Test of the Conserved Vector Current Hypothesis in $T = 1/2$ Mirror Transitions and New Determination of $|V_{ud}|$

O. Naviliat-Cuncic[1] and N. Severijns[2]

[1]LPC-Caen, ENSICAEN, Université de Caen Basse-Normandie, CNRS/IN2P3-ENSI, Caen, France
[2]Instituut voor Kern- en Stralingsfysica, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium
(Received 23 January 2009; published 8 April 2009)

The V_{ud} element of the Cabibbo-Kobayashi-Maskawa quark mixing matrix has traditionally been determined from the analysis of data in nuclear superallowed $0^+ \to 0^+$ transitions, neutron decay, and pion beta decay. After providing a new test of the conserved vector current hypothesis, we present here a new independent determination of $|V_{ud}|$ from a set of five $T = 1/2$ nuclear mirror transitions. The extracted value, $|V_{ud}| = 0.9719 \pm 0.0017$, is at 1.2 combined standard deviations from the value obtained from superallowed $0^+ \to 0^+$ transitions and has a precision comparable to the value obtained from neutron decay experiments.

DOI: 10.1103/PhysRevLett.102.142302 PACS numbers: 24.80.+y, 12.15.Hh, 23.40.−s

The unitarity conditions of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1,2] provide sensitive means to test the consistency of the three generation standard electroweak model and to search for new physics beyond. A stringent test is obtained from the elements of the first row

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1, \quad (1)$$

where V_{uj} denotes the amplitude of the quark mass eigenstate j into the quark weak eigenstate d', in the standard notation [3]. The accuracy in the verification of this condition is due to the dominant values and errors of the V_{ud} and V_{us} elements, the first being obtained from weak decay processes involving the lightest quarks, and the second from K-meson decays.

Three traditional sources to extract $|V_{ud}|$ from experiments have been considered during the past decades, namely, nuclear superallowed $0^+ \to 0^+$ pure Fermi transitions, neutron decay and pion beta decay, and the status of the determination of $|V_{ud}|$ from these sources has regularly been reviewed [4–6].

The corrected partial half-lives, $\mathcal{F}t$, of nine nuclear superallowed $0^+ \to 0^+$ transitions have been studied in great detail in the past and the set has recently been extended to include a total of 13 transitions [7]. Measurements of lifetimes, masses, and branching ratios reached precisions such that the required inputs for the determination of $\mathcal{F}t$ values are obtained at a level of few parts in 10^{-4}. The most recent survey yields [7]

$$|V_{ud}| = 0.974\,25(22) \quad (\text{superallowed } 0^+ \to 0^+), \quad (2)$$

where the error is dominated by uncertainties in theoretical radiative corrections.

Neutron decay involves both the vector and the axial-vector interactions so that the determination of $|V_{ud}|$, although free of nuclear structure corrections, requires here the analysis of at least two observables. The most precise determinations have so far been obtained by combining the neutron lifetime with the beta asymmetry parameter. The first attempt to determine $|V_{ud}|$ using only neutron decay data [8] yielded the value $|V_{ud}| = 0.9790(30)$. The present world average recommended value for the neutron lifetime, $\tau_n = 885.7(8)$ s [3], combined with the world average value for the beta asymmetry parameter, $A_n = -0.1173(13)$ [3], yields

$$|V_{ud}| = 0.9746(19) \quad (\text{neutron decay}). \quad (3)$$

The improvement in the error by a factor of about 1.5 over almost two decades shows the difficulty of the associated experiments [9,10]. Other values have, however, been reported [10,11] by taking selected results of the most precise experimental data, but those do not account for the spread in the existing results [3].

Finally, the absolute pion beta decay rate provides a clean observable for the determination of $|V_{ud}|$. The main experimental difficulty arises from the very weak (10^{-8}) branching of the beta decay channel. The most recent experimental determination yields [12]

$$|V_{ud}| = 0.9728(30) \quad (\text{pion decay}). \quad (4)$$

We consider here a new source for the determination of $|V_{ud}|$, namely, the beta decay transitions between $T = 1/2$ isospin doublets in mirror nuclei. Three such transitions (including neutron decay) have been considered in the past for a test of the Cabibbo angle universality [13], but errors in the experimental data led then to speculations about a possible cancellation of the Cabibbo angle in some decays.

Mirror transitions are mixed (Fermi–Gamow-Teller) and, like neutron decay, are driven by the vector and axial-vector interactions. Since the axial-vector current is not conserved in nuclear decays [14,15], the extraction of $|V_{ud}|$ proceeds in analogy with neutron decay, except for corrections associated with the nuclear system. The corrections for the determination of the $\mathcal{F}t$ values in these transitions have recently been surveyed [16] and were obtained with sufficient precision for their consideration in the analysis reported here. We use then below the results of this survey but include some new updated values. We adopt also the definitions and notations given there unless possible ambiguities require it otherwise.

The vector part of the corrected statistical decay rate function is given by [16]

$$\mathcal{F}t \equiv f_V t(1+\delta_R')(1+\delta_{\rm NS}^V-\delta_C^V), \tag{5}$$

where f_V is the uncorrected statistical rate function, δ_R' denotes nuclear dependent radiative corrections obtained from QED calculations, $\delta_{\rm NS}^V$ are nuclear structure corrections, and δ_C^V are isospin symmetry breaking corrections for the vector contribution. For mixed transitions, $\mathcal{F}t$ is related to V_{ud} by [16]

$$\mathcal{F}t = \frac{K}{G_F^2 V_{ud}^2}\frac{1}{C_V^2|M_F^0|^2(1+\Delta_R^V)[1+(f_A/f_V)\rho'^2]}, \tag{6}$$

where $K/(\hbar c)^6 = 2\pi^3 \ln 2\hbar/(m_e c^2)^5$ and has the value $K/(\hbar c)^6 = 8120.278(4)\times 10^{-10}$ GeV^{-4} s, $G_F/(\hbar c)^3 = 1.16637(1)\times 10^{-5}$ GeV^{-2} is the Fermi constant [3], $C_V = 1$ is the vector coupling constant, Δ_R^V is a transition-independent radiative correction [17], f_A is the statistical rate function for the axial-vector part of the interaction, and ρ' is the Gamow-Teller to Fermi mixing ratio. This ratio is defined by [16]

$$\rho' = \frac{C_A M_{GT}^0}{C_V M_F^0}\left[\frac{(1+\delta_{\rm NS}^A-\delta_C^A)(1+\Delta_R^A)}{(1+\delta_{\rm NS}^V-\delta_C^V)(1+\Delta_R^V)}\right]^{1/2} \tag{7}$$

$$\equiv \left[\frac{(1+\delta_{\rm NS}^A)}{(1+\delta_{\rm NS}^V)}\right]^{1/2}\rho, \tag{8}$$

where the square root in Eq. (7) contains the nuclear structure, isospin symmetry breaking, and nucleus independent radiative corrections for the vector and axial-vector contributions, C_A is the axial-vector coupling constant, and M_F^0 and $M_{\rm GT}^0$ are the isospin symmetry limit values of the Fermi and Gamow-Teller matrix elements, with $|M_F^0|^2 = 1$ for the $T_i = T_f = 1/2$ mirror transitions. In the following it is assumed that $\rho' \approx \rho$.

From the terms in Eq. (6) we define the quantity

$$\mathcal{F}t_0 = \mathcal{F}t C_V^2 |M_F^0|^2[1+(f_A/f_V)\rho^2], \tag{9}$$

which groups all terms associated with a given nuclear decay. Since the remaining terms are fundamental physical constants (G_F and K) and parameters associated with the electroweak interaction (V_{ud} and Δ_R^V) the $\mathcal{F}t_0$ values should be constant and furthermore independent of the isospin within the transition, provided the vector coupling be constant, as results from the conserved vector current (CVC) hypothesis [18].

With the corrected $\mathcal{F}t$ values from the recent compilation [16], $|V_{ud}|$ can be obtained from Eq. (6) using another observable measured with sufficient precision to deduce ρ. In the present analysis we consider three correlation coefficients: the beta-neutrino angular correlation $a_{\beta\nu}$, the beta asymmetry parameter A_β, and the neutrino asymmetry parameter B_ν. For β^+ mirror transitions, their expressions in the limit of zero momentum transfer are [19]

$$a_{\beta\nu}(0) = (1-\rho^2/3)/(1+\rho^2), \tag{10}$$

$$A_\beta(0) = \frac{\rho^2 - 2\rho\sqrt{J(J+1)}}{(1+\rho^2)(J+1)}, \tag{11}$$

$$B_\nu(0) = -\frac{\rho^2 + 2\rho\sqrt{J(J+1)}}{(1+\rho^2)(J+1)}, \tag{12}$$

where J denotes the spin of the initial and final states in the transition. At a precision level of about 1%, the impact of recoil effects has, however, to be considered. To first order in recoil, assuming time reversal invariance and the absence of second class currents [20], the correlation coefficients for a β^+ transition within a common isotopic multiplet are given by [21]

$$a_{\beta\nu}(E) = f_2(E)/f_1(E), \tag{13}$$

$$A_\beta(E) = f_4(E)/f_1(E), \tag{14}$$

$$B_\nu(E) = h_6(E)/f_1(E), \tag{15}$$

with the spectral functions [22]

$$f_1(E) = a^2 + c^2 - \frac{2E_0}{3M}(c^2+cb) + \frac{2E}{3M}(3a^2+5c^2+2cb) - \frac{2m_e^2}{3EM}(c^2+cb), \tag{16}$$

$$f_2(E) = a^2 - \frac{1}{3}c^2 + \frac{2E_0}{3M}(c^2+cb) - \frac{4E}{3M}(3c^2+cb), \tag{17}$$

$$f_4(E) = -\left(\frac{J}{J+1}\right)^{1/2}\left[2ac - \frac{2E_0}{3M}(ac+ab) + \frac{2E}{3M}(7ac+ab)\right] + \left(\frac{1}{J+1}\right) \times \left[c^2 - \frac{2E_0}{3M}(c^2+cb) + \frac{E}{3M}(11c^2+5cb)\right], \tag{18}$$

and

$$h_6(E) = -\left(\frac{J}{J+1}\right)^{1/2}\left[2ac + \frac{E}{M}(5ac+ab) - \frac{m_e^2}{EM}(ac+ab)\right] - \left(\frac{1}{J+1}\right) \times \left[c^2 - \frac{E_0}{M}(c^2+cb) + \frac{E}{2M}(7c^2+3cb) - \frac{m_e^2}{2EM}(c^2+cb)\right]. \tag{19}$$

Here E and E_0 denote, respectively, the total and the total maximal positron energies, M is the average mass of the mother and daughter isotopes, and m_e is the electron mass. In this notation [21] a, b, and c designate, respectively, the Fermi, weak magnetism, and Gamow-Teller form factors,

$$a = C_V M_F, \qquad c = C_A M_{\rm GT}, \tag{20}$$

and

$$b = A[(J+1)/J]^{1/2} M_F \mu, \qquad (21)$$

where A is the mass number and $\mu = [\mu(T_3) - \mu(T_3')]/(T_3 - T_3')$ the isovector contribution to the magnetic moment, with T_3 the third component of the isospin (in the convention where $T_3 = +1/2$ for a proton) and $\mu(T_3)$ and $\mu(T_3')$ the magnetic moments of the mother and daughter nuclei.

With the experimental data for the correlation coefficients, Eqs. (13) to (15) are then solved to yield the values of $\rho = c/a$, the signs of which were taken to be the same as in Ref. [16].

The data included in the present analysis are summarized in Table I. The mirror transitions are those in ^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar, and ^{37}K. The beta asymmetry parameter A_β has also been measured in ^{17}F [33], but the sensitivity of this result to ρ turns out to be very weak so that it was not included here. The values for E used in Eqs. (13) to (15) and listed in Table I are averages determined from the experimental conditions.

The consideration of recoil effects in the determination of the $\mathcal{F}t$ values was found to have a negligible impact. Electromagnetic corrections [34] other than the dominant Coulomb effects contained in the energy-dependent Fermi function $F(Z, E)$ and included in the $f_{V,A}$ factors, were verified to be negligible at the present level of precision.

The beta asymmetry parameter in ^{19}Ne decay has been measured twice [28,35]. The value reported in Ref. [35], $A_\beta = -0.03603(83)$, is more precise than the result quoted in Ref. [28] but was not included here since the result has never been published. In ^{35}Ar decay, the beta asymmetry parameter has reliably been measured twice, with the results $A_\beta = 0.49(10)$ [30] and $A_\beta = 0.427(23)$ [31]. The weighted mean of these (Table I), is dominated by the most recent result. Except for ^{19}Ne, recoil corrections appeared not to have a significant impact in the determination of ρ. For ^{19}Ne, Eq. (11) yields $\rho = 1.6015(45)$ what differs from the value obtained from Eq. (14), and quoted in Table I, by about half a standard deviation.

A fit by a constant of the $\mathcal{F}t_0$ values listed in Table I and presented in Fig. 1 yields

$$\overline{\mathcal{F}t}_0 = 6173 \pm 22 \text{ s}, \qquad (22)$$

with $\chi^2/\nu = 0.75$. This provides a test of CVC in these mirror transitions at the 3.6×10^{-3} level. It is the first consistent test of CVC in a set of nuclear decays other than superallowed pure Fermi transitions. This value is to be compared with $2\mathcal{F}t(0^+ \to 0^+) = 6143.7(17)$ s [7] obtained from superallowed pure Fermi transitions. Because of their larger uncertainties, the values from ^{29}P and ^{37}K do not contribute significantly to the result in Eq. (22).

Following this test of CVC, the value of $|V_{ud}|$ can be obtained from

$$V_{ud}^2 = \frac{K}{\overline{\mathcal{F}t}_0 G_F^2 (1 + \Delta_R^V)}, \qquad (23)$$

where $\Delta_R^V = 2.361(38)\%$ [17], yielding

$$|V_{ud}| = 0.9719(17) \quad \text{(nuclear mirror transitions)}. \qquad (24)$$

This result is more precise than the value obtained from

TABLE I. Input data used to determine the values of ρ and $\mathcal{F}t_0$.

	^{19}Ne	^{21}Na	^{29}P	^{35}Ar	^{37}K
J	1/2	3/2	1/2	3/2	3/2
$\mathcal{F}t$ [s][a]	1720.3(30)	4085(12)	4809(19)	5688.6(72)	4562(28)
f_A/f_V[b]	1.0143(29)	1.0180(36)	1.0223(45)	0.9894(21)	1.0046(9)
E_0 [MeV][c]	2.728 31(17)	3.036 58(70)	4.431 45(60)	5.455 14(70)	5.636 46(23)
E [MeV][d]	0.511	1.60	2.39	3.14	3.35
M [amu][e]	19.000 141 99(9)	20.995 750 9(4)	28.979 147 65(30)	34.972 055 1(4)	36.970 076 11(12)
b[f]	−148.5605(26)	82.6366(27)	89.920(15)	−8.5704(90)	−44.99(24)
$a_{\beta\nu}$		0.5502(60)[g]			
A_β	−0.0391(14)[h]		0.681(86)[i]	0.430(22)[j]	
B_ν					−0.755(24)[k]
ρ	1.5995(45)	−0.7136(72)	−0.593(104)	−0.279(16)	0.561(27)
$\mathcal{F}t_0$ [s]	6184(30)	6202(48)	6537(606)	6128(49)	6006(146)

[a] From Ref. [16] and for ^{19}Ne using the mass excess from Refs. [23,24].
[b] From Ref. [16] and using a 20% relative error on the deviation of f_A/f_V from unity as adopted there.
[c] Total end point energy of the decay from Ref. [25] and for ^{19}Ne using the mass excess from Refs. [23,24].
[d] Average total energy weighted over the beta spectrum considered in the extraction of the correlation coefficient from experiment.
[e] Average mass of the mother and daughter nuclei, using data from Ref. [25] and for ^{19}Ne from Ref. [23].
[f] Calculated with the magnetic moments listed in Ref. [26].
[g] From Ref. [27].
[h] Value for $E = m_e$ from Ref. [28].
[i] From Ref. [29].
[j] Weighted mean of values from Refs. [30,31].
[k] From Ref. [32].

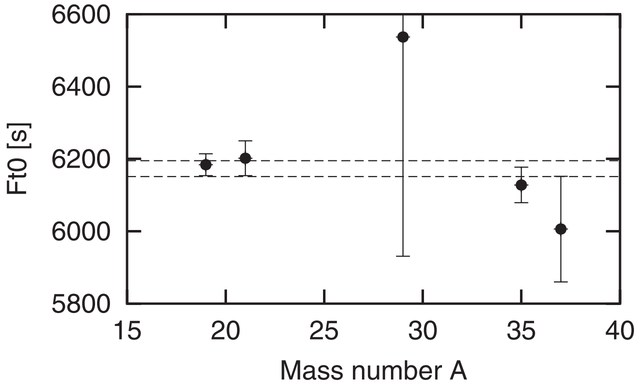

FIG. 1. $\mathcal{F}t_0$ values deduced for five mirror transitions as a function of the mass number of the mirror nuclei. The horizontal band shows the $\pm 1\sigma$ limits of the result from the fit.

pion decay, Eq. (4), it has a similar precision to the value obtained from neutron decay, Eq. (3), and is consistent within 1.2 combined standard deviations with the value obtained from nuclear superallowed $0^+ \rightarrow 0^+$ transitions, Eq. (2). Such a result is remarkable considering that none of the experimental inputs included in the analysis above was obtained from a measurement explicitly motivated by the determination of $|V_{ud}|$. This shows that nuclear mirror transitions provide an additional sensitive source for the determination of this matrix element and deserve therefore further theoretical studies and experimental investigations to improve the required inputs. The error on the value quoted in Eq. (24) is dominated by those in the experimental data, and more specifically by the errors on the mixing ratios ρ. The prospects and sensitivities for new precision correlation measurements to improve the determinations of the mixing ratios in mirror transitions were discussed elsewhere [36].

In conclusion, we have performed a new test of the conserved vector current hypothesis using data from mirror transitions in ^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar, and ^{37}K. Furthermore, a new independent value of the V_{ud} element of the CKM matrix was obtained. The result provides the second most precise determination of $|V_{ud}|$ and demonstrates that nuclear mirror transitions constitute a sensitive source to this end. Additional theoretical studies as well as precise determinations of the experimental inputs, and, in particular, of the correlation coefficients, are desirable for further improvements.

We acknowledge discussions with B. Blank, I. S. Towner, S. Triambak, and C. Volpe.

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[4] I. S. Towner and J. C. Hardy, in *Proceedings of the 5th International WEIN Symposium on Physics Beyond the Standard Model*, edited by P. Herczeg, C. M. Hoffman, and H. V. Klapdor (World Scientific, Singapore, 1999), p. 338.
[5] I. S. Towner and J. C. Hardy, J. Phys. G **29**, 197 (2003).
[6] J. C. Hardy, arXiv:hep-ph/0703165v1.
[7] J. C. Hardy and I. S. Towner, arXiv:nucl-ex/0812.1202v1.
[8] D. Thompson, J. Phys. G **16**, 1423 (1990).
[9] J. S. Nico and W. M. Snow, Annu. Rev. Nucl. Part. Sci. **55**, 27 (2005).
[10] H. Abele, Prog. Part. Nucl. Phys. **60**, 1 (2008) and references therein.
[11] H. Abele *et al.*, Phys. Rev. Lett. **88**, 211801 (2002).
[12] D. Pocanic *et al.*, Phys. Rev. Lett. **93**, 181803 (2004).
[13] J. C. Hardy and I. S. Towner, Phys. Lett. B **58**, 261 (1975).
[14] Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).
[15] M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).
[16] N. Severijns, M. Tandecki, T. Phalet, and I. S. Towner, Phys. Rev. C **78**, 055501 (2008).
[17] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **96**, 032002 (2006).
[18] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[19] N. Severijns, M. Beck, and O. Naviliat-Cuncic, Rev. Mod. Phys. **78**, 991 (2006).
[20] For a review, see L. Grenacs, Annu. Rev. Nucl. Part. Sci. **35**, 455 (1985).
[21] B. R. Holstein and S. B. Treiman, Phys. Rev. C **3**, 1921 (1971); B. R. Holstein, Phys. Rev. C **4**, 764 (1971).
[22] The signs of terms linear in c in Eqs. (16)–(19) are reversed with respect to Ref. [21] which uses a different representation of the Dirac matrices.
[23] W. Geithner *et al.*, Phys. Rev. Lett. **101**, 252502 (2008).
[24] A. Herlert (private communication).
[25] G. Audi, O. Bersillon, J. Blachot, and H. Wapstra, Nucl. Phys. **A729**, 3 (2003).
[26] N. J. Stone, At. Data Nucl. Data Tables **90**, 75 (2005). Note that the value of the magnetic moment for ^{21}Na quoted there should in fact read $+2.386\,30(10)\mu_N$.
[27] P. A. Vetter, J. R. Abo-Shaeer, S. J. Freedman, and R. Maruyama, Phys. Rev. C **77**, 035502 (2008).
[28] F. P. Calaprice, S. J. Freedman, W. C. Mead, and H. C. Vantine, Phys. Rev. Lett. **35**, 1566 (1975).
[29] G. S. Masson and P. A. Quin, Phys. Rev. C **42**, 1110 (1990).
[30] J. D. Garnett, E. D. Commins, K. T. Lesko, and E. B. Norman, Phys. Rev. Lett. **60**, 499 (1988).
[31] A. Converse *et al.*, Phys. Lett. B **304**, 60 (1993).
[32] D. Melconian *et al.*, Phys. Lett. B **649**, 370 (2007).
[33] N. Severijns, J. Wouters, J. Vanhaverbeke, and L. Vanneste, Phys. Rev. Lett. **63**, 1050 (1989).
[34] B. R. Holstein, Rev. Mod. Phys. **46**, 789 (1974); **48**, 673 (1976); B. R. Holstein, Phys. Rev. C **9**, 1742 (1974).
[35] D. F. Schreiber, Ph.D. thesis, Princeton University, 1983.
[36] O. Naviliat-Cuncic and N. Severijns, Eur. J. Phys. A (to be published).