Fractional Statistics and Quantum Scaling Properties of the Hubbard Chain with Bond-Charge Interaction

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We present a detailed study of the ground state and low-temperature properties of the integrable Hubbard model with bond-charge interaction, including its conducting properties and scaling behavior near the U-driven quantum phase transitions. Remarkably, the model displays fractional statistical properties, which enlighten the nature of various physical properties, such as the fractional elementary excitations, and give rise to a disordered condensate and phase separation in k space, as well as to a topological change in the generalized Fermi surface at half filling.

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Several variants of the Hubbard model with a hopping amplitude t that depends on the presence of particles of opposite spins on neighboring sites, i.e., models with correlated hopping or bond-charge interaction, in addition to the on-site coupling U, have been proposed to account for special features of materials and phenomena. In particular, we mention polyacetylene [1], high-temperature superconductors [2], superconductivity [3–7], including other properties of quasi-one-dimensional systems, e.g., Bechgaard salts [8,9], metallic ferromagnetism and metal-insulator transition [7,10], and entanglement properties in quantum information [11].

The simplest case one may consider is defined on a linear chain of L sites [4–12]:

$$\mathcal{H}_x = -t\sum_{\langle i,j\rangle,\sigma}[1 - x(n_{i\bar{\sigma}} + n_{j\bar{\sigma}})]c_{i\sigma}^\dagger c_{j\sigma} + U\sum_i n_{i\uparrow}n_{i\downarrow} - h\sum_i(n_{i\uparrow} - n_{i\downarrow}), \tag{1}$$

where $c_{i\sigma}$ ($c_{i\sigma}^\dagger$) are electron annihilation (creation) operators, $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$, $\sigma = \uparrow, \downarrow$, $\bar{\sigma} = -\sigma$, h is the magnetic field, and x is the bond-charge interaction parameter. Despite the integrability of the model at $x = 1$ [4–7,9,11,12], some relevant features still lack a proper description.

In this work, we undertake a detailed study of the ground state (GS) and low-temperature ($\beta = 1/k_BT$) properties of $\mathcal{H}_{x=1}$, including its conducting properties and scaling behavior near the U-driven quantum phase transitions in the rich (U/t, n) phase diagram, where n is the particle density. Remarkably, we show that $\mathcal{H}_{x=1}$ displays fractional statistical properties, which enlighten our understanding of this strongly correlated electron system, in particular, of the underlying nature of various physical properties, such as the elementary excitations, the disordered condensate, the phase separation in k space, and the predicted [5] exotic nonconducting line at $n = 1$, for any U, in spite of a vanishing charge gap.

Fractional exclusion statistics.—It has been noticed [9] that the correlated hopping term allows to split the four possible local states of the Hubbard model into two disjoint sets: $A = \{|\uparrow\rangle, |\downarrow\rangle\}$ and $B = \{|\uparrow\downarrow\rangle, |0\rangle\}$. By exploiting fundamental aspects of the Hilbert space in light of these Sutherland species, it has been shown that the spectrum [7,9] and the grand-partition function [9] read $E = \sum_k \varepsilon_k n_k + UN_{\uparrow\downarrow} - h(N_\uparrow - N_\downarrow)$, $k = 2\pi m/L (m = -L/2 + 1, \ldots, L/2)$, where $n_k = 0, 1$, $\varepsilon_k = -2t\cos k$, $N_\uparrow$ ($N_\downarrow$) is the number of electrons with spin $\uparrow$ ($\downarrow$) at singly occupied sites, $N_{\uparrow\downarrow}$ is the number of doubly occupied sites and $N = N_\uparrow + N_\downarrow + 2N_{\uparrow\downarrow}$ is the total number of particles; $\mathcal{Z} = [1 + e^{\beta(2\mu - U)}]^L \prod_k [1 + e^{-\beta(\varepsilon_k - \mu^*)}]$, where μ is the chemical potential and $\mu^* = \mu + \frac{1}{\beta}\ln[\frac{2\cosh(\beta h)}{1+e^{\beta(2\mu-U)}}]$ is the renormalized chemical potential. Above, apart from the Zeeman term, E is given by the spinless fermion contribution [7,9] plus a Coulomb term with conserved $N_{\uparrow\downarrow}$.

Surprisingly, we can show that the grand-canonical free energy, $\Omega(\beta, \mu, h)$, reads

$$\Omega = -\frac{1}{\beta}\sum_k \ln(1 + e^{-\beta\varepsilon_{k,1}} + e^{-\beta\varepsilon_{k,2}} + e^{-\beta\varepsilon_{k,3}}), \tag{2}$$

where $\varepsilon_{k,1} = \varepsilon_k - h - \mu$, $\varepsilon_{k,2} = \varepsilon_k + h - \mu$, and $\varepsilon_{k,3} = U - 2\mu$. Therefore, insofar as the thermodynamic properties are concerned, the $\mathcal{H}_{x=1}$ model is mapped onto an ideal gas of three species of exclusons, obeying fractional *exclusion* statistics [13,14]. Their average occupation number $\langle n_{k,\alpha}\rangle$, $\alpha = 1, 2, 3$, where $\langle N_\uparrow\rangle \equiv \sum_k\langle n_{k,1}\rangle$, $\langle N_\downarrow\rangle \equiv \sum_k\langle n_{k,2}\rangle$, $\langle N_{\uparrow\downarrow}\rangle \equiv \sum_k\langle n_{k,3}\rangle$, are derived by solving the set of equations $\langle N_\uparrow\rangle + \langle N_\downarrow\rangle + 2\langle N_{\uparrow\downarrow}\rangle = \frac{1}{\beta}\frac{\partial \ln Z}{\partial\mu}$, $\langle N_\uparrow\rangle - \langle N_\downarrow\rangle = \frac{1}{\beta}\frac{\partial \ln Z}{\partial h}$, and $\langle N_{\uparrow\downarrow}\rangle = -\frac{1}{\beta}\frac{\partial \ln Z}{\partial U}$. In fact, we find

$$\langle n_{k,\alpha=1,2}\rangle = \frac{1 \pm \tanh(\beta h)}{2}\langle n_k\rangle, \tag{3}$$

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$$\langle n_{k,3}\rangle = \frac{1-\langle n_k\rangle}{e^{\beta(U-2\mu)}+1}, \tag{4}$$

where $\langle n_k\rangle = 1/[e^{\beta(\varepsilon_k-\mu^*)}+1] = \langle n_{k,1}\rangle + \langle n_{k,2}\rangle$ is the renormalized Fermi distribution, in agreement with the spinless Fermi gas picture [4,7,9]. However, here the mapping is clearly associated with the fractional character of the species 1 and 2, as evidenced by the factor $1/2$ multiplying the average occupation number per orbital, $\langle n_k\rangle$. Besides, the factor $1-\langle n_k\rangle$ in Eq. (4) excludes the possibility of simultaneous occupation in k space of species 3 and species 1 or 2. A noticeable feature of the mapping is that the statistical matrix is exactly the one found for the Hubbard model with standard hopping and infinite-range interaction [15,16]:

$$[g]_{kk';\alpha\alpha'} = \delta_{kk'}\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}, \tag{5}$$

although in that case the third fractional species displays dispersive behavior: $\varepsilon_{k,3}\to 2\varepsilon_k + U - 2\mu$. Further, we can also show that (5) is the statistical matrix for a related model with pair hopping [17], with $\varepsilon_{k,3}\to\varepsilon_k + U - 2\mu$.

The proof of the mapping follows from the fact that Ω can be written as $\Omega = -\frac{1}{\beta}\sum_{k,\alpha}\ln(1+w_{k,\alpha}^{-1})$, where $w_{k,\alpha}$ satisfy the Haldane-Wu distribution $(1+w_{k,\alpha})\prod_{k',\lambda}(\frac{w_{k',\lambda}}{1+w_{k',\lambda}})^{g_{k'k;\lambda\alpha}} = e^{\beta\varepsilon_{k,\alpha}}$, with $w_{k,1} = e^{\beta\varepsilon_{k,1}}$, $w_{k,2} = (1+w_{k,1})e^{\beta(\varepsilon_{k,2}-\varepsilon_{k,1})}$, and $w_{k,3} = (1+w_{k,2})e^{\beta(\varepsilon_{k,3}-\varepsilon_{k,2})}$. We also verify that the fractional species $\langle n_{k,\alpha}\rangle$ satisfy the exclusion relation [13,14] $\langle n_{k,\alpha}\rangle w_{k,\alpha} = 1 - \sum_{k',\lambda} g_{kk';\alpha\lambda}\langle n_{k',\lambda}\rangle$, where $\langle n_{k,\alpha}\rangle = \frac{e^{-\beta\varepsilon_{k,\alpha}}}{1+\sum_{\lambda=1}^{3}e^{-\beta\varepsilon_{k,\lambda}}}$, in agreement with Eqs. (3) and (4). In particular, the spectrum of $\mathcal{H}_{x=1}$ can be written in terms of the fractional elementary excitations (FEE): $E-\mu N = \sum_{k,\alpha}\varepsilon_{k,\alpha}\langle n_{k,\alpha}\rangle$.

Ground state and low-T properties.—In order to clarify the zero-field GS phases of the model, it will prove very helpful to examine the dependence of the FEE properties on U and n (the scaling properties of the transitions shall be examined below). For $U < -4t$ ($\equiv U_{c1}$), the system is a disordered Mott insulator, with $E/L = Un/2$, $\mu = U/2$, and charge gap $\mu_+ - \mu_- \equiv \Delta_1 = U_{c1} - U$ [7], despite the nonzero η-pairing correlation $\lim_{|i-j|\to\infty}\langle\eta_i^\dagger\eta_j\rangle = (\langle N_{\uparrow\downarrow}\rangle/L)^2$, where $\eta = \sum_j c_{j\downarrow}c_{j\uparrow}$, [4–7]. Further, as shown in Fig. 1(a) for $U = -6t$ and $n = 1$, the gap $\Delta_1/2 = t$ separates the disordered condensate of exclusons 3, defined by the flat energy level $\varepsilon_{k,3} = (U-2\mu)_{\mu=U/2} = 0$ and $\langle n_{k,3}\rangle = n/2$, $\forall\ k$, from the bottom of the empty degenerate dispersive bands of exclusons 1 and 2 $\varepsilon_{k,1} = \varepsilon_{k,2} = \varepsilon_k - U/2$. In fact, Δ_1 is the excitation energy, at fixed N, of the elementary process: excluson $3\to$ excluson $1+$ excluson 2. We also emphasize that, due to the fractional occupation of species 1 and 2 [see Eq. (3)], the twofold spin degenerate dispersive bands are equivalent to a single band of spinless fermions. In the disordered condensate, static ($\uparrow\downarrow$)-pairs and holes are randomly distributed, both in direct and reciprocal spaces, with entropy $S = -(\partial F/\partial T)_{N,T=0} = k_B\ln[L!/\langle N_{\uparrow\downarrow}\rangle!(L-\langle N_{\uparrow\downarrow}\rangle)!]$, where $F = \mu N + \Omega$ is the Helmholtz free energy and $\mu \simeq \frac{U}{2} + \frac{k_BT}{2}\ln(\frac{n}{2-n})$. Further, the charge compressibility is singular $\kappa^{-1} = \partial^2(E/L)/\partial n^2 = 0$ (adding exclusons 3 costs no energy). In this phase, thermal excitation of exclusons 3 dominates several low-T responses. In particular, it is worth mentioning that the canonical specific heat, $C = T(\partial S/\partial T)_N$, reads

$$\frac{C}{L} \simeq \frac{k_B\Delta_1^2}{8}\sqrt{\frac{n(2-n)}{\pi t(k_BT)^3}}\exp\left(-\frac{\Delta_1}{2k_BT}\right). \tag{6}$$

For $U > 4t$ and $n = 1$ the system is a Mott insulator and a plot of the FEE with a gap $\Delta_2/2 = (U-4t)/2$ ($=t$) is shown in Fig. 1(b), where species 1 and 2 fill the band of spinless fermions. It is interesting to notice that the specific heat inside the Mott phase [9] $\frac{C}{L} \simeq \frac{k_B\Delta_2^2}{8\sqrt{\pi t(k_BT)^3}}\exp(-\frac{\Delta_2}{2k_BT})$, is obtained from Eq. (6) by setting $n = 1$ and replacing $U\to -U$.

For $U_{c1} < U < -4t\cos(n\pi)$ ($\equiv U_{c2}$), the system is metallic, although at $n = 1$ the net dc current is zero [5]. In any case, by eliminating μ in favor of $\mu(n,T)$, the three fractional species coexist in equilibrium with GS entropy, due to spin degeneracy and disorder, given by

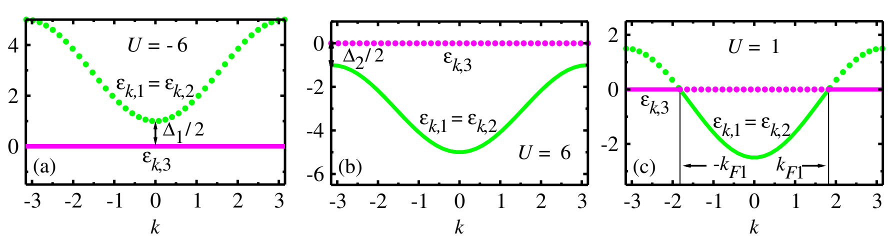

FIG. 1 (color online). Fractional elementary excitations at $n = 1$, $h = 0$, and $t\equiv 1$. (a) Disordered condensate at $\varepsilon_{k,3} = 0$ and $U = -6$; the empty degenerate band of exclusons 1 and 2 are shown above the gap $\Delta_1/2 = 1$. (b) Mott insulator phase for $U = 6$; a gap $\Delta_2/2 = 1$ splits the full effective spinless band from the empty flat level $\varepsilon_{k,3} = 0$. (c) Metallic phase for $U = 1$, characterized by the coexistence of the three fractional species; at $n = 1$ the net dc current is zero [5].

$$S = -\left(\frac{\partial F}{\partial T}\right)_{N,T=0} = k_B \ln\left[\frac{2^{(\langle N_\uparrow\rangle + \langle N_\downarrow\rangle)} L_{\rm eff}!}{\langle N_{\uparrow\downarrow}\rangle!(L_{\rm eff} - \langle N_{\uparrow\downarrow}\rangle)!}\right], \quad (7)$$

where $L_{\rm eff} = L - (\langle N_\uparrow\rangle + \langle N_\downarrow\rangle)$, valid in all phases. The total energy per site is $\frac{E}{L} = -\frac{2t}{\pi}\sqrt{1-(U/4t)^2} + \frac{U}{2} \times [n - \frac{1}{\pi}\arccos(-U/4t)]$ [7], and $\mu = U/2$ as in the former case. A plot of the FEE is shown in Fig. 1(c) for $U = t$ and $n = 1$; we stress that $\kappa^{-1} = 0$ is the signature of the phase separation in k space. In fact, as clearly shown in Fig. 1(c), species 1 and 2 are restricted to the interval $[-k_{F1}, k_{F1}]$ where $k_{F1} = \pi(\langle N_\uparrow\rangle + \langle N_\downarrow\rangle)/L$, whereas the disordered flat level of exclusons 3 is confined to k vectors of the complementary set $(-\pi, -k_{F1}) \cup (k_{F1}, \pi]$. Moreover, for $U > U_{c2}$ and $n < 1$ (the case $n > 1$ is obtained by particle-hole symmetry), the GS remains metallic with $\kappa^{-1} \neq 0$ since species 3 is absent, the spinless band is partially filled and the system is well described by the infinite-U Hubbard chain [4].

Topology of the generalized Fermi surface.—In Ref. [5], it was shown that the GS Drude weight D_c at $n = 1$ is exactly zero, notwithstanding the absence of charge gap for $-4t < U < 4t$. In fact, we can decompose the current density $J_{x=1} = -(\partial \mathcal{H}_{x=1}/\partial\phi)_{\phi=0}$ [after the Peierls transformation $c_{j\sigma} \to e^{-ij\phi}c_{j\sigma}$ in (1)] into two components: $J_{x=1} = it\sum_{j,\sigma}\{(1-2n_{j\bar\sigma})(1-n_{j\bar\sigma}-n_{j+1\bar\sigma}) \times c^\dagger_{j+1\sigma}c_{j\sigma} - (1-2n_{j+1\bar\sigma})(1-n_{j\bar\sigma}-n_{j+1\bar\sigma})c^\dagger_{j\sigma}c_{j+1\sigma}\} + 2it\sum_{j,\sigma}n_{j\bar\sigma}n_{j+1\bar\sigma}(c^\dagger_{j\sigma}c_{j+1\sigma} - c^\dagger_{j+1\sigma}c_{j\sigma}) \equiv J_S + J_D$. J_S is associated with the transport of exclusons 1 and 2 only and mapped onto the current density of the spinless Fermi gas, while excluson 3 is transported by J_D in opposite direction to J_S, thus nullifying the net effect at $n = 1$. This confirms that the vanishing behavior of $D_c \sim (n-1)^2$ ($-4t < U < 4t$, $n \to 1$) [5] does not signal a standard density-driven metal-insulator transition (MIT) [18], once we also verify that F is analytic and obeys a Sommerfeld-like expansion in the vicinity of $n = 1$. However, we must conciliate the above results with the FEE plot shown in Fig. 1(c).

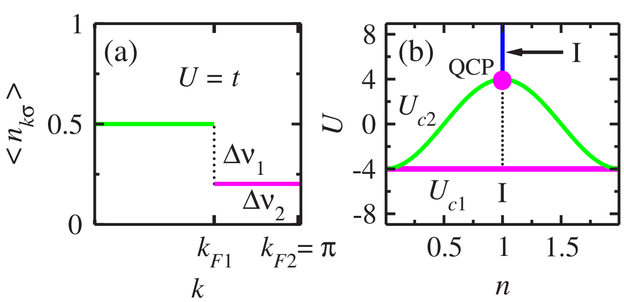

FIG. 2 (color online). (a) Fractional average number of electrons of spin $\sigma = \uparrow, \downarrow$ at $n = 3/4$ and $U = t$, displaying step singularity at the wave vector k_{F1}. (b) GS phase diagram in $h = 0$ and $t \equiv 1$. Capital letters I denote insulating phases, otherwise the GS is metallic. However, at $n = 1$, $D_c = 0$ (dotted line). Line U_{c1} and the QCP are associated with MIT, while U_{c2} separates distinct metallic phases.

Following Ref. [19], we make use of concepts of generalized Fermi surface and Luttinger theorem proposed for electronic systems exhibiting non-Fermi liquid behavior, including fractionalization effect on the average particle number, i.e., $\langle n_{k\sigma}\rangle < 1$ at $T = 0$. The Fermi surface of $\mathcal{H}_{x=1}$ is defined by the k vectors that mark singularities at $T = 0$ in $\langle n_{k\uparrow}\rangle \equiv \langle n_{k,1}\rangle + \langle n_{k,3}\rangle$ and $\langle n_{k\downarrow}\rangle \equiv \langle n_{k,2}\rangle + \langle n_{k,3}\rangle$ (in our case, step discontinuities). Figure 2(a) shows $\langle n_{k\sigma}\rangle$ for $U = t$ and $n = 3/4$, where the Fermi surface is given by the k set $\{\pm\pi, \pm k_{F1}\}$. The indices [19] characterizing the singularities read

$$\Delta\nu_1 = \lim_{\eta\to 0^+}[\langle n_{k_{F1}-\eta,\sigma}\rangle - \langle n_{k_{F1}+\eta,\sigma}\rangle] = \frac{\pi(1-n)}{2(\pi - k_{F1})},$$
$$\Delta\nu_2 = \lim_{\eta\to 0^+}\langle n_{\pi-\eta,\sigma}\rangle = \frac{n\pi - k_{F1}}{2(\pi - k_{F1})}, \quad (8)$$

which depend on n and U. Further, in terms of the Fermi surface topology, the total number of particles of spin σ is given by the generalized Luttinger theorem [19]:

$$\frac{2\pi N(\sigma)}{L} = \int_{-\pi}^{\pi}\langle n_{k\sigma}\rangle dk = 2[(\Delta\nu_1)k_{F1} + (\Delta\nu_2)\pi]. \quad (9)$$

However, once $\Delta\nu_1 = 0$ and $\Delta\nu_2 = 1/2$ at $n = 1$, the Fermi surface undergoes a topological change since it is now reduced to the vectors $k = \pm\pi$ with $\langle n_{k\uparrow}\rangle = \langle n_{k\downarrow}\rangle = 1/2$, $\forall\ k$. Thus, on average, there is one carrier per orbital k and the system is nonconducting. We can also predict that D_c is zero for any T. In fact, $\langle n_{k\uparrow}\rangle = \langle n_{k\downarrow}\rangle = \frac{e^{-\beta(\varepsilon_k-\mu)} + e^{-\beta(U-2\mu)}}{1+2e^{-\beta(\varepsilon_k-\mu)} + e^{-\beta(U-2\mu)}} = \frac{1}{2}$, $\forall\ k$, since at half filling and $T > 0$, $\mu = U/2$ exactly.

Scaling properties.—We now provide a scaling analysis of the U-driven quantum phase transitions exhibited by $\mathcal{H}_{x=1}$. In the vicinity of a quantum critical point (QCP), the singular part of F, $F_{\rm sing}(T, h; U - U_c)$, can be written [18] either as $F_{\rm sing} = |U - U_c|^{2-\alpha}F_U(\frac{T}{|U-U_c|^{\nu z}}, \frac{h}{|U-U_c|^{\beta+\gamma}})$ or as $F_{\rm sing} = T^{1+(d/z)}F_T(\frac{|U-U_c|}{T^{1/(\nu z)}}, \frac{h}{T})$ if $k_B T$ dominates the energy scale, where F_U, F_T are scaling functions, and α, β, γ, ν, z are critical exponents satisfying the relations $\nu z = \beta + \gamma$ and $2 - \alpha = \nu(d + z)$.

In Fig. 2(b), the various phases of $\mathcal{H}_{x=1}$ are depicted. The line $U = U_{c1}$ is the quantum critical line of the U-driven MIT, which is attained by letting $\Delta_1 \to 0^+$ in Fig. 1(a) at fixed n, with order parameter $m_{U_{c1}} = \frac{(\langle N_\uparrow\rangle + \langle N_\downarrow\rangle)_{T=0}}{L}$. In the metallic side and $h = 0$, we find $F_{\rm sing} = -\frac{(U-U_{c1})^{3/2}}{\sqrt{t}}F_{U_{c1}}(x)$, $x = k_B T/(U - U_{c1})$, which implies the critical exponents of the QCP of the spinless Fermi gas in $d = 1$ [18] $\alpha = \beta = \gamma = \nu = 1/2$ and $z = \delta = 2$ (see below). The exponent $z = 2$ is confirmed by noticing that $F_{\rm sing}$ is dominated by gapless excitations a round the bottom of the dispersive band of exclusons 1 and 2: $\varepsilon_{k,\alpha=1,2} = (\varepsilon_k - U/2)_{U=U_{c1}} \simeq tk^2 \sim k^z$. Further, the GS critical behavior of the order parameter reads $m_{U_{c1}}(h=0; U-U_{c1}) \simeq \frac{1}{\pi}(\frac{U-U_{c1}}{t})^{1/2}$, $U \rightarrow U_{c1}^{+}$; $m_{U_{c1}}(h; U=U_{c1}) \simeq \frac{1}{\pi}(\frac{h}{t})^{1/2}$. It is also worth mentioning that the QCP of the MIT attained by letting $\Delta_2 \rightarrow 0^{+}$ at fixed $n=1$ in Fig. 1(b), and order parameter $m_{\rm QCP} = \frac{\langle N_{\uparrow\downarrow}\rangle_{T=0}}{L}$, is also in the same universality class of the quantum critical line $U=U_{c1}$. On the other hand, when k_BT dominates the energy scale, the quantum critical behavior of the specific heat along the line $U=U_{c1}$ reads $\frac{C}{L} \simeq \frac{3k_B}{2\pi} \times (\frac{k_BT}{t})^{1/2} \int_0^{+\infty} \ln[1+\sqrt{n(2-n)}e^{-x^2}]dx \sim T^{d/z}$. Further, the above expression is also valid for the QCP at $U=4t$ by setting $n=1$.

The quantum critical line $U=U_{c2}$ separates distinct metallic phases with order parameter $m_{U_{c2}} = \frac{\langle N_{\uparrow\downarrow}\rangle_{T=0}}{L}$. By assuming that $U_{c2}-U>0$, the scaling part of F can be found by a Sommerfeld-like expansion $F_{\rm sing} = -\frac{(U_{c2}-U)^2}{t} F_{U_{c2}}(x)$, $x=[k_BT/(U_{c2}-U)]\ln[(U_{c2}-U)/t]$, $h=0$, which implies $\alpha=\gamma=0$ and $\beta=\nu=z=\delta=1$ (see below), with logarithmic corrections consistent with $d=z$ $(=1)$ [18]. In contrast with the previous case, $F_{\rm sing}$ is dominated by gapless excitations around $k=k_{F1}=\pi(\langle N_{\uparrow}\rangle + \langle N_{\downarrow}\rangle)_{U=U_{c2}}/L = n\pi$ in the dispersive band of exclusons 1 and 2: $\varepsilon_{k,\alpha=1,2} = (\varepsilon_k - U/2)_{U=U_{c2}} \simeq (2t\sin n\pi)(k-n\pi) \sim (k-n\pi)^z$. The GS critical behavior of $m_{U_{c2}}$ reads $m_{U_{c2}}(h_{U_{c2}}=0; U-U_{c2}) \simeq \frac{U_{c2}-U}{8\pi t\sin(n\pi)}$, $U \rightarrow U_{c2}^{-}$; $m_{U_{c2}}(h_{U_{c2}}; U=U_{c2}) \simeq \frac{h_{U_{c2}}}{8\pi t\sin(n\pi)}$, $h_{U_{c2}} \rightarrow 0^{+}$, where $h_{U_{c2}}$ is the scaling field coupled to $m_{U_{c2}}$ through the term $-h_{U_{c2}}\sum_i n_{i\uparrow}n_{i\downarrow}$ added to $\mathcal{H}_{x=1}$. In addition, the quantum critical behavior of the specific heat along the critical line $U=U_{c2}$ reads $\frac{C}{L} \simeq \frac{k_B^2T}{8\pi t\sin(n\pi)}\ln^2(\frac{k_BT}{t}) \sim T^{d/z}$.

Finally, the scaling prediction $L^{-1}(\partial S/\partial U)_N \sim |U_c - U|^{1-\alpha-\nu z}$, which manifests itself as $[\simeq \frac{k_B \ln[n(2-n)]}{4\pi\sqrt{2t(U-U_{c1})}}$, $U \rightarrow U_{c1}^{+}]$ and $[\simeq \frac{k_B}{8\pi t\sin(n\pi)} \ln(\frac{U_{c2}-U}{t})$, $U \rightarrow U_{c2}^{-}]$, is in agreement with both the power law and logarithmic singularities derived in the framework of quantum-information theory [11]. Noticeably, the amplitude of $L^{-1}(\partial S/\partial U)_N$, $U \rightarrow U_{c1}^{+}$ vanishes at $n=1$, consistent with the U-independent entropy at $n=1$: $S=k_BN\ln 2$.

In conclusion, we reported on a quite complete study of the GS and low-T properties of the integrable Hubbard model with bond-charge interaction. In particular, we analyzed its conducting properties and presented a detailed description of the scaling behavior near a quantum phase transition between two distinct metallic phases, with quantum dynamic exponent $z=1$, and near both a QCP and a quantum critical line of MIT, with $z=2$. Remarkably, the model exhibits fractional statistical properties, which are manifest in the nature of the fractional elementary excitations, with nontrivial implications. Notably, we mention the disordered condensate, the phase separation in k space, and the topological change in the generalized Fermi surface at half filling.

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