Dark Matter from R^2 Gravity

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The modification of Einstein gravity at high energies is mandatory from a quantum approach. In this work, we point out that this modification necessarily introduces new degrees of freedom. We analyze the possibility that these new gravitational states can provide the main contribution to the nonbaryonic dark matter of the Universe. By following an effective field theory approach, we illustrate this idea with the first and simplest high energy modification of the Einstein-Hilbert action: R^2 gravity.

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Different astrophysical observations agree that the main amount of the matter content of our Universe is in the form of unknown particles that are not included in the standard model (SM). Typical candidates to account for the missing matter can be found in well motivated extensions of the electroweak sector. However, there is a fundamental sector in our model of particles and interactions, where the introduction of new degrees of freedom is not only well motivated, but absolutely necessary.

The nonunitarity and nonrenormalizability of the gravitational interaction described by the Einstein-Hilbert action (EHA) demands its modification at high energies. There are a number of proposed candidates for a *quantum gravity theory* (QGT). However, these QGTs not only need to overcome major formal problems, but also to provide some experimental predictions that can be observed to prove them. Most of these models are based on different developments of nonperturbative string theories. Each one of these scenarios predict the existence of a rich spectrum of new particles at high energies. The possibility that these states may constitute dark matter (DM) has been analyzed in different articles in these particular scenarios and/or with particular assumptions [1]. In this work, we point out that the introduction of new states is general and it does not depend on the particular ultraviolet (UV) completion of the gravitational interaction. Indeed, these states will typically interact with SM fields through Planck scale suppressed couplings and potentially work as DM.

To deal with this problem, we will adopt the basic approach of effective field theories, that identify higher energy corrections with higher derivative terms [2]. The first UV correction to the EHA that needs to be considered is given by four-derivative terms in the metric that preserve general covariance. Indeed, not only one-loop corrections from the graviton, but also from the SM, generate these terms [3]. Therefore, they have to be present, at least, in the effective theory of gravity (ETG) [2]. Interestingly, four-derivative gravity is renormalizable [4]. The general four-derivative action supports, in addition to the usual massless spin-two graviton, a massive spin-two and a massive scalar mode, with a total of 8 degrees of freedom (in the *physical* or *transverse* gauge [4,5]). However, this model cannot be trusted beyond small modifications of the EHA, because the massive spin-two gravitons are ghostlike particles that generate new unitarity violations, breaking of causality, and inadmissible instabilities [6].

In any case, in four dimensions, there is a nontrivial four-derivative extension of Einstein gravity that is free of ghosts and phenomenologically viable. It is the so called R^2 gravity since it is defined by the only addition of a term proportional to the square of the scalar curvature to the EHA. This term does not improve the UV behavior of Einstein gravity but extends the spectrum of the ETG in a consistent way. R^2 gravity only introduces one additional scalar degree of freedom, whose mass m_0 is given by the corresponding new constant in the action:

$$S_G = \int d^4x\sqrt{g}\left\{-\Lambda^4 - \frac{M_{\rm Pl}^2}{2}R + \frac{M_{\rm Pl}^2}{12m_0^2}R^2 + \ldots\right\}, \quad (1)$$

where $M_{\rm Pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV, $\Lambda \simeq 2.3 \times 10^{-3}$ eV, and the dots refer to higher energy corrections that must be present in the model to complete the UV limit. In this work, we will show that just the Action (1) can explain the late time cosmology since the first term can account for the dark energy (DE) content, while the third term is able to explain the DM one. The first term is just the standard cosmological constant, that we will neglect along our analysis. We will focus on the new phenomenology that introduces the third term when it can be identified with the observed DM (read [7] for different approaches to DM from modified gravity).

The R^2 term does not modify the standard Einstein equations (EEs) at low energies except for the mentioned introduction of a new spin-zero mode [8–10]. In this work we will argue that the energy stored in these new metric oscillations behaves exactly as cold DM and can explain the missing matter problem of the Universe. We want to emphasize that this new mode of the metric is an independent degree of freedom that eventually will cluster and generate a successful structure formation if it is produced in the proper amount.

In fact, if we impose to preserve standard gravity up to nuclear densities or Big Bang Nucleosynthesis (BBN) temperatures, the constraints on m_0 are just $m_0 \gtrsim 10^{-12}$ eV. In this Letter, we will discuss in detail the restrictions and possible signatures of the model. To write the action for the new degree of freedom of the metric in a canonical way, we can work directly [5] (*Jordan frame*) or through a conformal transformation [11] (*Einstein frame*). In both cases, in the limit in which $R \ll m_0^2$, the metric can be expanded perturvatively as

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \frac{2}{M_{\rm Pl}} h_{\mu\nu} - \sqrt{\frac{2}{3}} \frac{1}{M_{\rm Pl}} \phi \hat{g}_{\mu\nu}, \tag{2}$$

where $\hat{g}_{\mu\nu}$ is its classical background solution, $h_{\mu\nu}$ takes into account the standard 2 degrees of freedom associated with the spin-two (traceless) graviton, and ϕ corresponds to the new mode. This scalar field has associated a canonical kinetic term with the mass m_0 as we have already commented.

We will deduce the couplings of this scalar graviton with the SM fields by supposing that gravity is minimally coupled to matter (in the Jordan frame). In such a case, there is a linear coupling to matter through the trace of the standard energy-momentum tensor [5]: $\mathcal{L}_{\phi - T_{\mu\nu}} = \phi T^\mu_\mu / (M_{\rm Pl}\sqrt{6})$. It implies that the couplings with massive SM particles are given at tree level. In particular, the three body interactions are given by:

$$\mathcal{L}^{\text{tree-level}}_{\phi\text{-SM}} = \frac{1}{M_{\rm Pl}\sqrt{6}} \phi \Big\{ 2m_\Phi^2 \Phi^2 - \nabla_\mu \Phi \nabla^\mu \Phi - 2m_W^2 W_\mu^+ W^{-\mu} - m_Z^2 Z_\mu Z^\mu + \sum m_\psi \bar{\psi}\psi \Big\} \tag{3}$$

with the Higgs (Φ), electroweak gauge bosons, and (Dirac) fermions (ψ), respectively. In contrast with what has been claimed in previous studies, this field does couple to photons and gluons due to the *conformal anomaly* induced at one loop by charged fermions and gauge bosons. We find (following notation from [12]):

$$\mathcal{L}^{\text{1-loop}}_{\phi\text{-SM}} = \frac{1}{M_{\rm Pl}\sqrt{6}} \phi \Big\{ \frac{\alpha_e c_e}{8\pi} F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_s c_s}{8\pi} G^a_{\mu\nu} G_a^{\mu\nu} \Big\}. \tag{4}$$

The particular value of the couplings (c_e and c_s) depends on the energy. We will be particularly interested in the coupling with photons, which leads to potential observational decays of ϕ. We will perform all the calculations restricting ourselves to the content of the SM but the exact values of the couplings depend also on heavier particles, charged with respect to these gauge interactions, that may extend the SM at high energies.

In principle, the above interactions of the scalar mode with the SM could produce a thermal abundance of ϕ at a very early stage of the Universe. However, it is expected that higher order corrections to Action (1) will be important at this point. In fact, it will typically take place at temperatures $T \gg \Lambda_G \equiv \sqrt{M_{\rm Pl} m_0}$, when we need to know the UV completion of the gravitational theory to study its dynamic. Nevertheless, there is at least another abundance source for this scalar mode that can be computed with Eq. (1). As other bosonic particles, such as axions [13], this field may have associated big abundances through the so called *misalignment mechanism*. There is no reason to expect that the initial value of the scalar field (ϕ_1) should coincide with the minimum of its potential ($\phi = 0$) if $H(T) \gg m_0$. But below the temperature T_1 for which $3H(T_1) \simeq m_0$, ϕ behaves as a standard scalar. It oscillates around the minimum. These oscillations correspond to a zero-momentum condensate whose initial number density, $n_\phi \sim m_0 \phi_1^2/2$ (where $\phi_1 = \sqrt{\langle \phi(T_1)^2 \rangle}$), will evolve as the typical one associated with standard nonrelativistic matter.

Taking into account that the number density of scalar particles scales as the entropy density of radiation ($s = 2\pi^2 g_{s1} T_1^3/45$) in an adiabatic expansion, we can write:

$$\Omega_\phi h^2 \simeq \frac{(n_\phi/s)(s_0/\gamma_{s1})}{\rho_{\rm crit}} m_0, \tag{5}$$

where $h \simeq 0.70$ is the Hubble parameter, $\rho_{\rm crit} \simeq 1.054 \times 10^4$ eV cm^{-3} is the critical density, $s_0 \simeq 2970$ cm^{-3} is the present entropy density of the radiation, and γ_{s1} is the factor that this entropy has increased in a comoving volume since the onset of scalar oscillations.

If we supposed a radiation dominated universe at T_1 ($3H_1 = \pi(g_{e1}/10)^{1/2} T_1^2/M_{\rm Pl}$), we can estimate T_1 by solving $m_0 = 3H_1(T_1)$:

$$T_1 \simeq 15.5 \text{ TeV} \left[\frac{m_0}{1 \text{ eV}}\right]^{1/2} \left[\frac{100}{g_{e1}}\right]^{1/4}, \tag{6}$$

and calculate the abundance as:

$$\Omega_\phi h^2 \simeq 0.86 \left[\frac{m_0}{1 \text{ eV}}\right]^{1/2} \left[\frac{\phi_1}{10^{12} \text{ GeV}}\right]^2 \left[\frac{100 g_{e1}^3}{(\gamma_{s1} g_{s1})^4}\right]^{1/4}, \tag{7}$$

where g_{e1} (g_{s1}) are the effective energy (entropy) number of relativistic degrees of freedom at T_1. We see that initial conditions of order of $\phi_1 \sim 10^{12}$ GeV can lead to the nonbaryonic DM (NBDM) abundance depending on the rest of the parameters and the early physics of the Universe (see Fig. 2). We can check that this result is consistent with a perturbative treatment of the background metric: $\|\Delta g_{\mu\nu}/\hat{g}_{\mu\nu}\| \lesssim 10^{-6}$, for the entire computation. In particular, we can check that the mass of the scalar field is approximately constant as we have assumed. In general, the mass of this scalar mode depends on the environment [10] (and Λ if it is not negligible). However, the above inequality is equivalent to $\phi/M_{\rm Pl} \lesssim 10^{-6}$ (or $R/m_0^2 \lesssim 10^{-6}$ in the Jordan frame). Therefore, it is evident that the scalar mode is minimally displaced from the bottom of its potential, where ordinary EEs are recovered and the mass of the scalar field is constant and equal to m_0.

On the other hand, Eq. (3) implies that the new scalar graviton mediates an attractive Yukawa force between two nonrelativistic particles of masses M_a and M_b:

$$V_{ab} = -\alpha \frac{1}{8\pi M_{\rm Pl}^2} \frac{M_a M_b}{r} e^{-m_0 r}, \tag{8}$$

with $\alpha = 1/3$ [5]. The nonobservation of such a force by torsion-balance experiments requires [14]:

$$m_0 \gtrsim 2.7 \times 10^{-3}\ \text{eV} \quad \text{at 95\%C.L.} \tag{9}$$

This is the most constraining lower bound on the mass of the scalar mode and it is independent of its misalignment or any other supposition about its abundance.

On the contrary, depending on its abundance, m_0 is constrained from above. The decay in e^+e^- is particularly interesting since it is the most constraining if ϕ constitutes the total NBDM. From (3), it is possible to calculate the ϕ decay rate into a generic pair fermion antifermion [15]. In particular, for the e^+e^- decay:

$$\Gamma_{\phi\to e^+e^-} \simeq \left[2.14 \times 10^{24} s \frac{r_e^2}{(r_e^2 - 1)^{3/2}}\right]^{-1}, \tag{10}$$

where $r_e = m_0/(2m_e)$. Restrictions are set by the observations of the SPI spectrometer on the INTEGRAL (International Gamma-ray Astrophysics Laboratory) satellite, which has measured a 511 keV line emission of $1.05 \pm 0.06 \times 10^{-3}$ photons $\text{cm}^{-2}\,\text{s}^{-1}$ from the Galactic center (GC) [16], confirming previous measurements. This 511 keV line flux is fully consistent with an e^+e^- annihilation spectrum although the source of positrons is unknown.

If $m_0 \gtrsim 1.2$ MeV, the scalar mode cannot constitute the total local DM since we should observe a bigger excess of the 511 line coming from the GC. On the other hand, decaying DM (DDM) has been already proposed in different works as a possible source of the inferred positrons if its mass is lighter than $M_{\rm DDM} \lesssim 10$ MeV [17] and its decay rate in e^+e^- verifies [18,19]:

$$\frac{\Omega_{\rm DDM} h^2 \Gamma_{\rm DDM}}{M_{\rm DDM}} \simeq [(0.2 - 4) \times 10^{27}\ \text{s MeV}]^{-1}. \tag{11}$$

The most important uncertainty for this interval comes from the dark halo profile, although a cuspy density is definitely needed (with a inner slope $\gamma \gtrsim 1.5$ [19]). If m_0 is tuned to $2m_e$ with an accuracy of 5%–10%, the line could be explained by R^2 gravity (see Fig. 1). The same gravitational DM can explain the 511 line with a less tuned mass (up to $m_0 \sim 10$ MeV) if $\phi_1 \sim 10^9$ GeV, i.e., with a lower abundance (Fig. 2). If $m_0 \gtrsim 10$ MeV, the gamma-ray spectrum that originated by in-flight annihilation of the positrons with interstellar electrons is even more constraining than the 511 keV photons [17].

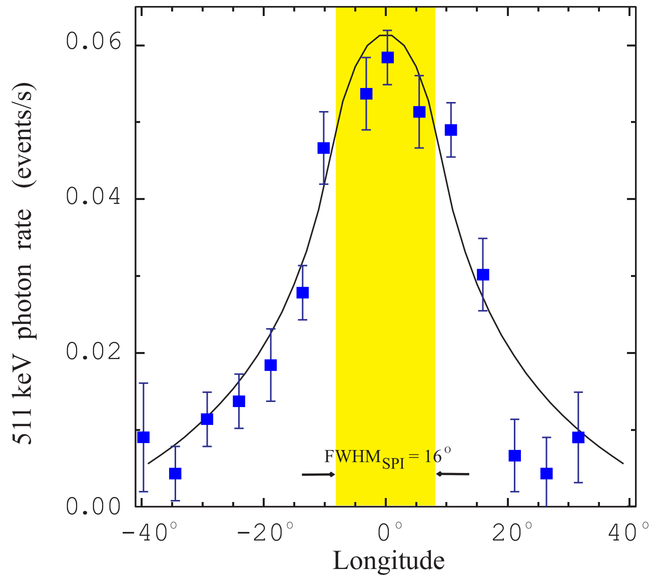

FIG. 1 (color online). The 511 keV line emission signal from R^2 gravity with $\Omega_\phi h^2 \simeq 0.11$ and $m_0 \simeq 1.15$ MeV, i.e. $\Gamma_{\phi\to e^+e^-} \simeq (2 \times 10^{26}\ \text{s})^{-1}$. We have taken into account the 16° field of view of the spectrometer, subtracted the flux at higher longitudes ($l \gtrsim 45°$), and assumed a positronium fraction of $p = 0.94$. We have used the dark matter halo profile described in [19] and the data from [25] (the analysis is described in more detail in [15]).

On the contrary, if $m_0 < 2m_e$, the only decay channel that may be observable is in two photons. We find [15]:

$$\Gamma_{\phi\to\gamma\gamma} \simeq \left|\frac{3c_e}{11}\right|^2 \left[2.5 \times 10^{29} s \left(\frac{1\ \text{MeV}}{m_0}\right)^3\right]^{-1}. \tag{12}$$

In particular, taking into account all SM charged particles and ϕ much lighter than all of them: $c_e = 11/3$. If $m_0 \lesssim 1$ MeV, it is difficult to detect these gravitational decays in the isotropic diffuse photon background (iDPB) [19,20]. The gamma-ray spectrum at high galactic latitudes can have contributions from galactic and extragalactic sources, but it seems well fitted at $E_\gamma \lesssim 1$ MeV by assuming active galactic nuclei (AGN) as main sources. The spectrum observed by COMPTEL (Compton Imaging Telescope) [21], SMM (Solar Maximum Mission) [22], and INTEGRAL [23], falls like a power law, with $dN/dE \sim E^{-2.4}$ [21], and dominates any possible signal from R^2 gravity if $m_0 \lesssim 1$ MeV.

However, a most promising analysis is associated with the search of gamma-ray lines at $E_\gamma = m_0/2$ from localized sources, as the GC. The iDPB is continuum since it suffers the cosmological redshift. But the monoenergetic photons originated by local sources may give a clear signal of R^2 gravity. INTEGRAL has performed a search for gamma-ray lines that originated within 13° from the GC over the energy ranges 0.08–8 MeV. It has not observed any

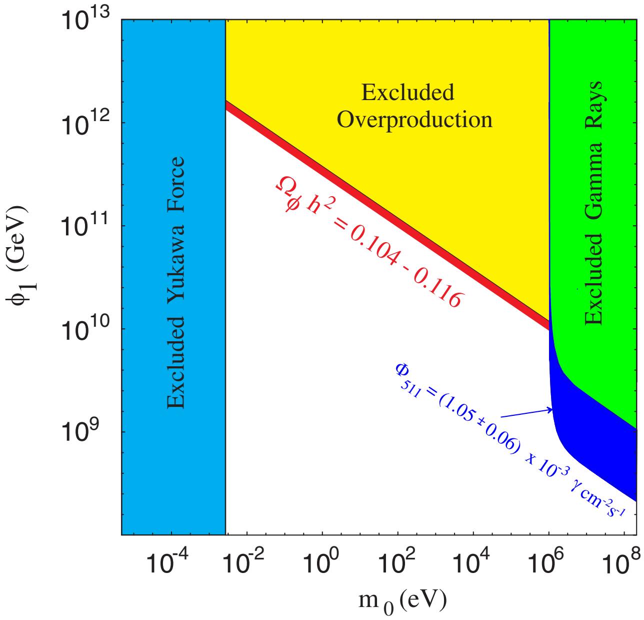

FIG. 2 (color online). Parameter space: m_0 is the mass of the new scalar mode and ϕ_1 is its misalignment when $3H \sim m_0$ (we assume $g_{e1} = g_{s1} \simeq 106.75$, and $\gamma_{s1} \simeq 1$). The left side is excluded by modifications of Newton's law. The right one is excluded by cosmic ray observations. In the limit of this region, R^2 gravity can account for the positron production in order to explain the 511 keV line coming from the GC confirmed by INTEGRAL [16] (up to $m_0 \sim 10$ MeV). The upper area is ruled out by DM overproduction. The diagonal line corresponds to the NBDM abundance fitted with WMAP data [26].

line below 511 keV up to upper flux limits of 10^{-5}–10^{-2} $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, depending on line width, energy, and exposure [24]. Unfortunately, these flux limits are, at least, 1 order of magnitude over the expected fluxes from ϕ decays with $m_0 \lesssim 1$ MeV, even for cuspy halos. The photon flux originated by R^2 gravity depends on m_0 as $\Phi_{E_\gamma = m_0/2} \propto m_0^2$. This strong dependence implies that only the heavier allowed region could be detected with reasonable improvements of present experiments [20].

In conclusion, we have studied the possibility that the DM origin resides in UV modifications of gravity. Although our results may seem particular of R^2 gravity, the low energy phenomenology of the studied scalar mode is ubiquitous in high energy corrections of the EHA coming from string theory, supersymmetry or extra dimensions; since all these models predict the existence of light scalars, that can be identified as pseudogoldstone bosons associated to the breaking of scale invariance. The instability of the deduced DM predicts a deviation from EEs at a density energy scale: 1 TeV $\lesssim \Lambda_G \lesssim 10^5$ TeV. Consequences for hierarchy interpretations, baryogenesis or inflation deserve further investigations.

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