Comment on "Turbulence-Condensate Interaction in Two Dimensions"

In a recent Letter [1], Xia *et al.* reported on experiments on two-dimensional turbulence which they interpreted in the light of the theory of Kraichnan [2]. According to this theory, turbulent energy which is injected at a forcing wave number k_f in a two-dimensional domain is transferred to smaller wave numbers, corresponding to larger spatial scales. At wave numbers between k_f and the wave number corresponding to the size of the confining domain, the theory of Kraichnan predicts a kinetic energy spectrum $E(k) = C\Pi^{2/3}k^{-5/3}$, where C is a constant and Π is the flux of energy from large to small wave numbers. The range $k \in [k_1, k_2]$ where the energy spectrum has this form is called the inertial range. In the experiments by Xia *et al.*, vortices were electromagnetically generated in a thin stratified layer of a NaCl solution, at forcing wave number $k_f \approx 400\ \mathrm{m}^{-1}$. Two experiments were reported: one with a strong mean flow (strong condensate) and one with a weak mean flow (weak condensate). In both experiments, the measured energy spectrum was found to be consistent with a $k^{-5/3}$ dependence between wave numbers k_1 and k_f, where k_1 was determined to 125 and 80 m^{-1} in the strong and the weak condensate, respectively.

A necessary condition for the existence of an inertial range is that the viscous dissipation in the inertial range is much smaller than the energy flux Π. The inertial range dissipation in the experiments can be written as the sum of the dissipation due to bulk viscosity

$$\epsilon_b = 2\nu \int_{k_1}^{k_2} k^2 E(k) dk = \frac{3}{2}\nu C \Pi^{2/3}(k_2^{4/3} - k_1^{4/3}) \quad (1)$$

and the dissipation due to linear bottom drag

$$\epsilon_d = \alpha \int_{k_1}^{k_2} E(k) dk = \frac{3}{2}\alpha C \Pi^{2/3}(k_1^{-2/3} - k_2^{-2/3}), \quad (2)$$

where ν is the kinematic viscosity and α is the drag coefficient. The energy flux was estimated by measuring third-order moments of velocity increments and using an analytical high Reynolds number formula [3], relating such moments to the energy flux. Based on these estimates and the measured energy spectra, the constant C was determined to 7.0 and 5.8 in the strong and weak condensate, respectively. Xia *et al.* [1] do not provide the estimated values of the energy flux in their report. In a private communication [4] to the author, the values $\Pi = 8.64 \times 10^{-7}\ \mathrm{m}^2\,\mathrm{s}^{-3}$ and $\Pi = 7.0 \times 10^{-6}\ \mathrm{m}^2\,\mathrm{s}^{-3}$ were given, for the strong and weak condensate, respectively. The drag coefficient α was measured to 0.15 and 0.3 s^{-1} in the strong and weak condensate, respectively. Choosing the upper limit of the inertial range as $k_2 = 300\ \mathrm{m}^{-1}$ and assuming that $\nu = 1.0 \times 10^{-6}\ \mathrm{m}^2\,\mathrm{s}^{-1}$ for the NaCl solution, we can now calculate the two contributions to the inertial range dissipation as $\epsilon_b \approx 1.5\Pi$ and $\epsilon_d \approx 2.9\Pi$ in the strong condensate and $\epsilon_b \approx 0.8\Pi$ and $\epsilon_d \approx 4.3\Pi$ in the weak condensate. In both experiments, the linear drag dissipation in the inertial range is considerably larger than the energy flux through the same range. In the strong condensate the inertial range dissipation due to bulk viscosity is larger than the energy flux, and in the weak condensate the same quantity is of the same order as the energy flux. These results are not consistent with the existence of an inertial range.

In conclusion, there are either errors in the given values of the quantities measured by Xia *et al.* or it is not possible to interpret their experiments in light of the theory of two-dimensional turbulence.

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[4] The values of Π used here are 8 times larger than the values first given to the author in a private communication from Michael Shats. In a subsequent communication, the author was informed that the values in the first communication should be multiplied by a factor of 8, due to a data evaluation error which just had been discovered.

