)]+\frac{17}{15}[N(1520)+N(1700)]-\frac{3}{5}N_{5/2}(1675)-\Delta_{1/2}(1620)$$
$$-\frac{1}{6}[N(1535)-N(1650)](\cos 2\theta_{N1}+\sin 2\theta_{N1})+[N(1520)-N(1700)]\left(\frac{13}{15}\cos 2\theta_{N3}-\frac{1}{3}\sqrt{\frac{5}{2}}\sin 2\theta_{N3}\right). \quad (6)$$

The right-hand side of Eq. (6) is plotted as a function of θ_{N1} in the right panel of Fig. 1, where it can be compared against the experimental value $\bar{\Lambda} = 1481.7 \pm 1.5$ MeV. Allowing for SU(3) breaking effects ∼100 MeV, this constraint is also compatible with the range for θ_{N1} obtained above from direct determinations of the mixing angles.

Combining the Eqs. (5) and (6) gives a determination of the mixing angles from hadron masses alone, in contrast to their usual determination from $N^* \rightarrow N\pi$ decays [14]. The dark shaded (green) area in Fig. 1 shows the allowed region for (θ_{N1}, θ_{N3}) compatible with a positive SU(3) breaking correction in $\bar{\Lambda}$ of 100 ± 30 MeV. One notes a good agreement between this determination of the mixing angles and that from $N^* \rightarrow N\pi$ strong decays.

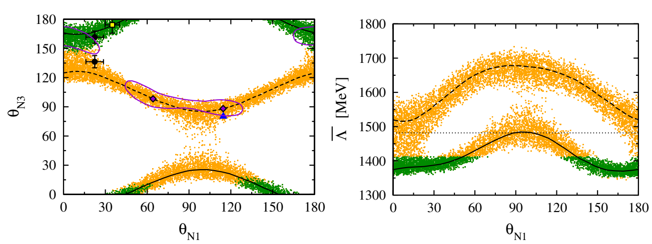

FIG. 1 (color online). Left: correlation in the (θ_{N1}, θ_{N3}) plane in the quark model with the most general two-body quark interactions. Right: prediction for the spin-weighted $\bar{\Lambda}$ mass in the SU(3) limit as a function of the θ_{N1} mixing angle, corresponding to the two solutions for θ_{N3}. The dark shaded (green) points correspond to $\bar{\Lambda} = \bar{\Lambda}_{\rm exp} - (100 \pm 30)$ MeV, with $\bar{\Lambda}_{\rm exp} = 1481.7 \pm 1.5$ MeV.

We derive next constraints on the spin-flavor structure of the quark interaction, which can discriminate between models of effective quark interactions. There are two popular models used in the literature, see Ref. [3] for a discussion in the context of the states considered here. The first model is the one-gluon-exchange model (OGE) [1] which includes operators in Table I without isospin dependence. Expressed in terms of the operator basis O_{1-10} this gives the constraints

$$C_1 = C_6 = C_7 = C_{10} = 0. \quad (7)$$

An alternative to the OGE model is the Goldstone boson exchange model (GBE) [8]. In this model, quark forces are mediated by Goldstone boson exchange, and the quark Hamiltonian contains all the operators in Table I which contain the flavor-dependent factor $t_i^a t_j^a$. The coefficients of the hadronic Hamiltonian Eq. (1) satisfy the constraints ($F = 3$ is the number of light quark flavors)

$$C_1 = \frac{F}{4}C_3, \qquad C_5 = C_9 = 0. \quad (8)$$

We would like to determine the coefficients C_i in the most general case and compare their values with the predictions of the two models Eqs. (7) and (8). However, since the rank of $\hat{M}_{ij}$ is 9, only the following combinations of coefficients can be determined from the available data: C_0, $C_1 - C_3/2$, $C_2 + C_3$, C_4, C_5, C_6, C_7, $C_8 + C_{10}/4$, $C_9 - 2C_{10}/3$. In particular, as the coefficients of the spin-orbit interaction terms C_{4-7} can be determined, we propose to use their values to discriminate between different models of quark interaction.

They can be compared with the hierarchy expected in each model. In the OGE model, the flavor-dependent operators have zero coefficients $C_{6,7} \sim 0 \ll |C_{4,5}|$, while in the GBE model, the spin-orbit interaction of the excited quark vanishes $C_5 \sim 0 \ll |C_{4,6}|$.

The coefficient $C_5 = 75.7 \pm 2.7$ MeV is fixed by the $\Lambda_{3/2} - \Lambda_{1/2}$ splitting [10]. This indicates the presence of the operators $s_i \pm s_j$ in the quark Hamiltonian, which is compatible with the OGE model.

A suppression of the coefficients $C_{6,7}$ would be further evidence for the OGE model. We show in Fig. 2 the coefficients of the spin-orbit operators $C_{6,7}$ as functions of θ_{N1}. Within errors, small values for C_7 are still allowed; however, no suppression is observed for C_6. This indicates the presence of the operators $(s_i \pm s_j)t_i^a t_j^a$ in the quark Hamiltonian. These results show that the quark Hamiltonian is a mix of the OGE and GBE interactions.

In the pure OGE model Eq. (7), the 7 nonvanishing coefficients C_i can be determined from the 7 nonstrange N^*, Δ^* masses (assuming only isospin symmetry but no specific form of the wave functions). This fixes the mixing angles, and the $\Lambda_{3/2} - \Lambda_{1/2}$ splitting, up to a 2-fold ambiguity. The allowed region for mixing angles is shown as the encircled (violet) region in Fig. 1 left, and the central values as diamonds $(\theta_{N1}, \theta_{N3}) = (64.2°, 98.2°)$, $(114.5°, 88.2°)$. Note that they are different from the angles obtained in the Isgur-Karl model $(31.7°, 173.6°)$ in Refs. [2,15].

The encircled (violet) region near $\theta_{N1} \sim 0$ is consistent with the determinations from strong decays and from the SU(3) universal relation Eq. (6), but is ruled out by the prediction for the Λ splitting, in agreement with the nonzero value of C_6 that can be read off from Fig. 2. This implies that the pure OGE model is disfavored. Note that this argument neglects possible long-distance contributions to the Λ splitting, due to the proximity of the $\Lambda(1405)$ to the KN threshold. Such threshold effects are not described by the quark Hamiltonian Eq. (2), and their presence could invalidate the prediction of the Λ splitting in the OGE model.

We discussed in this Letter the predictions of the constituent quark model with the most general spin-flavor two-body quark interactions, using a new relation between the spin-flavor structure of the quark interactions and the hadronic mass operator [5]. We find two universal relations among the hadronic parameters of the negative parity excited baryons, valid in any model with two-body quark interactions. They fix the mixing angles, and deviations from them can probe the presence of three-body quark interactions. We propose new constraints on the relative importance of the different spin-flavor structures in the quark interaction, without imposing any theoretical prejudice on the form of the quark interaction Hamiltonian and the hadronic wave functions. The precision of these constraints is limited by the uncertainty in the hadronic masses and mixing angles. In principle, such information can also be gained from lattice QCD, where the mixing angles can be related to the relative overlaps of the interpolating fields for the excited states.

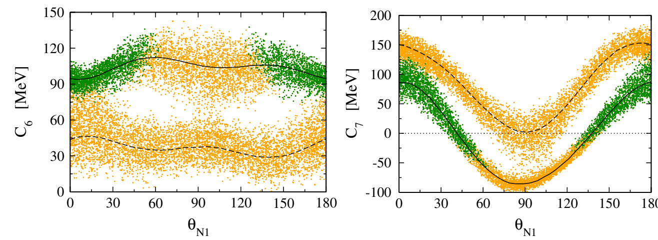

FIG. 2 (color online). The coefficients of the spin-orbit operators $C_{6,7}$ as functions of the mixing angle θ_{N1}, in the quark model with the most general two-body interactions. The dark shaded (green) area is obtained by imposing the $\bar{\Lambda}$ constraint Eq. (6).

We acknowledge useful discussions with W. Melnitchouk and D. Richards.

[1] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).
[2] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).
[3] N. Isgur, Phys. Rev. D **62**, 054026 (2000).
[4] R. L. Jaffe, Phys. Rev. D **15**, 281 (1977).
[5] D. Pirjol and C. Schat, Phys. Rev. D **78**, 034026 (2008).
[6] H. Collins and H. Georgi, Phys. Rev. D **59**, 094010 (1999).
[7] T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[8] L. Y. Glozman and D. O. Riska, Phys. Rep. **268**, 263 (1996).
[9] C. E. Carlson *et al.*, Phys. Rev. D **59**, 114008 (1999).
[10] C. L. Schat, J. L. Goity, and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); Phys. Rev. D **66**, 114014 (2002).
[11] J. L. Goity, C. Schat, and N. Scoccola, Phys. Rev. D **71**, 034016 (2005).
[12] N. N. Scoccola, J. L. Goity, and N. Matagne, Phys. Lett. B **663**, 222 (2008).
[13] D. Pirjol and C. Schat, Phys. Rev. D **67**, 096009 (2003); AIP Conf. Proc. **698**, 548 (2004).
[14] A. J. G. Hey, P. J. Litchfield, and R. J. Cashmore, Nucl. Phys. **B95**, 516 (1975); D. Faiman and D. E. Plane, Nucl. Phys. **B50**, 379 (1972).
[15] J. Chizma and G. Karl, Phys. Rev. D **68**, 054007 (2003).