Dilatation Symmetry in Higher Dimensions and the Vanishing of the Cosmological Constant

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A wide class of dilatation symmetric effective actions in higher dimensions leads to a vanishing four-dimensional cosmological constant. This requires no tuning of parameters and results from the absence of an allowed potential for the scalar dilaton field. The field equations admit many solutions with flat four-dimensional space and nonvanishing gauge couplings. In a more general setting, these are candidates for asymptotic states of cosmological runaway solutions, where dilatation symmetry is realized dynamically if a fixed point is approached as time goes to infinity. Dilatation anomalies during the runaway can lift the degeneracy of solutions and lead to an observable dynamical dark energy.

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Dilatation symmetry may play a crucial role in our understanding of cosmology [1]. If the quantum effective action is scale invariant, a spontaneous breaking of the dilatation symmetry by a nonzero value of the scalar dilaton field will lead to a massless Goldstone boson. In the presence of a dilatation anomaly, a potential and a small mass for this field are generated—the pseudo Goldstone boson becomes the cosmologically relevant "cosmon field." For many cosmological "runaway solutions" the dilatation anomaly vanishes asymptotically, as a fixed point is approached for time going to infinity [2]. As a consequence, the mass of the cosmon field decreases with time. Typically, it is of the order of the Hubble parameter [3]. If the fixed point occurs for a vanishing cosmological constant, rather interesting quintessence cosmologies follow from this scenario [1]. They describe tracker solutions with dynamical dark energy [1–4]. Our approach aims for an explanation for why the cosmon potential vanishes asymptotically rather than approaching a nonvanishing constant.

In four dimensions, dilatation symmetry alone cannot explain why the fixed point occurs for a vanishing cosmological constant. Within a dilatation symmetric standard model of elementary particle physics, every mass scale is replaced by a combination $h\chi$, with χ the scalar dilaton field and h some appropriate dimensionless coupling. Four-dimensional dilatation symmetry permits a polynomial potential $V(\chi) = \lambda\chi^4$. After Weyl scaling, this results in the Einstein frame as an effective cosmological constant proportional to the dimensionless coupling λ. In contrast, dilatation symmetry in dimension $d > 6$ does not allow anymore a polynomial potential. We will show in this Letter that, for a wide class of dilatation symmetric effective actions, this simple fact results in a vanishing four-dimensional cosmological constant without a tuning of parameters.

Consider the quantum effective action Γ for the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ and a dilaton field ξ in arbitrary dimension d. It includes all effects from quantum fluctuations, including those of effective four-dimensional fields after spontaneous compactification. The field equations or other constraints [2] that arise from the extremum condition for Γ are exact. Scale transformations (dilatations) act on the fields by multiplicative rescaling with powers of a constant ν, $\hat{g}_{\hat{\mu}\hat{\nu}} \to \nu^{-2}\hat{g}_{\hat{\mu}\hat{\nu}}$, $\xi \to \nu^{(d-2)/2}\xi$, while the coordinates $\hat{x}^{\hat{\mu}}$ remain unchanged. We will first explore the consequences of two simple assumptions: (i) the effective action is scale invariant; (ii) Γ can be written as a polynomial of ξ and the curvature scalar $\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ or their covariant derivatives. We emphasize that we do *not* assume that the effective action as a whole has these properties. It is sufficient that our assumptions hold for the fixed point when the dilatation symmetry violating terms have vanished for a runaway solution approaching the fixed point arbitrarily close.

With our assumptions the most general dilatation symmetric effective action reads, with $\hat{g} = \det(-\hat{g}_{\hat{\mu}\hat{\nu}})$,

$$\Gamma = \int_{\hat{x}} \hat{g}^{1/2}\left\{-\frac{1}{2}\xi^2\hat{R} + \frac{\zeta}{2}\partial^{\hat{\mu}}\xi\partial_{\hat{\mu}}\xi + F\right\}. \tag{1}$$

The first two terms are the higher-dimensional generalization of the Jordan-Brans-Dicke theory [5] in the absence of matter, while F contains higher powers of the curvature tensor. The most striking property of (1) is the absence of a potential for ξ. Indeed, a scale-invariant polynomial ξ^n, $n \in \mathbb{N}$, requires $n(d-2)/2 = d$. For $d = 4$ one has $n = 4$, and for $d = 6$ a cubic potential $\sim\xi^3$ is possible, but no solution exists for $d > 6$.

The field equations derived from Γ read

$$\zeta D^{\hat{\mu}}D_{\hat{\mu}}\xi + \hat{R}\xi = 0, \tag{2}$$

$$\xi^2(\hat{R}_{\hat{\mu}\hat{\nu}} - \tfrac{1}{2}\hat{R}\hat{g}_{\hat{\mu}\hat{\nu}}) = T^{(\xi)}_{\hat{\mu}\hat{\nu}} + T^{(F)}_{\hat{\mu}\hat{\nu}}, \tag{3}$$

with $T^{(\xi)}_{\hat{\mu}\hat{\nu}}$ involving derivatives of ξ. For geometries with singularities the extremum condition for Γ yields further "brane constraints" [6]. For example, an extremum with respect to the infinitesimal variation $\xi \to \xi[1 + \epsilon(x)]$ requires a vanishing boundary term

$$\int_y \partial_{\hat{\mu}}(\hat{g}^{1/2}\xi\partial^{\hat{\mu}}\xi) = 0, \tag{4}$$

with $\int_y$ indicating the integration over the "internal coordinates" y^α, while x^μ denotes the four-dimensional coordinates.

Neglecting first the contribution $T^{(F)}_{\hat\mu\hat\nu}$ from F, the field equations have a simple class of static solutions, namely,

$$\xi = \xi_0 = \text{const}, \qquad \hat R_{\hat\mu\hat\nu} = 0. \tag{5}$$

These are candidates for the asymptotic limits of time-dependent cosmological solutions as $t \to \infty$. The condition of Ricci flatness $\hat R_{\hat\mu\hat\nu} = 0$ still allows many different geometries. Among them is d-dimensional Minkowski space. More interesting for the real world are geometries which are a direct product of flat four-dimensional Minkowski space and a Ricci-flat D-dimensional internal space ($D = d - 4$) with finite volume Ω_D. Such geometries admit dimensional reduction to an effective four-dimensional theory. If we keep only the four-dimensional metric $g^{(4)}_{\mu\nu}(x)$ and a scalar dilaton field $\chi(x) = \Omega_D^{1/2}\xi(x)$, the reduced four-dimensional effective action Γ_4 exhibits an effective four-dimensional dilatation symmetry

$$\Gamma^{(4)} = \int_x (g^{(4)})^{1/2}\left\{-\frac{1}{2}\chi^2 R^{(4)} + \frac{\zeta}{2}\partial^\mu\chi\partial_\mu\chi\right\}. \tag{6}$$

No term $\sim\lambda\chi^4$ appears—the effective four-dimensional cosmological constant vanishes.

If internal space has isometries, $\Gamma^{(4)}$ can be extended to include the gauge bosons of the corresponding local gauge symmetry. For finite Ω_D the dimensionless gauge couplings are finite and nonzero. A simple example for a Ricci-flat internal space is a D-dimensional torus with isometry $U(1)^D$, but there are many more possibilities, including spaces with non-Abelian isometries. Beyond a polynomial approximation to the effective action, the four-dimensional gauge coupling may be running, according to a gauge and dilatation invariant kinetic term $\sim F^{\mu\nu}K(-D^\mu D_\mu/\chi^2)F_{\mu\nu}$. For asymptotically free theories this can result in particle masses proportional to a "confinement scale" Λ_c which scales $\sim\chi$. Ratios of particle masses and the effective Planck mass χ do not depend on χ in this case. Solutions (5) with finite Ω_D share therefore many aspects of a satisfactory asymptotic state of cosmology—namely, flat space and particle physics with constant dimensionless couplings and mass ratios.

In even dimensions we may include a suitable polynomial F of the curvature tensor $\hat R_{\hat\mu\hat\nu\hat\rho\hat\sigma}$ and its covariant derivatives. Consider first the case where F can be written as a polynomial of $\hat R_{\hat\mu\hat\nu}$ and its covariant derivatives. The solution (5) persists. The variation of F always contains terms linear in $\hat R_{\hat\mu\hat\nu}$ and its covariant derivatives and therefore gives no contribution if $\hat R_{\hat\mu\hat\nu} = 0$. This general form of F contains a large number of different invariants, with different dimensionless couplings τ_i. We have therefore established solutions for which the effective four-dimensional cosmological constant vanishes for arbitrary τ_i, involving no tuning of parameters. The most general form of F can be written as a polynomial of $\hat R$, the traceless part of the Ricci tensor $\hat H_{\hat\mu\hat\nu} = \hat R_{\mu\nu} - \frac{1}{d}\hat R\hat g_{\hat\mu\hat\nu}$, and the totally antisymmetric part of the curvature tensor $\hat C_{\hat\mu\hat\nu\hat\rho\hat\sigma} = \hat R_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$, as well as covariant derivatives thereof. Flat space geometries $\hat R_{\hat\mu\hat\nu\hat\rho\hat\sigma} = 0$, $\xi = \xi_0$ always solve the field equations.

We next turn to a general discussion of "quasistatic" geometries, for which the internal geometry and ξ are independent of x^μ, while the four-dimensional space-time is maximally symmetric. The four-dimensional Ricci tensor obeys $R^{(4)}_{\mu\nu} = \Lambda g^{(4)}_{\mu\nu}$, where positive (negative) Λ corresponds to (anti–)de Sitter space and a vanishing cosmological constant $\Lambda = 0$ to flat Minkowski space. For this discussion we weaken our assumptions and require no longer a polynomial dependence of F on the curvature tensor. For this very general setting, we find as a central result of this Letter that all stable extrema of the effective action with nonzero ξ and finite characteristic length scale l of internal space have a vanishing cosmological constant. More precisely, we can show that for $|\Lambda| \ll \chi^2$ no stable extrema of Γ with $\Lambda \neq 0$ exist in this case. Stable quasistatic solutions single out a vanishing cosmological constant.

The direct product solutions are not the only interesting geometries. There may be singular solutions with warping [7–10], corresponding to a brane [8,11] or a "zero warp" [7] sitting at the singularity. Such spaces may be interesting because they can lead to chiral fermions after dimensional reduction [12]. The most general quasistatic geometry

$$\hat g_{\hat\mu\hat\nu} = \begin{pmatrix} \sigma(y)g^{(4)}_{\mu\nu}(x), & 0 \\ 0, & g^{(D)}_{\alpha\beta}(y) \end{pmatrix} \tag{7}$$

involves the warp factor $\sigma(y)$ and the internal metric $g_{\alpha\beta}(y)$. For singular solutions one can always obtain local solutions with $\Lambda = 0$, but often also neighboring solutions with $\Lambda \neq 0$ exist [7–9]. We find that some of the solutions with $\Lambda = 0$ (flat four-dimensional geometry) are consistent extrema of the action [6]. In contrast, neighboring solutions with a small $|\Lambda| \ll \chi^2$ are inconsistent and do not correspond to extrema of Γ (unless the fields are constrained by additional physics which fixes the strength of the singularity [6]). For singular spaces the remarkable property that a vanishing cosmological constant is singled out originates from the extremum conditions which go beyond the higher-dimensional field equations. Additional "brane constraints" [6] are due to Eq. (4) and to similar boundary terms for the metric. For regular geometries, the brane constraints are obeyed automatically.

We restrict our discussion to solutions with local four-dimensional gravity, in the sense that an effective four-dimensional action can be expanded in powers of the curvature tensor. In this case, an extremum of the higher-dimensional action must also be an extremum of the effective four-dimensional action. The latter is obtained from Γ [Eq. (1)] by inserting given solutions of the higher-dimensional field equations $\xi(y)$, $\sigma(y)$, and $g^{(D)}_{\alpha\beta}(y)$ according to the ansatz (7) and integrating over the internal coordinates y. In particular, we can consider $\Gamma^{(4)}$ as a functional of the variable $g^{(4)}_{\mu\nu}$:

$$\Gamma^{(4)} = \int_x (g^{(4)})^{1/2} W, \quad W = V - \frac{\chi^2}{2} R^{(4)} + \tilde{H}(R^{(4)}_{\mu\nu\rho\sigma}). \quad (8)$$

Here we identify V with the effective four-dimensional cosmological constant and χ with the effective Planck mass (in the Jordan frame), and $\tilde{H}$ contains higher orders of the four-dimensional curvature tensor. For a maximally symmetric four-dimensional space, one finds for the four-dimensional Lagrangian W

$$W = V - 2\Lambda\chi^2 + \Lambda^2 \hat{H}(\Lambda), \qquad \lim_{\Lambda\to 0} \Lambda \hat{H}(\Lambda) = 0. \quad (9)$$

Both V and χ depend on the geometry of internal space and on the warping.

In addition to $g^{(4)}_{\mu\nu}$ we may also keep the normalization of ξ and $g^{(D)}_{\alpha\beta}$ as free variables. These degrees of freedom can be expressed in terms of a characteristic length scale l for internal space and a characteristic average $\bar{\xi}$ for ξ:

$$\int_y (g^{(D)})^{1/2} \sigma^2 = l^D, \qquad \int_y (g^{(D)})^{1/2} \sigma \xi^2 = l^D \bar{\xi}^2. \quad (10)$$

We are interested in extrema where both l and $\bar{\xi}$ are finite and nonzero. For the metric (7) the d-dimensional curvature scalar obeys $\hat{R} = R^{(\mathrm{int})} + R^{(4)}/\sigma$ such that

$$\chi^2 = l^D \bar{\xi}^2 - 2\tilde{G} l^{-2}, \qquad \tilde{G} = l^2 \int_y (g^{(D)})^{1/2} \sigma G. \quad (11)$$

Here G arises from the expansion of F in the four-dimensional curvature tensor $F(\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}) = F(R^{(\mathrm{int})}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}) + G R^{(4)}/\sigma + \cdots$, and $\tilde{G}$ is dimensionless. Similarly, we can write

$$V = \tilde{Q} \bar{\xi}^2 l^{D-2} + \tilde{F} l^{-4}, \quad (12)$$

with dimensionless quantities

$$\tilde{Q} = \frac{1}{2} \bar{\xi}^{-2} l^{2-D} \int_y (g^{(D)})^{1/2} \sigma^2 (\zeta \partial^\alpha \xi \partial_\alpha \xi - \xi^2 R^{(\mathrm{int})}),$$
$$\tilde{F} = l^4 \int_y (g^{(D)})^{1/2} \sigma^2 F(R^{(\mathrm{int})}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}). \quad (13)$$

Equation (9) yields for arbitrary quasistatic geometries

$$W = \bar{\xi}^2 [\tilde{Q} l^{D-2} - 2\Lambda l^D] + \frac{\tilde{F}}{l^4} + \frac{4\tilde{G}\Lambda}{l^2} + \Lambda^2 \hat{H}(\Lambda l^2). \quad (14)$$

An extremum of Γ must be an extremum of $W(\bar{\xi}, l)$, provided that $g^{(4)}_{\mu\nu}$ and therefore Λ are kept fixed. Stability requires $\tilde{Q} \geq 0$, $\tilde{F} \geq 0$, $\chi^2 > 0$, and the absence of negative eigenvalues for the second variations of W. For $|\Lambda| \ll \chi^2$ we can further neglect the term $\Lambda^2 \hat{H}$. A general analysis of the stable extrema of W shows two "phases." A "flat phase" with $\Lambda = 0$ is possible for all $\tilde{F}$, $\tilde{Q}$, and $\tilde{G}$. For $\tilde{F} > 0$ it occurs, however, for $l \to \infty$. A "nonflat phase" with $\Lambda \neq 0$ is possible only in a restricted range of $(\tilde{F}, \tilde{Q}, \tilde{G})$, with $\tilde{G} < 0$, $\tilde{Q} > 0$, and requires $\bar{\xi} = 0$. All stable extrema with $\bar{\xi} > 0$ belong to the flat phase with $\Lambda = 0$. Furthermore, a finite volume $l < \infty$ requires $\tilde{F} = \tilde{Q} = 0$. The solutions in the flat phase remain intact if we add the term $\Lambda^2 \hat{H}$. It is conceivable that in the presence of $\hat{H} \neq 0$ new stable extrema become possible with $|\Lambda| \approx \chi^2$. In units of the scale $l^D \bar{\xi}^2$, the possible values of Λ show a gap between zero and a value of order unity.

The phase structure of the possible stable extrema of Γ has important consequences for the stability of Λ with respect to parameter changes of the higher-dimensional action. Let us start with a specific form of Γ for which solutions in the flat phase with $\xi > 0$, $l < \infty$ are explicitly known. An example is a polynomial F which contains at least two powers of $\hat{R}_{\hat{\mu}\hat{\nu}}$ and direct product solutions of four-dimensional Minkowski space and Ricci-flat internal space, for which $\Lambda = 0$, $\tilde{F} = \tilde{Q} = 0$ can be easily verified. We next modify Γ by a small change δF of the ξ-independent term. Either no extremum with $\xi > 0$, $l < \infty$ exists anymore in the presence of δF (this could happen if δF induces an instability for all such solutions), or the modified Γ still admits stable extrema with $\xi > 0$, $l < \infty$. Then the flat phase persists and $\Lambda = 0$ remains preserved—a jump to a value $\Lambda \approx \chi^2$ is not possible for a continuous change of F (except for special points where χ^2 vanishes). We may evaluate the contribution of δF to $\tilde{F}$, i.e., $\delta\tilde{F}$, for the original extremum of Γ (i.e., in the absence of δF). It is not necessary that $\delta\tilde{F}$ vanishes. If not, the internal geometry and $\sigma(y)$, $\xi(y)$ will readjust such that, for the new extremum in the presence of the term δF, one again finds $\tilde{F} = 0$. This procedure can be continued to "explore" the parameter space of effective actions for which stable extrema with $\xi > 0$, $l < \infty$ exist—they all have $\Lambda = 0$. Typically, this parameter space covers arbitrary F which are consistent with general stability criteria, since flat internal space with finite Ω_D and $\xi = \xi_0$ always provides for a possible extremum with $\xi > 0$, $l < \infty$.

The readjustment of $\tilde{F}$ to zero can be associated with a readjustment of the cosmological constant. [Note that the readjustment of $\tilde{Q}$ follows automatically, since for any configuration (7) with $g^{(4)}_{\mu\nu} = \eta_{\mu\nu}$ the field equation (2) for ξ and the extremum condition (4) imply $\tilde{Q} = 0$ [6].] In this respect we emphasize an important difference between higher-dimensional theories and the four-dimensional setting. In general, $\tilde{F}$ is a functional of higher-dimensional fields $\alpha(y)$ that describe the changes of $g^{(D)}_{\alpha\beta}$, σ, and ξ. The extremum condition $\delta\tilde{F}/\delta\alpha(y)$ amounts to field equations with derivatives of α. Local solutions have free integration constants which may be adapted to obey $\tilde{F} = 0$. There is no need for a very special form of $\tilde{F}[\alpha(y)]$. In other words, we have infinitely many four-dimensional fields in order to achieve the readjustment. This contrasts with a finite number of homogeneous four-dimensional fields α_i, where the conditions $\partial\tilde{F}/\partial\alpha_i = 0$, $\tilde{F} = 0$, can be met simultaneously only for a special choice of $\tilde{F}$.

These arguments equally apply for the role of particular "Casimir contributions" to Λ from some effective four-dimensional quantum fluctuations or from the QCD condensate $\Lambda_{\rm QCD}$ or the Fermi scale. In a dilatation symmetric setting these effects typically are $\sim l^{-4}$ and therefore give a contribution to $\tilde{F}$. In a general covariant setting they therefore contribute to F, perhaps in a nonpolynomial form. Indeed, the "compactification scale" l^{-1} acts as an effective ultraviolet cutoff for the validity of a four-dimensional description and sets the scale for Casimir effects. It also sets the initial scale for the four-dimensional running of couplings, such that $\Lambda_{\rm QCD} \sim l^{-1}$, etc.

We have seen that there are many other contributions to $\tilde{F}$ from geometrical degrees of freedom. If a stable extremum with $\bar{\xi} > 0$, $l < \infty$ exists, all of these contributions must cancel precisely by virtue of the higher-dimensional field equations, resulting in $\tilde{F} = 0$. Tiny adjustments of infinitely many four-dimensional scalar fields are sufficient for this purpose. Typically, such a field A has a mass term $\sim \chi^2 A^2$ and couples linearly to the low energy degrees of freedom, as $\chi \varphi^\dagger \varphi A$ for the coupling to the Higgs field φ. A change of $\varphi \to \varphi + \delta\varphi$ results in a change of $V = \Lambda\chi^2$ of the order $\delta V \sim \varphi^3 \delta\varphi$ due to quartic interactions $\lambda\varphi^4$, etc. It is easy to verify that the resulting change $\delta A \sim \varphi\delta\varphi/\chi$ also contributes $\delta V \sim \varphi^3 \delta\varphi$, allowing for compensation.

Dilatation symmetry is not expected to be an exact quantum symmetry. Even for a scale-invariant classical action, the missing dilatation invariance of the measure in the functional integral induces a dilatation anomaly. The issue of the dilatation anomaly can be understood by performing a Weyl scaling of the d-dimensional metric $\hat{g}_{\hat{\mu}\hat{\nu}} = w^2 \tilde{g}_{\hat{\mu}\hat{\nu}}$, $w = M_d \xi^{-2/(d-2)}$. With $\tilde{R}$ the curvature scalar computed from the metric $\tilde{g}_{\hat{\mu}\hat{\nu}}$ in the Einstein frame, the effective action (1) reads in the new fields

$$\Gamma = \int \tilde{g}^{1/2} \left\{ -\frac{M_d^{d-2}}{2} (\tilde{R} - \tilde{\zeta} \partial^{\hat{\mu}} \ln\xi \partial_{\hat{\mu}} \ln\xi) + \tilde{F} \right\}, \quad (15)$$

where $\ln\xi$ stands for $\ln(\xi/M_d^{(d-2)/2})$. In the Einstein frame (15) the dilatations act as shifts in the dilaton field $\delta \sim M_d \ln\xi$, while $\tilde{g}_{\hat{\mu}\hat{\nu}}$ is invariant. Quantization in the Einstein frame preserves the dilatation symmetry as an exact quantum symmetry, since a functional measure for δ is invariant under a global shift $\delta \to \delta + \alpha$. However, we propose that the functional measure is defined in terms of the original variables ξ and $\hat{g}_{\hat{\mu}\hat{\nu}}$. The Weyl scaling to a measure for δ and $\tilde{g}_{\hat{\mu}\hat{\nu}}$ involves a Jacobian $\sim w^f$ for every space-time point $\hat{x}$ and therefore contributes to Γ an anomalous piece $\Gamma_{\rm an} \sim \Sigma_{\hat{x}} \ln\xi^2$. Regularization of the sum over all space-time points introduces a mass scale μ which explicitly breaks dilatation symmetry: $\Gamma_{\rm an} = \int_{\hat{x}} \hat{g}^{1/2} \mu^d (\ln\xi^2 + {\rm const})$, where $\hat{g}^{1/2}$ arises from the requirement of general covariance of the regularization.

To demonstrate the effect of the anomaly we add in the bracket in Eq. (1) a term $\hat{V}_{\rm an} = \mu^d$. After dimensional reduction the anomaly adds to V in Eq. (8) a term $V_{\rm an} = \mu^d l^D$. In order to discuss cosmological solutions it is convenient to perform a four-dimensional Weyl scaling with $w_4 = M/\chi$, such that in terms of the new metric for the Einstein frame one has $\Gamma^{(4)} = \int g^{1/2}(-M^2 R/2 + U)$. For $\tilde{F} = \tilde{Q} = 0$ the effective potential reads

$$U = M^4 V_{\rm an} \chi^{-4} = M^4 \gamma^{D/2} (\mu/\chi)^d, \quad (16)$$

where we have introduced $\gamma = \chi^2 l^2 = \omega^2 - 2\tilde{G}$. Realistic asymptotic solutions should lead to a constant value ω. Cosmology corresponds then to an increase of χ for $t \to \infty$, resulting in a decrease of the effective cosmological constant $\sim U/M^2$. This yields a typical quintessence cosmology, with an exponentially decreasing potential for the cosmon field $\varphi \sim \ln(\chi/M)$ [1]. The effect of the anomaly vanishes for $t \to \infty$ such that the system tends indeed to one of the quasistatic extrema of a dilatation symmetric effective action.

We conclude that a cosmological runaway towards a fixed point, where dilatation symmetry is realized dynamically on a quantum level, offers exciting prospects for a solution of the cosmological constant problem. The asymptotic value of the cosmological constant vanishes for a wide class of dilatation symmetric asymptotic states, without any tuning of parameters.

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